

Forward and Inverse Modeling of EEG and MEG data

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Overview

Motivation and background

Forward modeling

- Source model

- Volume conductor model

Inverse modeling - biophysical models

- Single and multiple dipole fitting

- Distributed source models

- Beamforming methods

Inverse modeling - independent components

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Motivation 1

Strong points of EEG and MEG

Temporal resolution (~ 1 ms)

Characterize individual components of ERP

Oscillatory activity

Disentangle dynamics of cortical networks

Weak points of EEG and MEG

Measurement on outside of brain

Overlap of components

Low spatial resolution

Motivation 2

If you find a ERP/ERF component, you want to characterize it in physiological terms

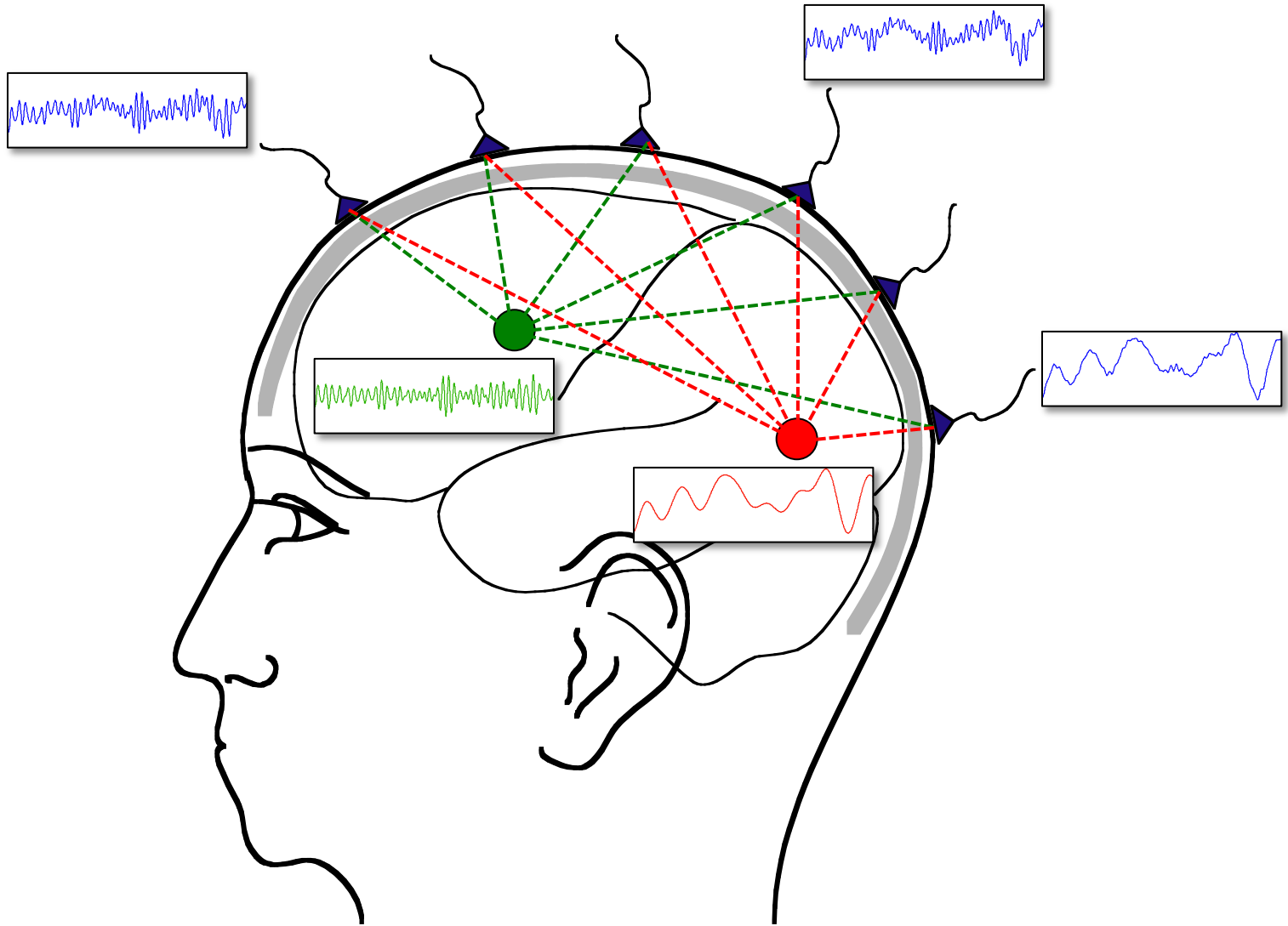
Time or frequency are the “natural” characteristics

“Location” requires interpretation of the scalp topography

Forward and inverse modeling helps to interpret the topography

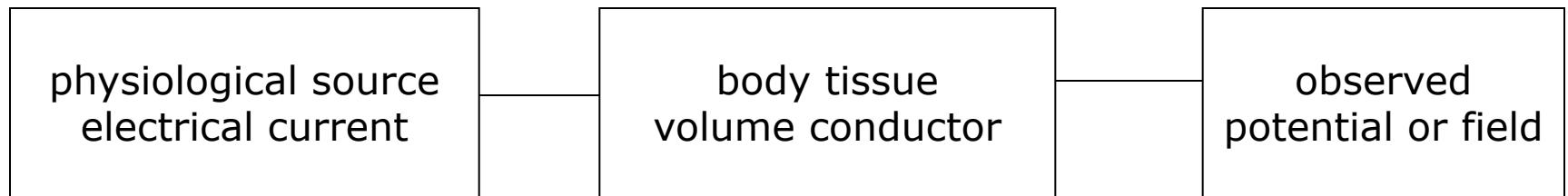
Forward and inverse modeling helps to disentangle overlapping source timeseries

Superposition of source activity



Biophysical source modelling: overview

forward model



inverse model

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Single and multiple dipole fitting

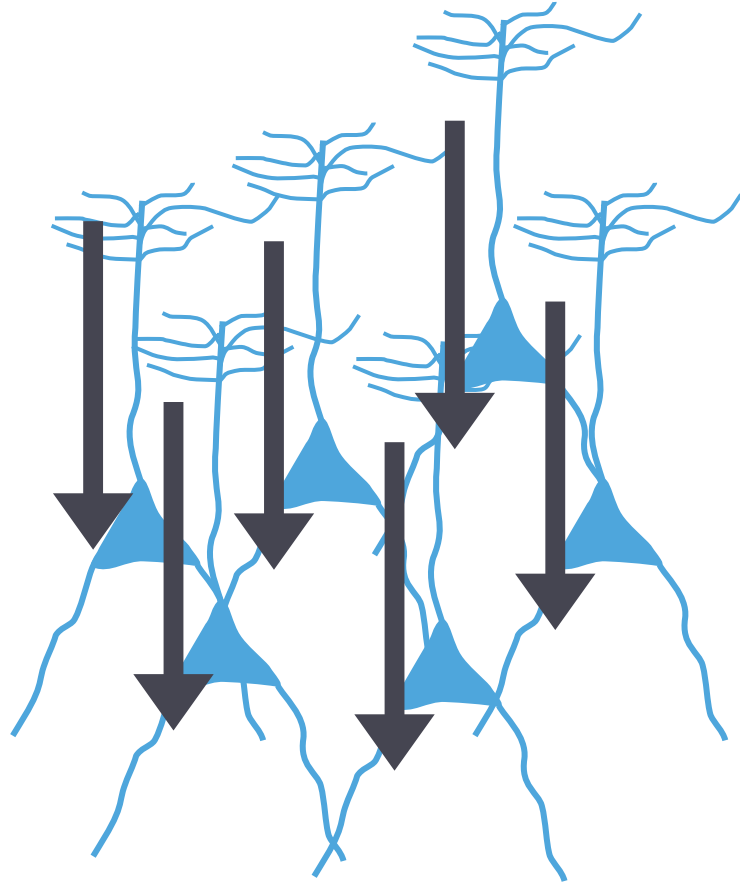
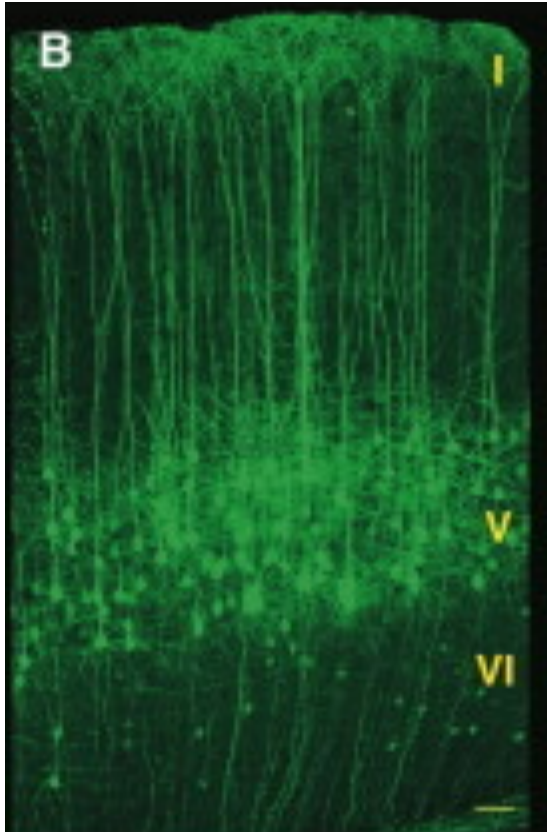
Distributed source models

Beamforming methods

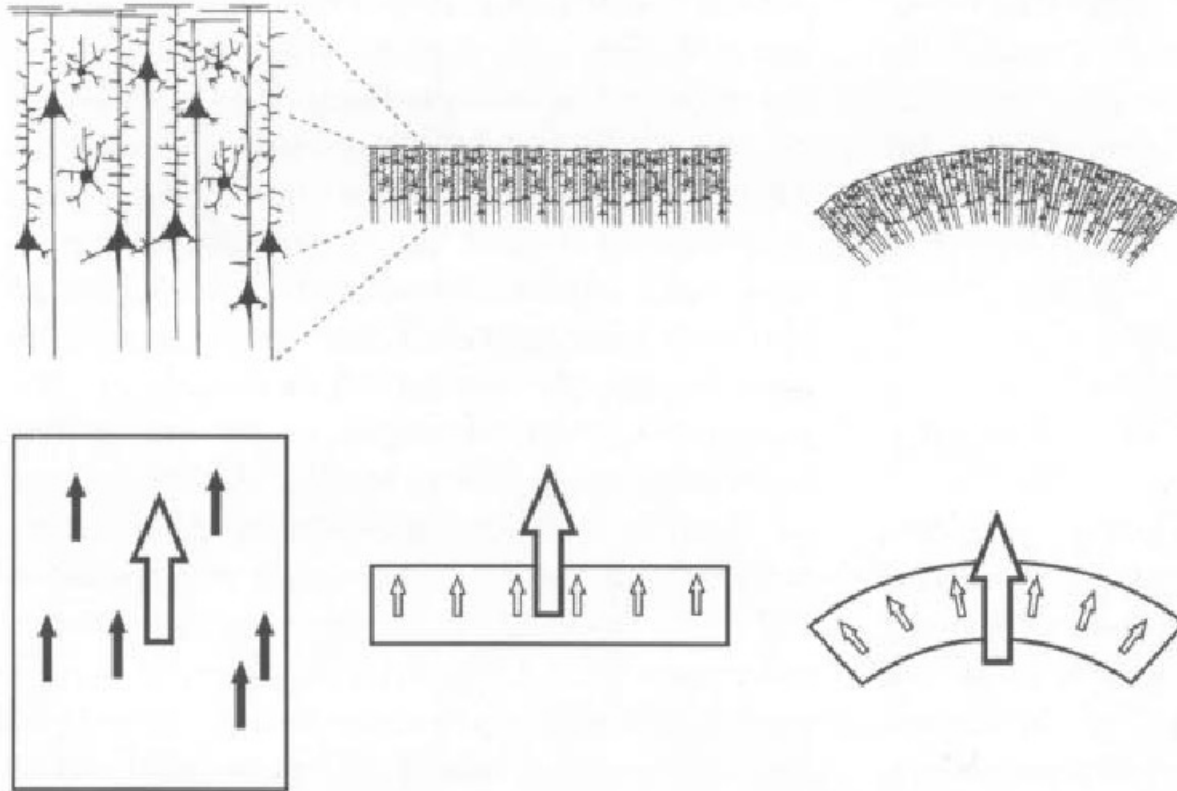
Inverse modeling - independent components

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What produces the electric current



Equivalent current dipoles



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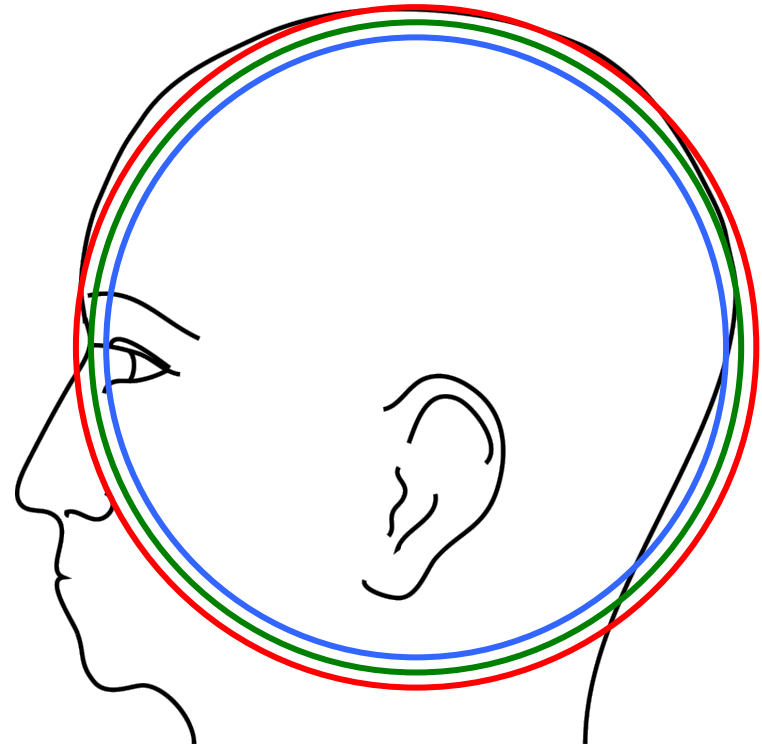
Volume conductor

described electrical properties of tissue

describes geometrical model of the head

describes **how** the currents flow, not where they originate from

same volume conductor for EEG as for MEG, but also for tDCS, tACS, TMS, ...



Volume conductor

Computational methods for volume conduction problem that allow for realistic geometries

BEM *Boundary Element Method*

FEM *Finite Element Method*

FDM *Finite Difference Method*

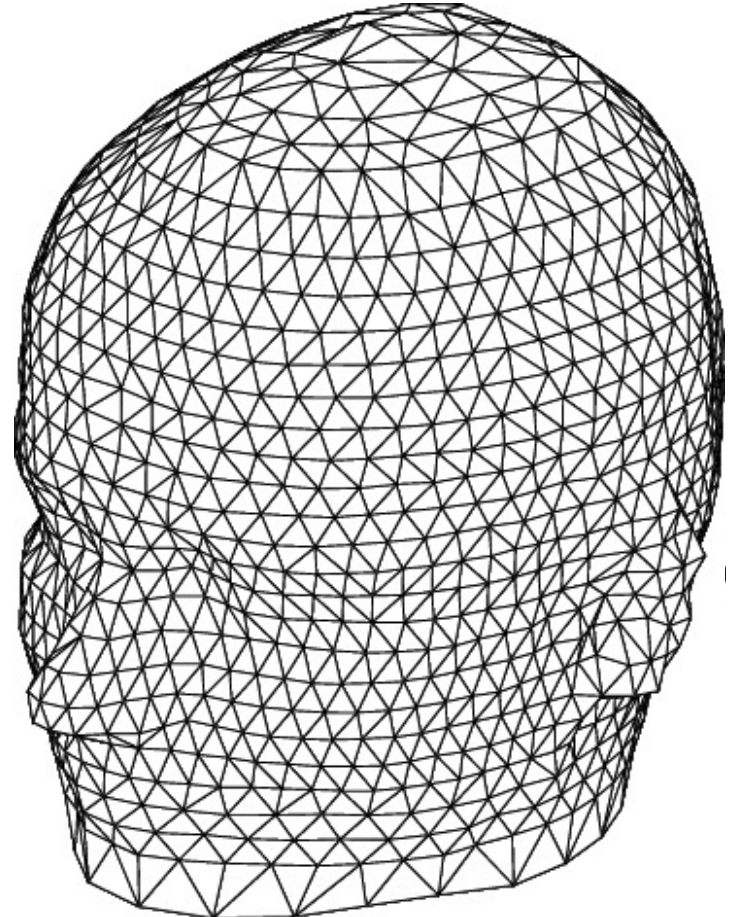
Volume conductor: Boundary Element Method

Each compartment is
homogenous
isotropic

Important tissues

skin
skull
brain
(CSF)

Triangulated surfaces
describe boundaries



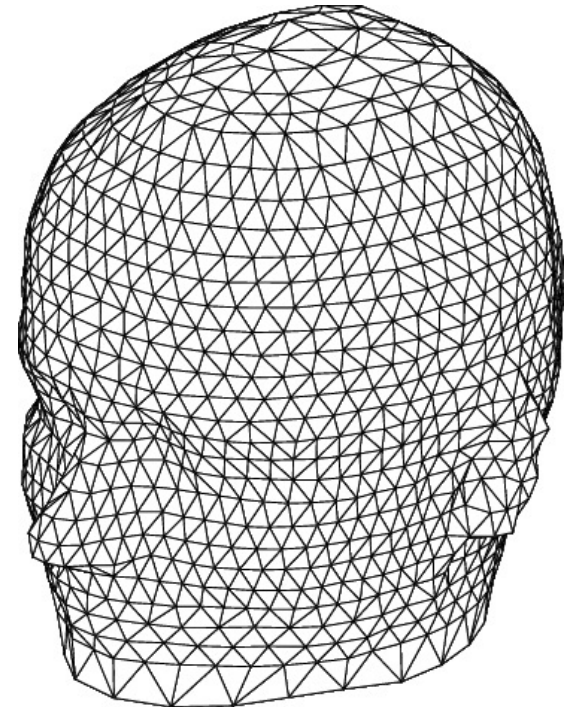
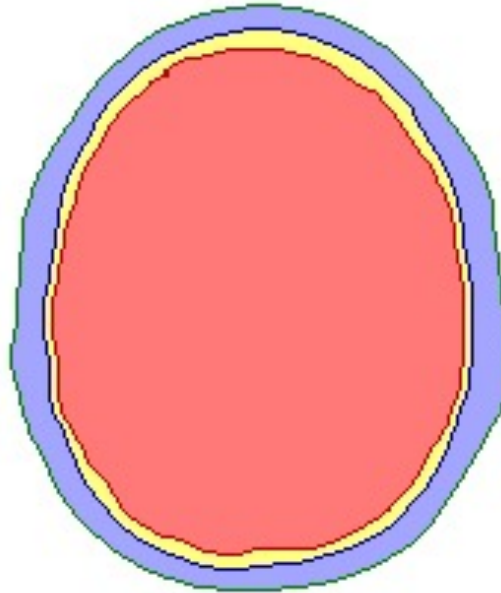
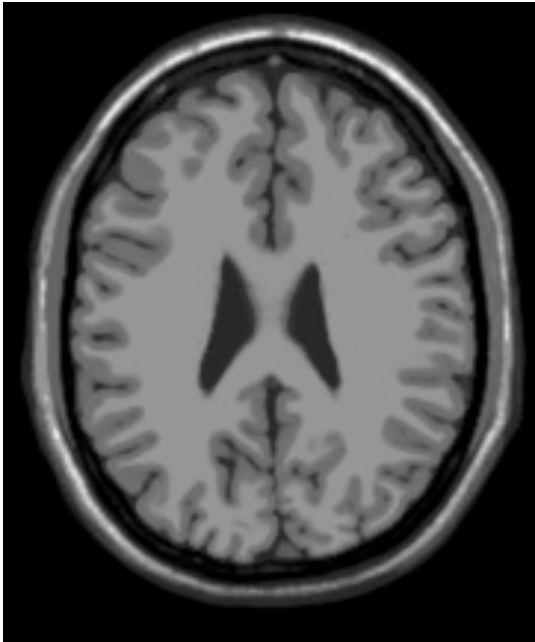
Volume conductor: Boundary Element Method

Construction of geometry

segmentation in different tissue types

extract surface description

downsample to reasonable number of triangles



Volume conductor: Boundary Element Method

Construction of geometry

- segmentation in different tissue types

- extract surface description

- downsample to reasonable number of triangles

Computation of model

- independent of source model

- only one lengthy computation

- fast during application to real data

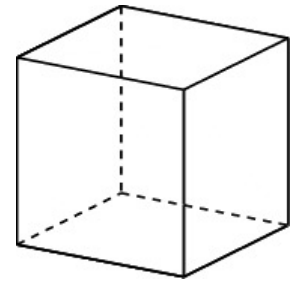
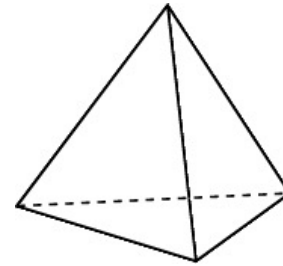
Can also include more complex geometrical details

- ventricles

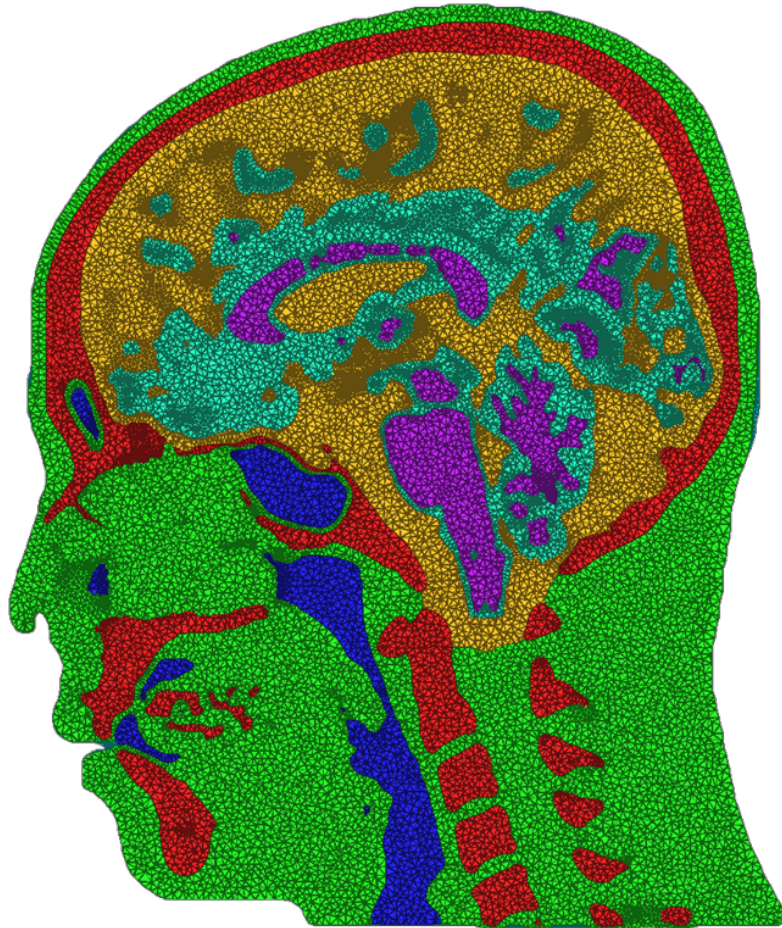
- holes in skull

Volume conductor: Finite Element Method

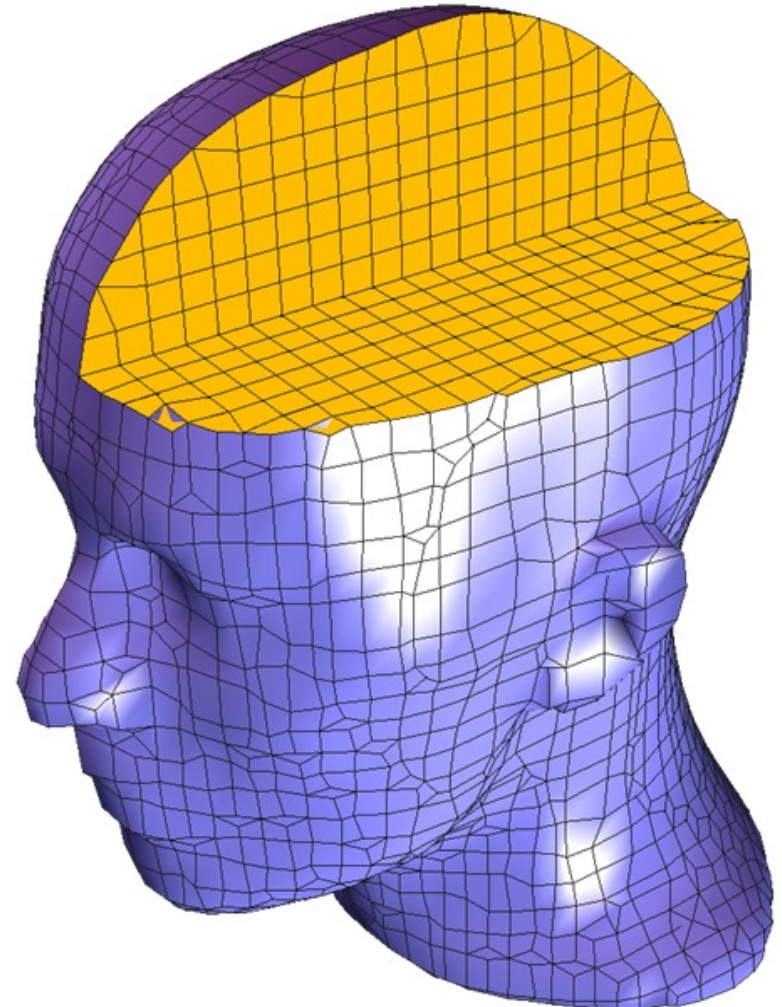
Tesselation of 3D volume in tetraeders or hexaheders



Volume conductor: Finite Element Method



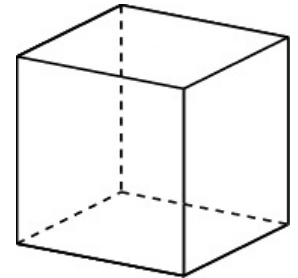
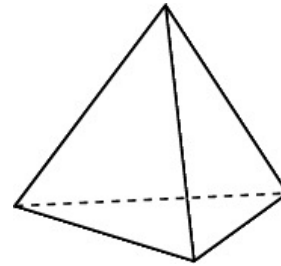
tetraeders



hexaheders

Volume conductor: Finite Element Method

Tesselation of 3D volume in tetraeders or hexaheders



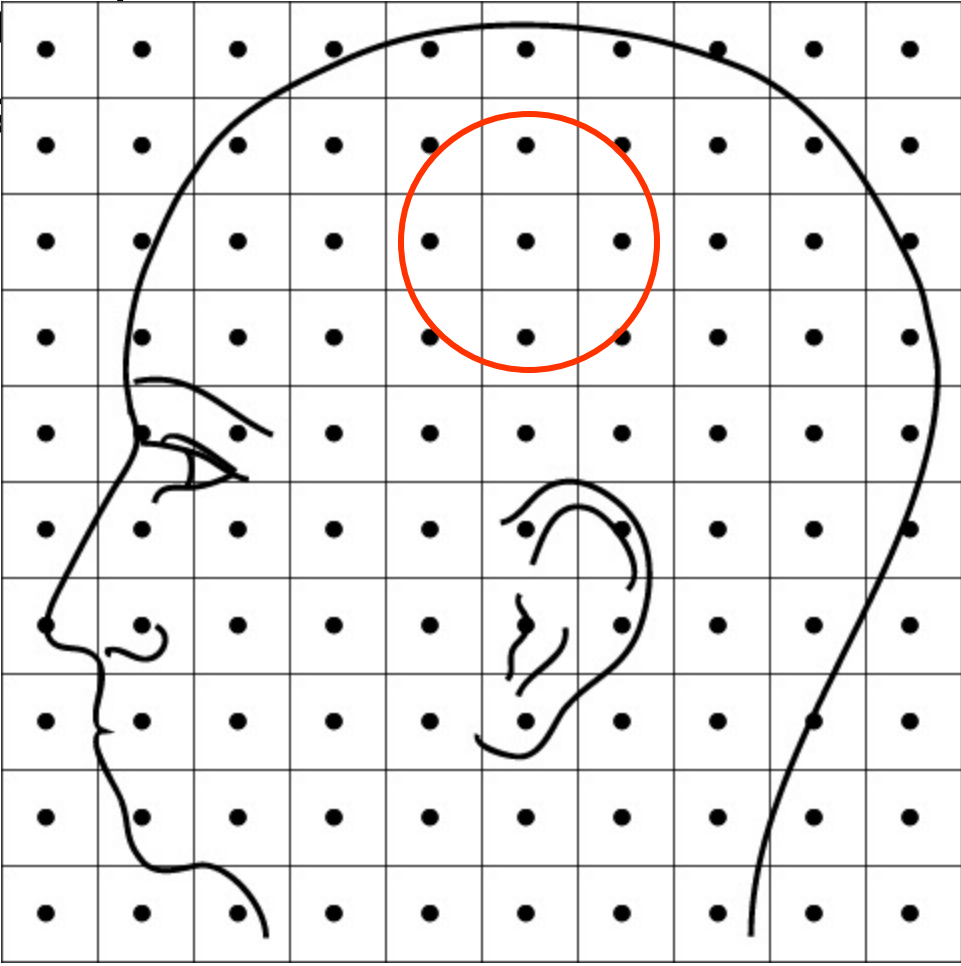
Each element can have its own conductivity

FEM is the most accurate numerical method but computationally quite expensive

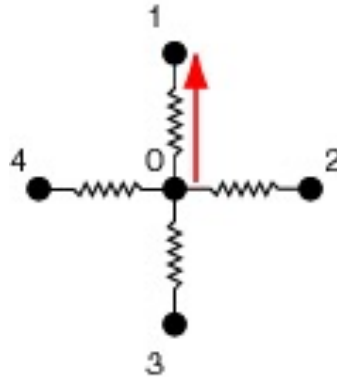
Geometrical processing not as simple as BEM

Volume conductor: Finite Difference Method

Easy to con
Not very us



Volume conductor: Finite Difference Method



$$\left. \begin{array}{l} I_1 + I_2 + I_3 + I_4 = 0 \\ V = I * R \end{array} \right\} \Rightarrow$$

$$\Delta V_1 / R_1 + \Delta V_2 / R_2 + \Delta V_3 / R_3 + \Delta V_4 / R_4 = 0 \quad \Rightarrow$$

$$(V_1 - V_0) / R_1 + (V_2 - V_0) / R_2 + (V_3 - V_0) / R_3 + (V_4 - V_0) / R_4 = 0$$

Volume conductor: Finite Difference Method

Unknown potential V_i at each node

approx. $100 \times 100 \times 100 = 1.000.000$ unknowns

Linear equation for each node

approx. $100 \times 100 \times 100 = 1.000.000$ linear equations

Add a source/sink

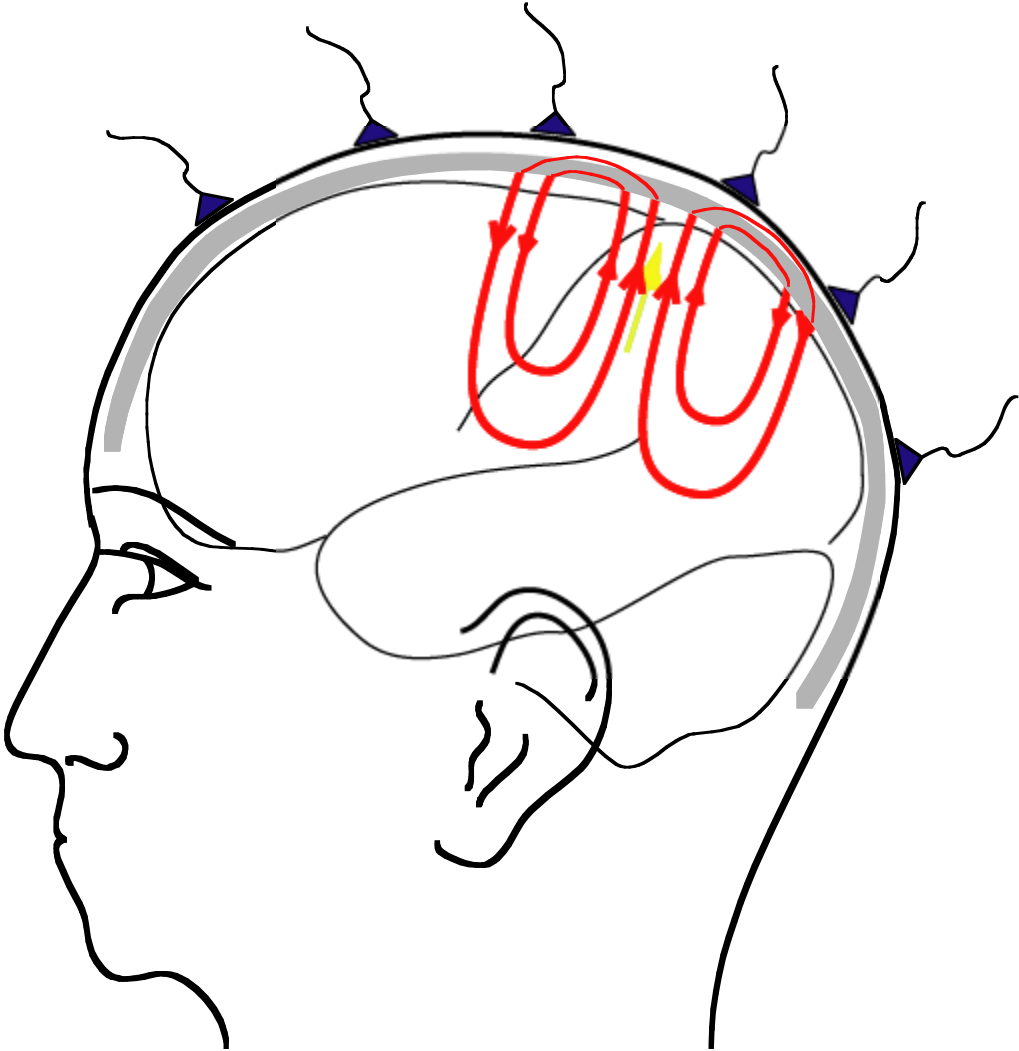
sum of currents is zero for all nodes, except

sum of current is I_+ for a certain node

sum of current is I_- for another node

Solve for unknown potential

EEG volume conduction



EEG volume conduction

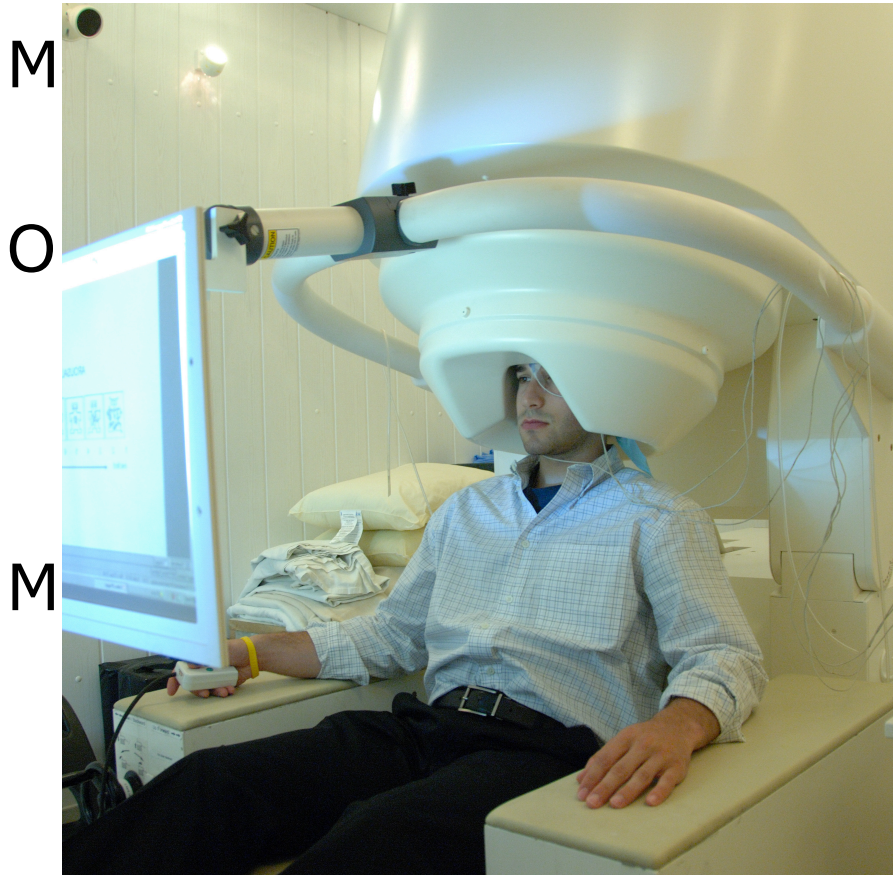
Potential difference between electrodes
corresponds to current flowing through skin

Only tiny fraction of current passes through skull

Therefore the model should describe the skull and
skin **as accurately as possible**

MEG volume conduction

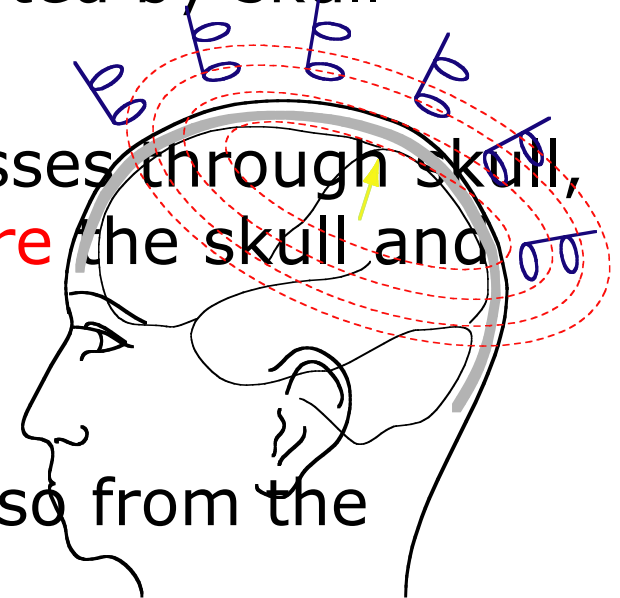
MEG measures magnetic field over the scalp



distorted by skull

passes through skull,
ignore the skull and

but also from the



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- Single and multiple dipole fitting

- Distributed source models

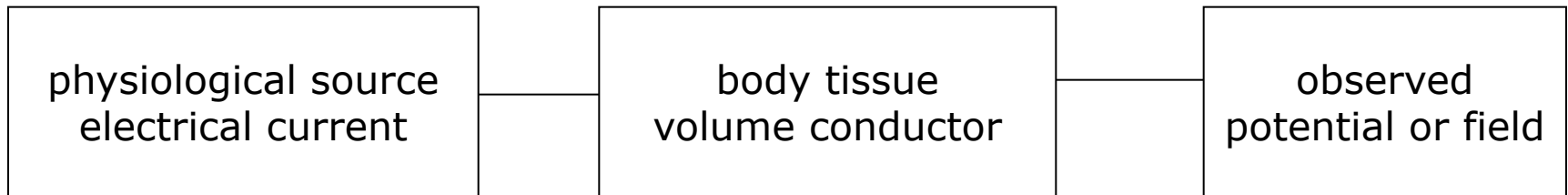
- Beamforming methods

Inverse modeling - independent components

Summary

Biophysical source modelling: overview

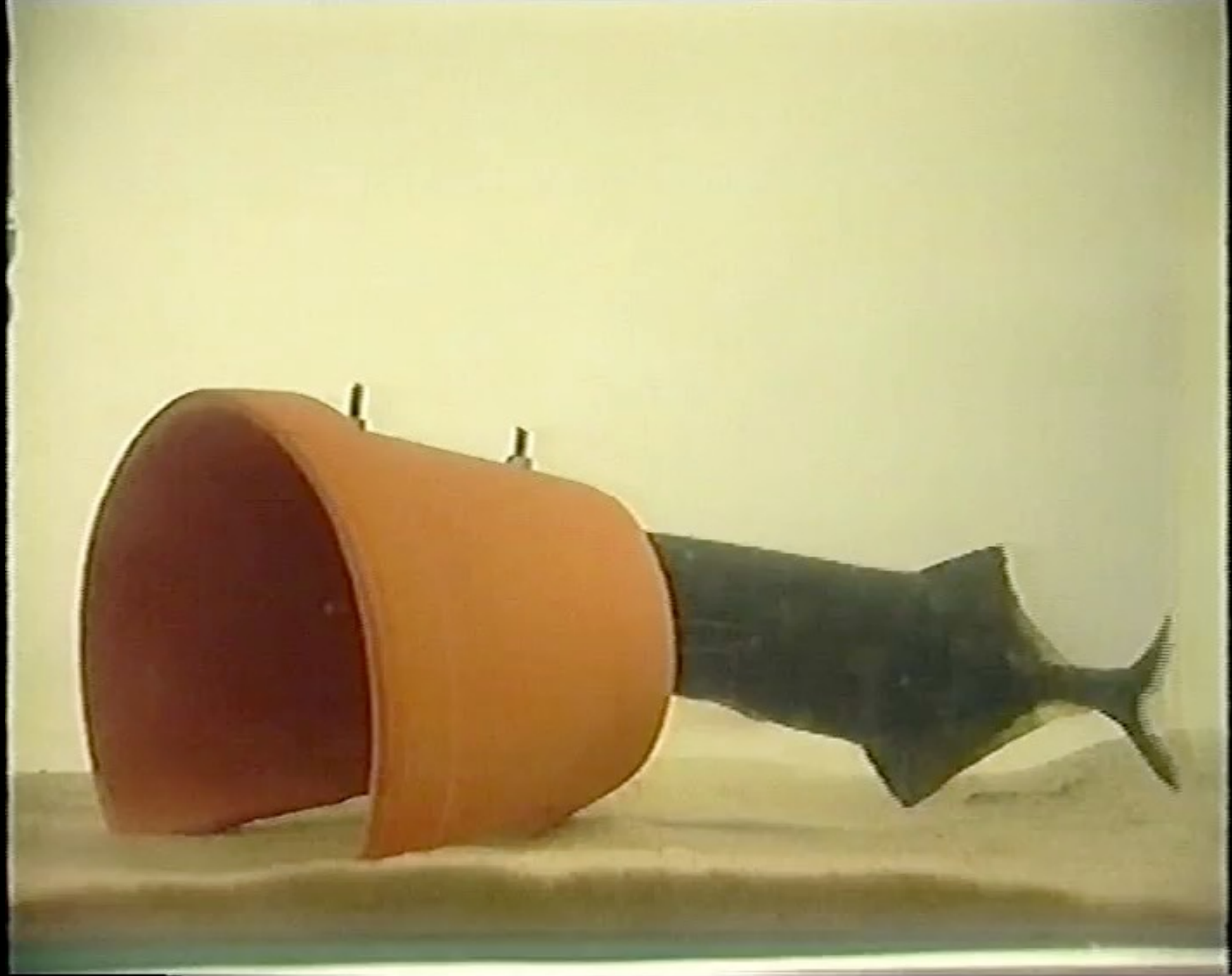
forward model



inverse model



Inverse localization: demo



Inverse methods

Single and multiple dipole models

Minimize error between model and measured potential/field

Distributed source models

Perfect fit of model to the measured potential/field

Additional constraint on source smoothness, power or amplitude

Spatial filtering

Scan the whole brain with a single dipole and compute the filter output at every location

Beamforming (e.g. LCMV, SAM, DICS)

Multiple Signal Classification (MUSIC)

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- Single and multiple dipole fitting**

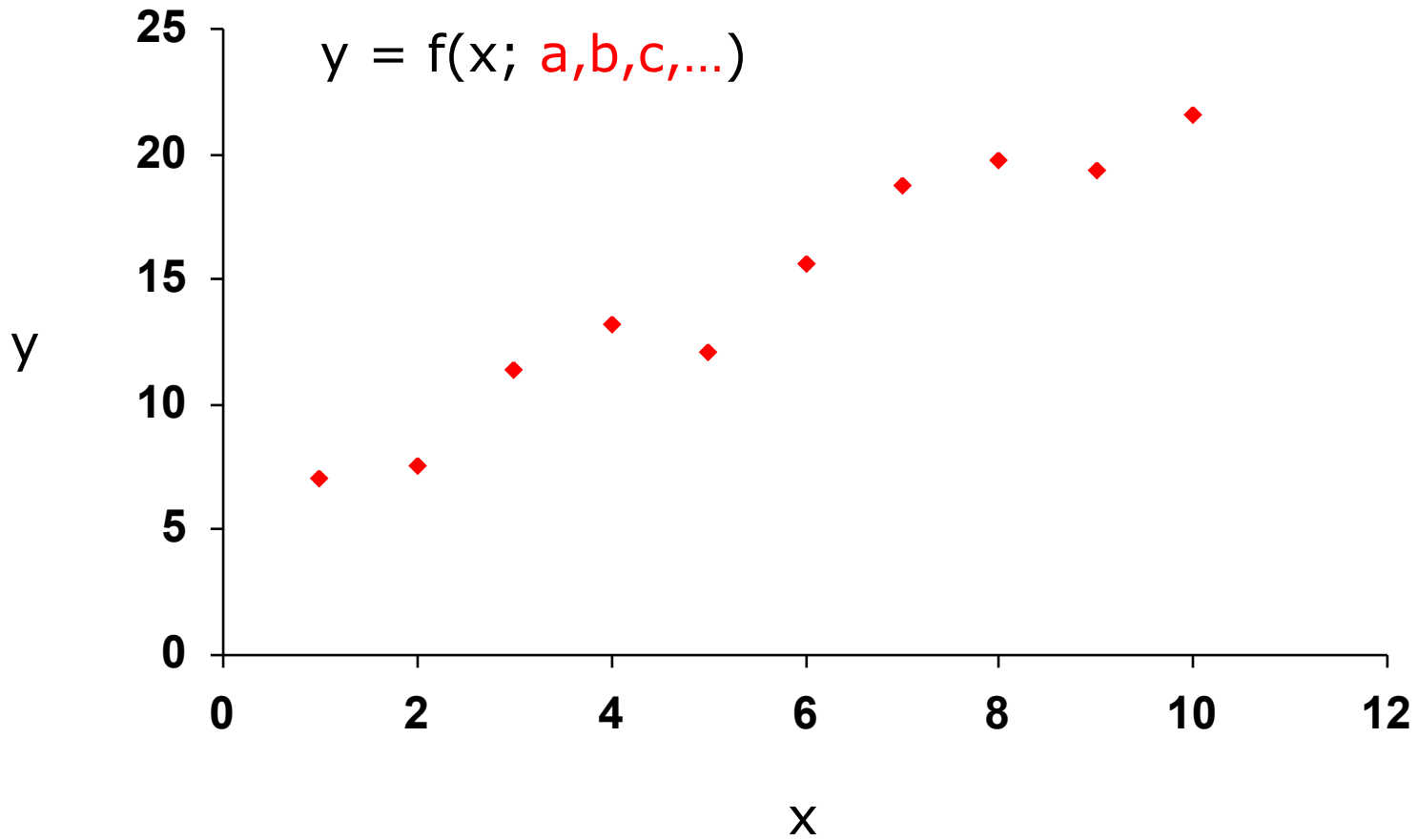
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Summary

Single or multiple dipole models - Parameter estimation



Parameter estimation: dipole parameters

source model with
few parameters

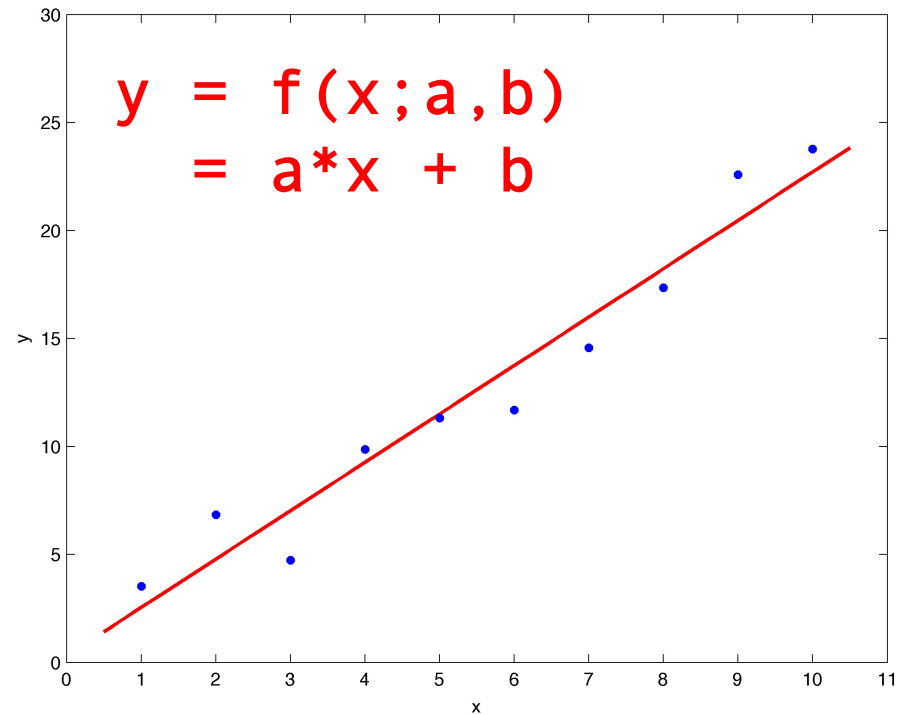
position

orientation

strength

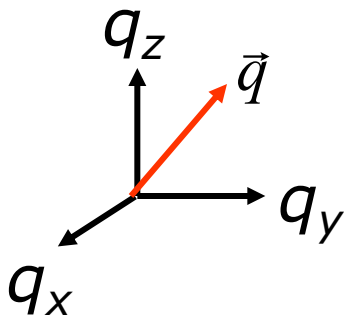
compute the model
data

minimize difference
between actual and
model data



Linear parameters: estimation

$$Y = G_x q_x + G_y q_y + G_z q_z = \begin{bmatrix} G_{x,1} & G_{y,1} & G_{z,1} \\ G_{x,2} & G_{y,2} & G_{z,2} \\ \vdots & \vdots & \vdots \\ G_{x,N} & G_{y,N} & G_{z,N} \end{bmatrix} \cdot \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} = \mathbf{G} \cdot \vec{q}$$



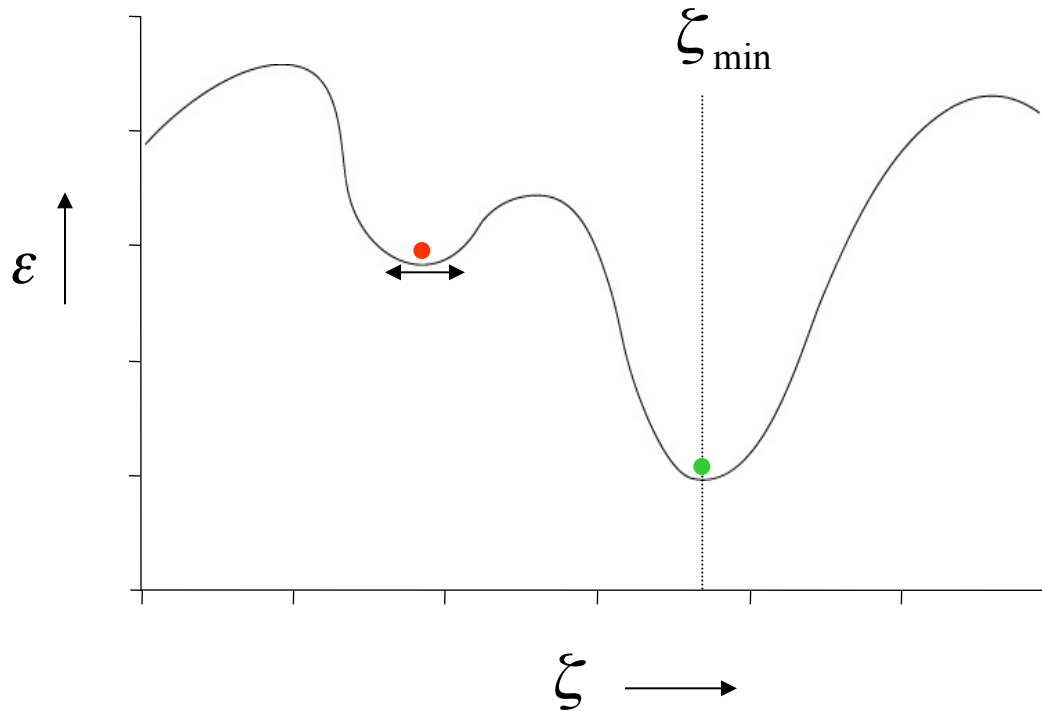
$$Y = \mathbf{G} \cdot \vec{q}$$
$$= \mathbf{G}(\zeta) \cdot \vec{q}$$

$$\vec{q} = \mathbf{G}^{-1} \cdot Y$$

Non-linear parameters

$$error(\zeta) = \sum_{i=1}^N (Y_i(\zeta) - V_i)^2 \Rightarrow \min_{\zeta} (error(\zeta))$$

$$\zeta = a, b, c, \dots$$



Non-linear parameters: grid search

One dimension, e.g. location along medial-lateral

100 possible locations

Two dimensions, e.g. med-lat + inf-sup

$100 \times 100 = 10.000$

Three dimensions

$100 \times 100 \times 100 = 1.000.000 = 10^6$

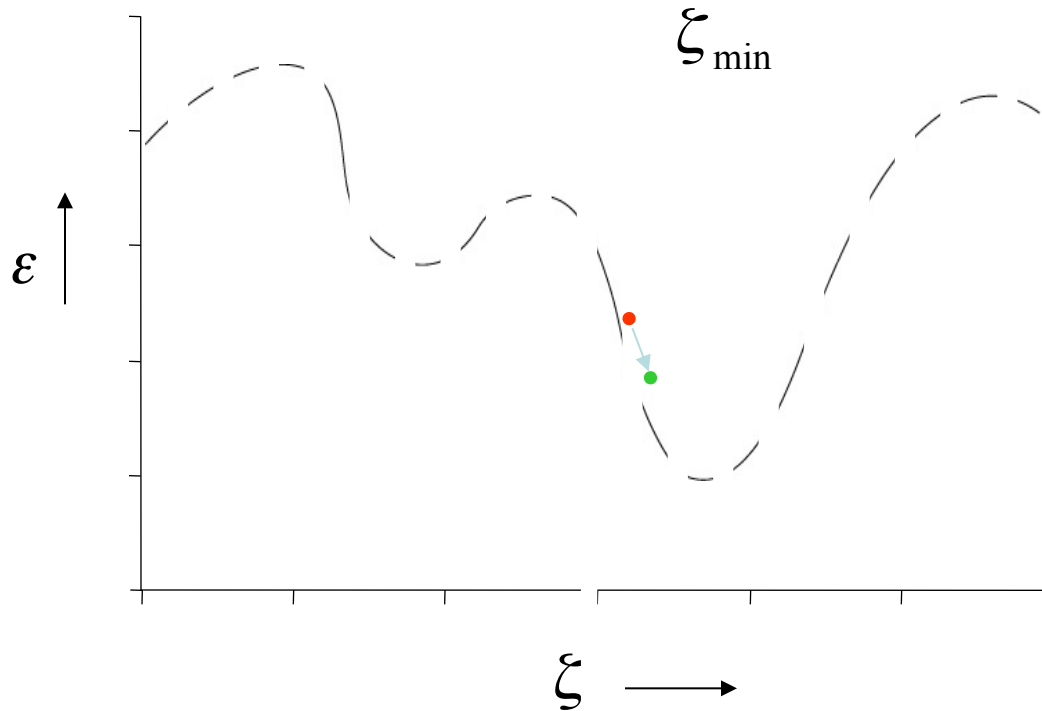
Two dipoles, each with three dimensions

$100 \times 100 \times 100 \times 100 \times 100 \times 100 = 10^{12}$

Non-linear parameters: gradient descent optimization

$$error(\zeta) = \sum_{i=1}^N (Y_i(\zeta) - V_i)^2 \Rightarrow \min_{\zeta} (error(\zeta))$$

$$\zeta = a, b, c, \dots$$



Single or multiple dipole models - Strategies

Single dipole:

scan the whole brain, followed by iterative optimization

Two dipoles:

scan with symmetric pair, use that as starting point for iterative optimization

More dipoles:

sequential dipole fitting

Sequential dipole fitting to explain spread of activity

Assume that activity starts “small”

explain earliest ERP component with single equivalent current dipole

Assume later activity to be more widespread

add ECDs to explain later ERP components

estimate position of new dipoles

re-estimate the activity of all dipoles

Iterative and interactive (hence subjective)
process

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Distributed source model

Position of the source is **not estimated** as such
Pre-defined grid (3D volume or on cortical sheet)

Strength is estimated

In principle easy to solve, however...

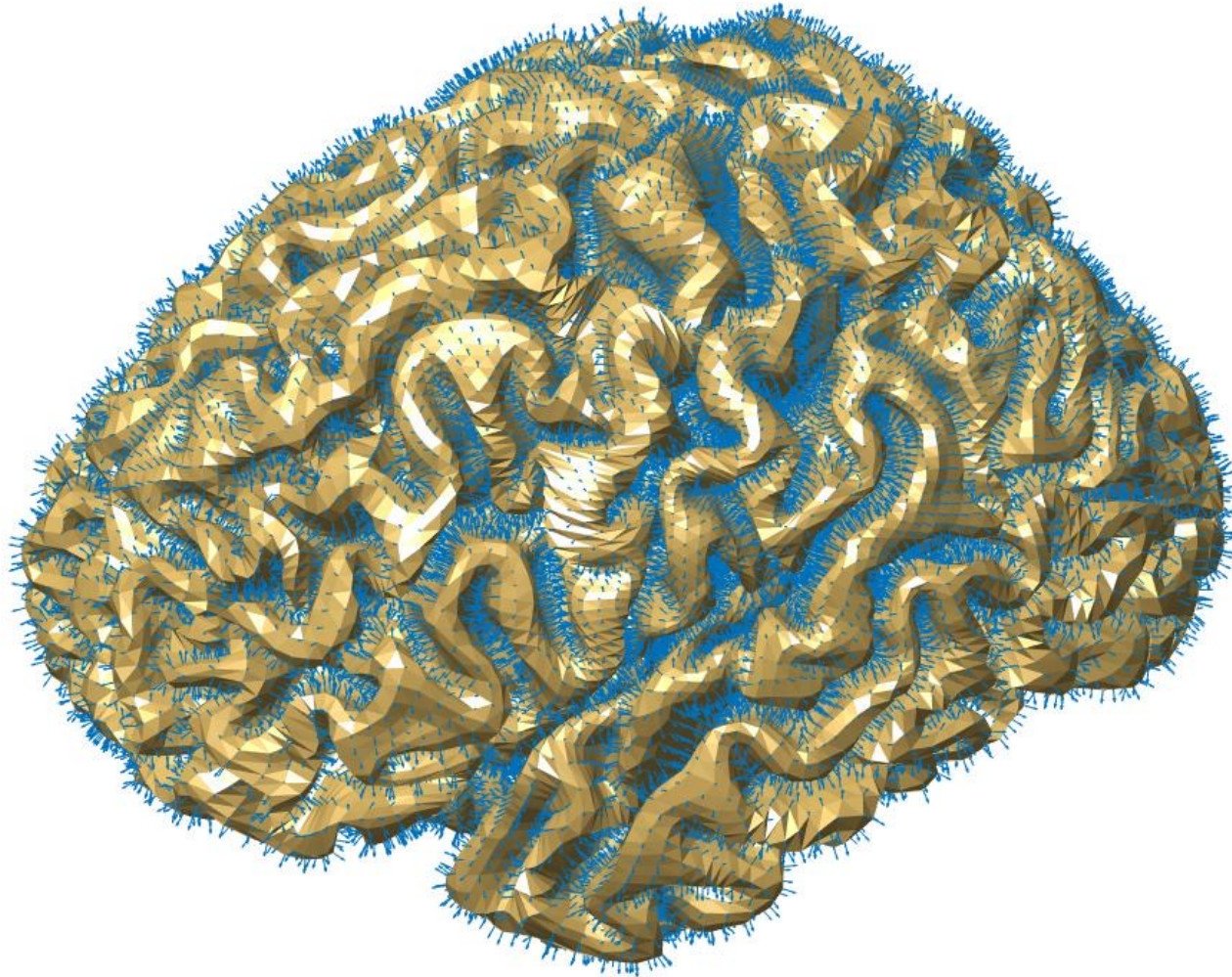
More “unknowns” than “knowns”

Infinite number of solutions can explain the data perfectly

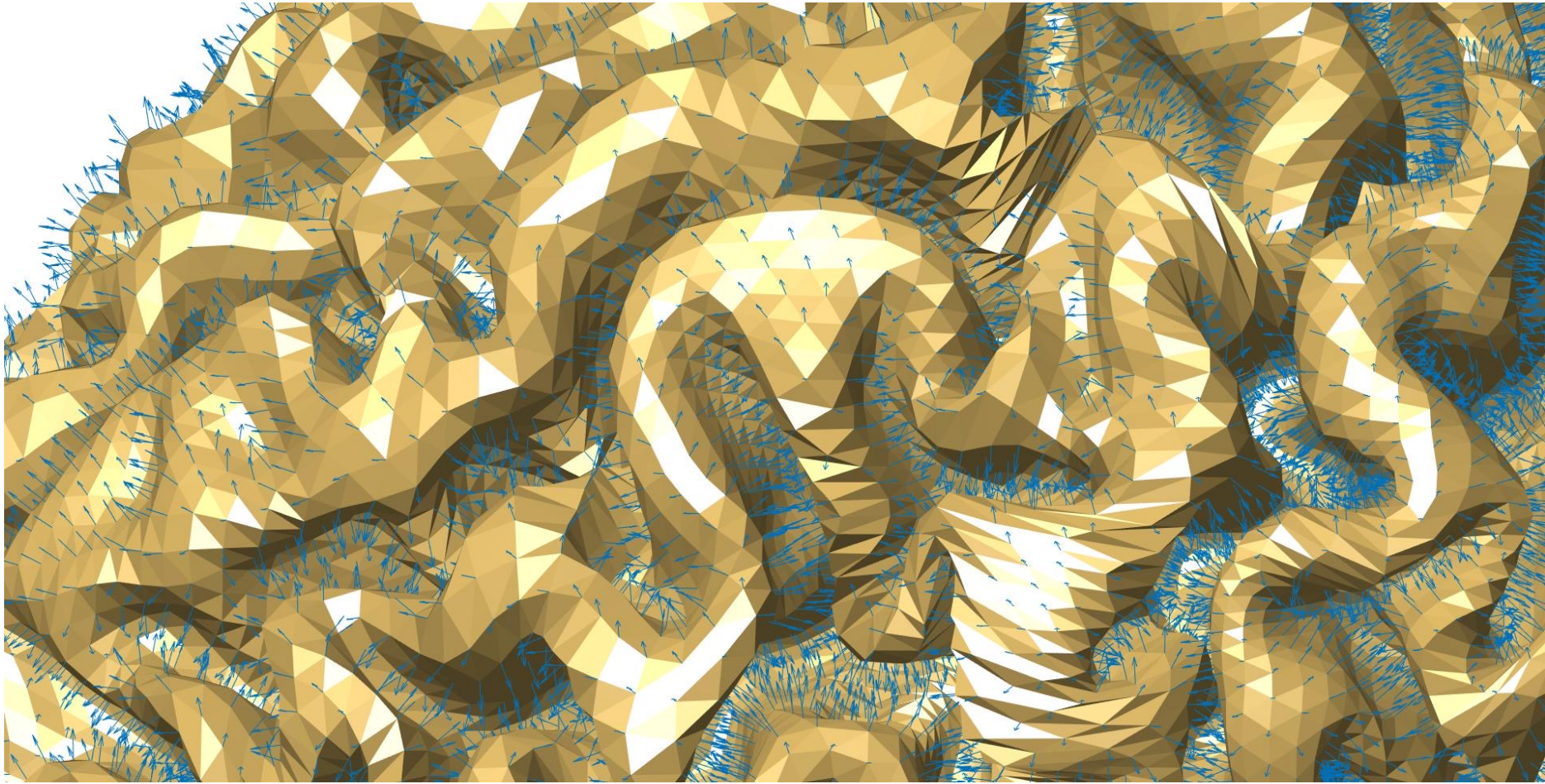
Additional constraints required

Linear estimation problem

Distributed source model



Distributed source model



Distributed source model: linear estimation

$$Y = G_1 q_1 + G_2 q_2 + \dots = \begin{bmatrix} G_{1,1} & G_{2,1} & \dots \\ G_{1,2} & G_{2,2} & \dots \\ \vdots & \vdots & \ddots \\ G_{1,N} & G_{2,N} & \dots \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ \vdots \end{bmatrix} = \mathbf{G} \cdot \vec{q}$$

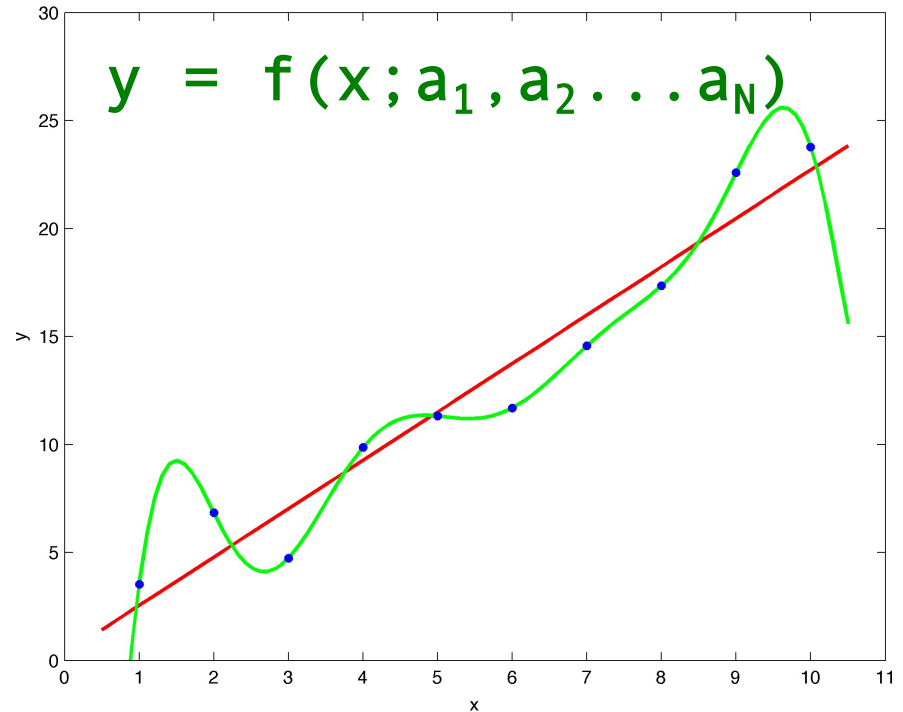
$$\vec{q} = \mathbf{G}^{-1} \cdot Y$$

Distributed source model: linear estimation

distributed source model
with **many dipoles**
throughout the whole
brain

estimate the strength of
all dipoles

data and noise can be
perfectly explained



Distributed source model: minimum-norm and regularization


$$V = G \cdot q + \text{Noise}$$

$$\min_q \{ \|V - G \cdot q\|^2 \} = 0 \quad !!$$

Regularized linear estimation:

$$\rightarrow \min_q \{ \|V - G \cdot q\|^2 + \lambda \cdot \|D \cdot q\|^2 \}$$


mismatch with data


mismatch with prior
assumptions

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Spatial filtering with beamforming

Position of the source is **not estimated** as such

Loop over a pre-defined grid

Manipulate filter properties, not source properties

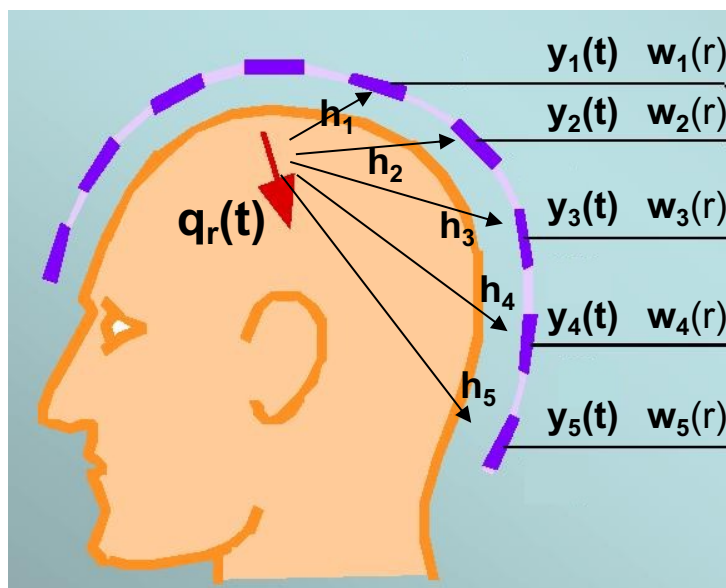
No explicit assumptions about source constraints
(implicit: single dipole)

Assumption that sources that contribute to the data
should be uncorrelated

Beamformer: the question

What is the activity of a source \mathbf{q} , at a location \mathbf{r} , given the data \mathbf{y} ?

We estimate \mathbf{q} with a spatial filter \mathbf{w}



$$\hat{q}_r(t) = \mathbf{w}(r)^T \mathbf{y}(t)$$

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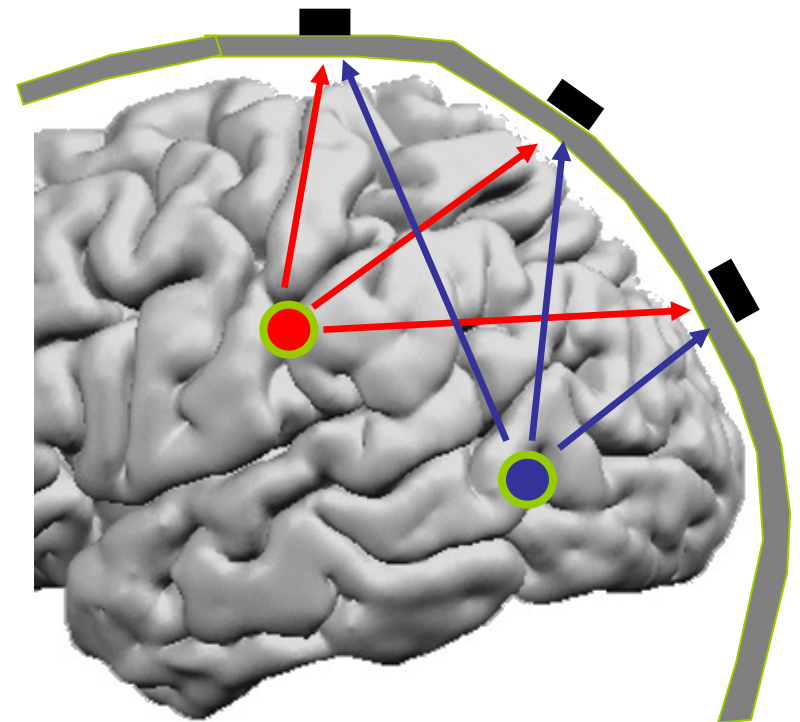
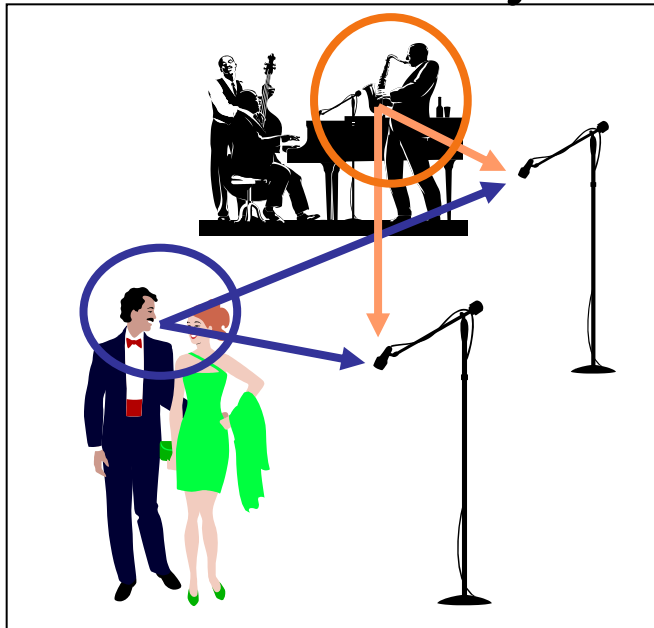
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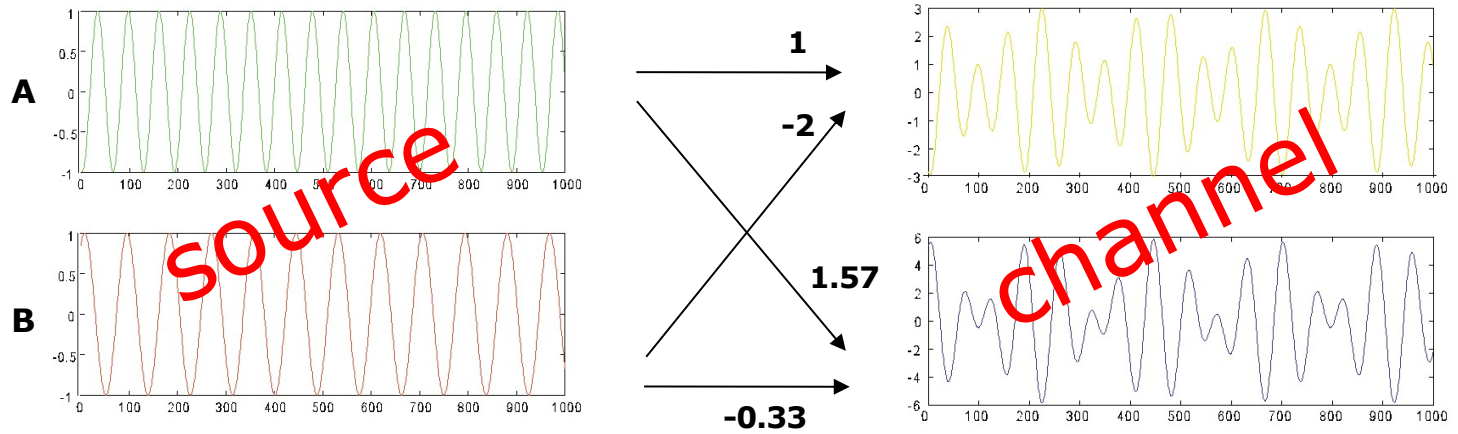
Independent component analysis

Mixture of Brain source activity

Cocktail Party



Independent component analysis



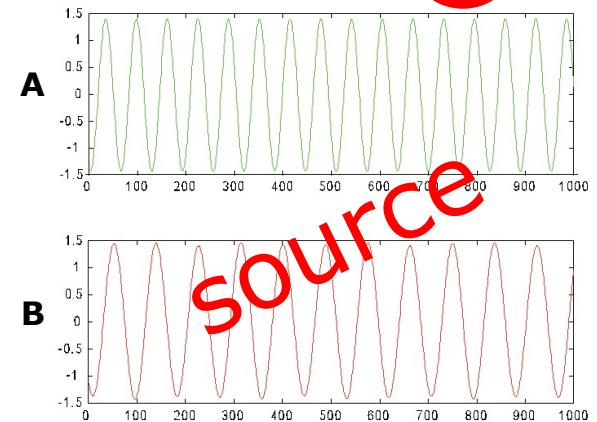
$$X=[A;B]$$

Linear Combination

$$Y=W^{-1}X$$

ICA

$$X=WY$$



Estimating source timecourse activity using independent component analysis

$$Y = G_1X_1 + G_2X_2 + \dots + G_nX_n + \text{noise}$$

n typically the same as the number of channels

$$Y = G (X + \text{noise})$$

*includes line-noise, EOG, ECG and other
noise that is visible on all channels*

$X' = W Y$, where W maximizes the independence of X'

rows of W^{-1} correspond to G_1, G_2, \dots

Source modelling of independent components

Components have (maximal) independent timecourses

Unmixing of timeseries has already been taken care of

Assumption: components correspond to compact spatial patches (or bilateral patches)

Use simple biophysical dipole models to model the spatial component topographies

It can be challenging to match ICA sources over subjects

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Summary 1

Forward modelling

Required for the interpretation of scalp topographies

Different methods with varying accuracy

Inverse modelling

Estimate source location and timecourse from data

Assumptions on source locations

Single or multiple point-like source

Distributed source

Assumptions on source timecourse

Uncorrelated (and dipolar)

Independent

Summary 2

Source analysis is not only about the “where”
but also about untangling the “what” and
“when”

timecourse of activity
-> ERP

spectral characteristics
-> power spectrum

temporal changes in power
-> time-frequency response (TFR)

spatial distribution of activity over the head
-> source reconstruction

