Connectivity analysis of electrophysiological data

Jan Mathijs Schoffelen

Donders Institute, Radboud University, Nijmegen, NL

M/EEG signal characteristics considered during analysis

timecourse of activity
-> ERP

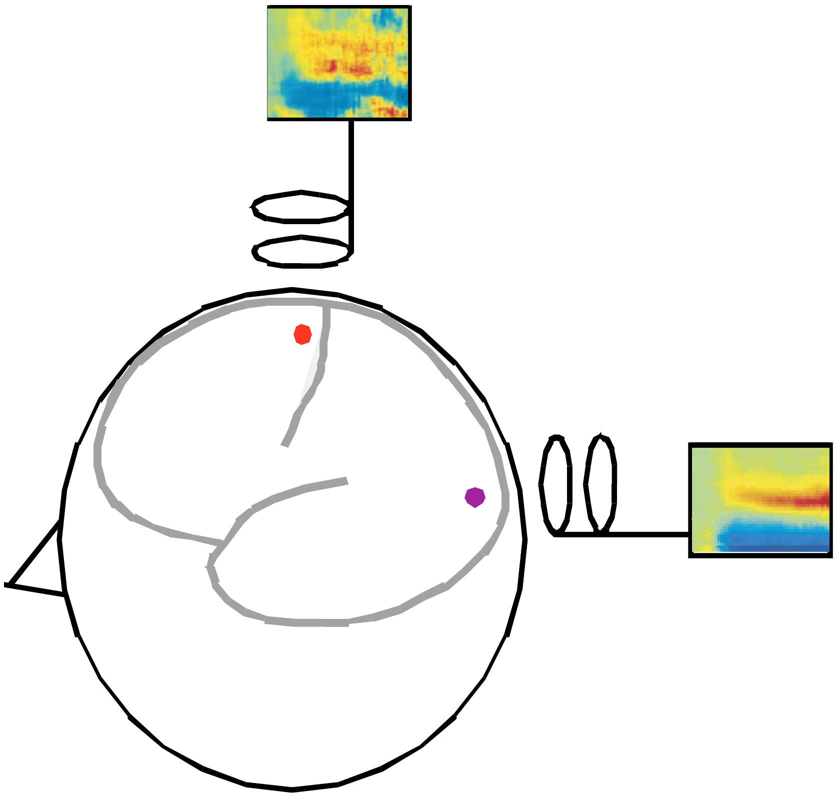
spectral characteristics
-> power spectrum

temporal changes in power
-> time-frequency response (TFR)

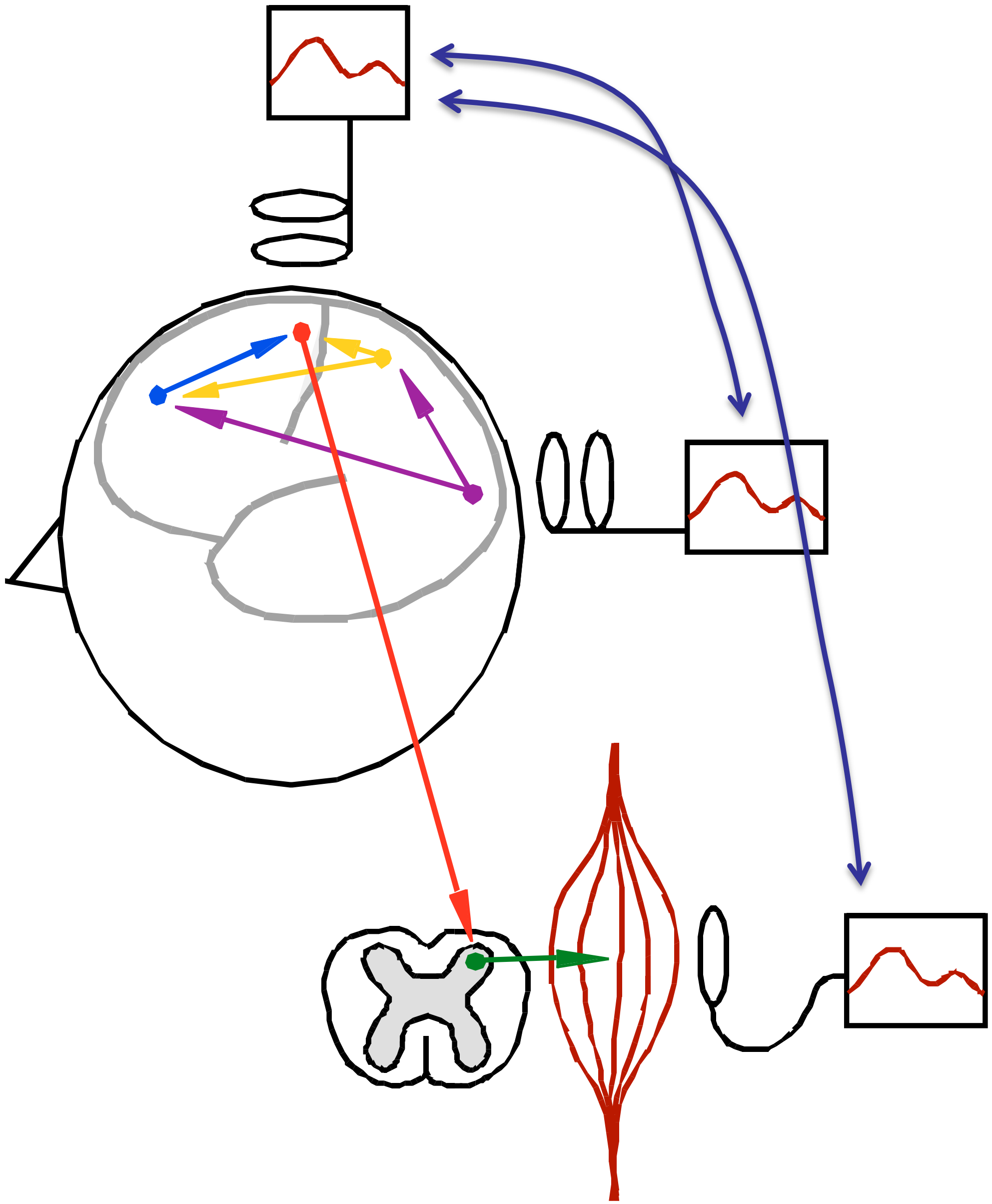
spatial distribution of activity
-> M/EEG source reconstruction
-> directly from iEEG recordings

Brain-level time courses and spectral details

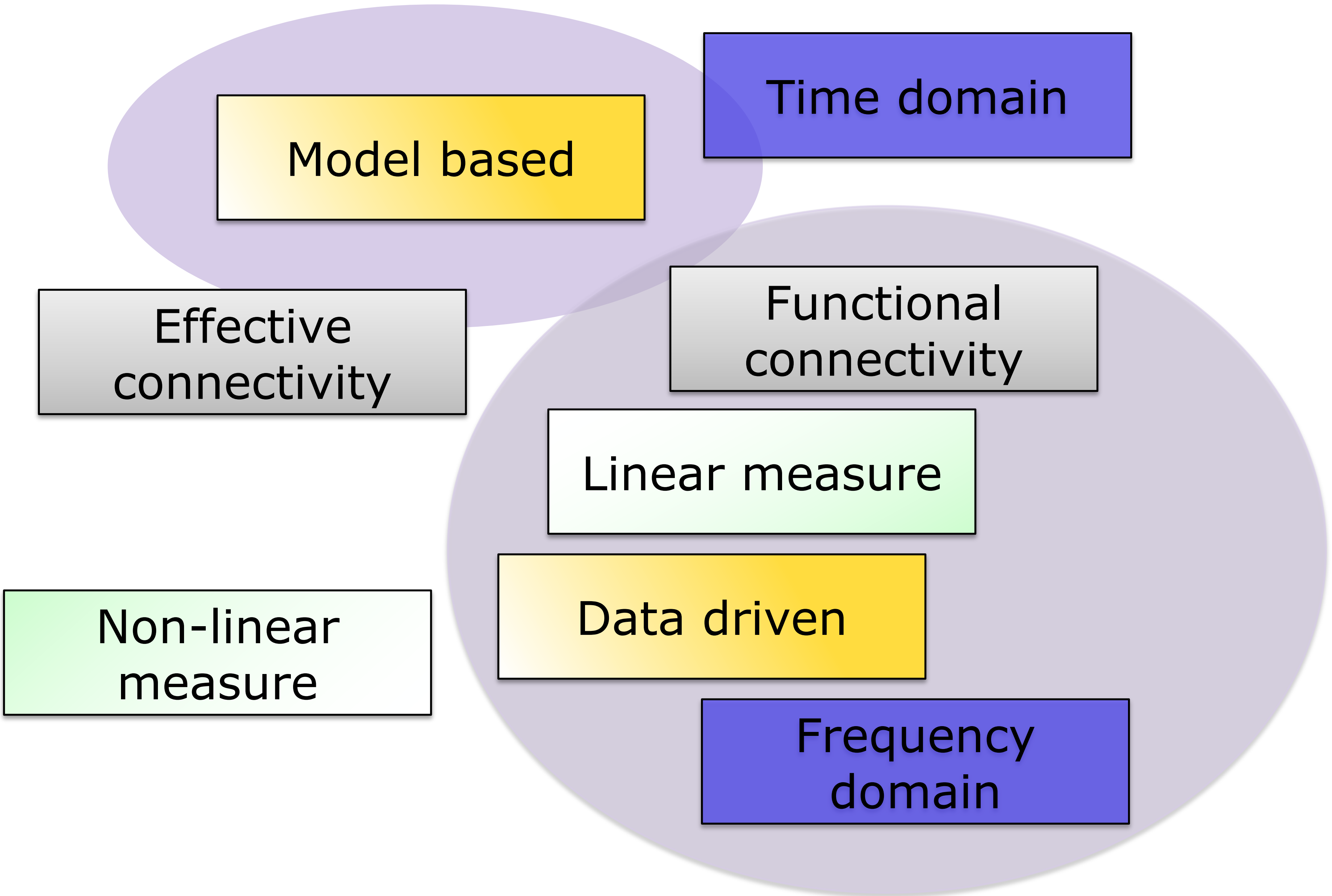
Univariate analysis



Connectivity analysis: Beyond univariate analysis



Measures of connectivity



Measures of frequency domain connectivity

Coherence coefficient

Directed transfer function

Phase locking value

Phase lag index

Phase synchronization

Imaginary part of coherency

Partial directed coherence

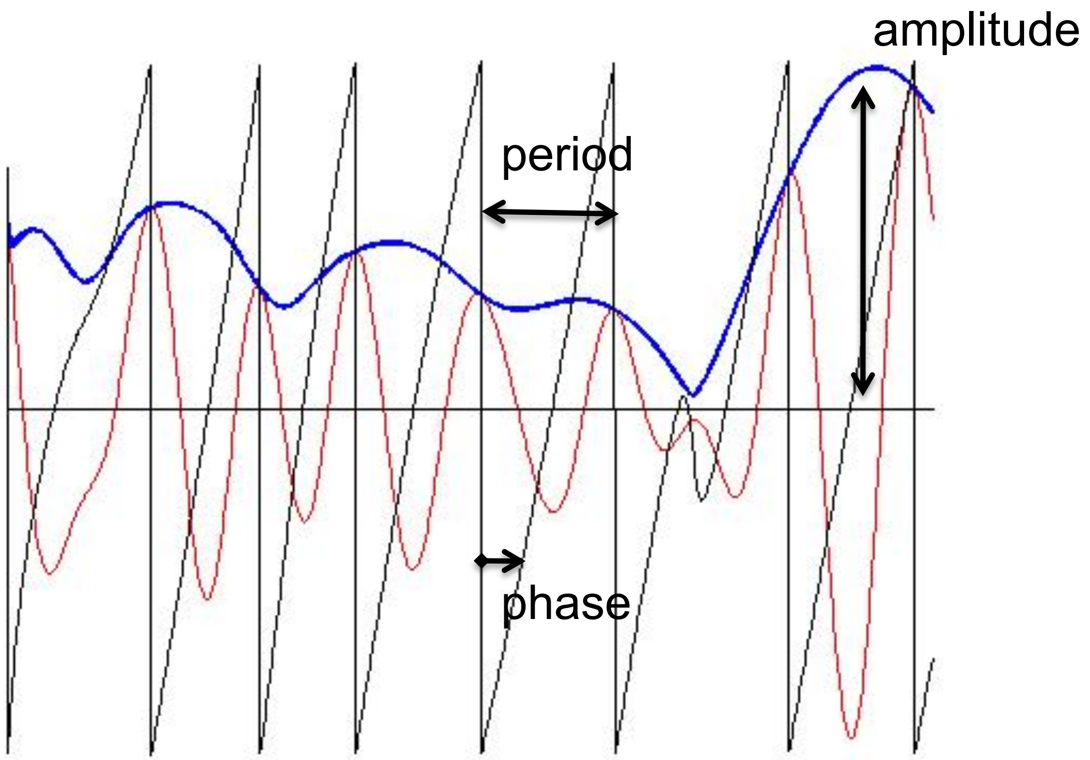
Pairwise phase consistency

Phase slope index

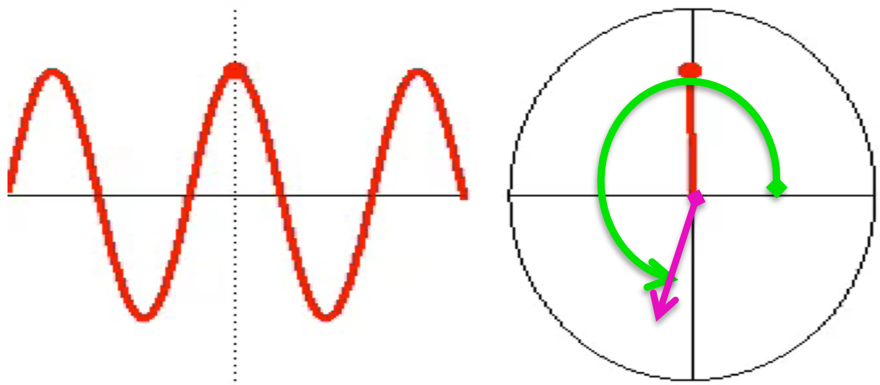
Synchronization likelihood

Frequency domain granger causality

What constitutes an oscillation? (recap)



What constitutes an oscillation? (the movie)



$$x = Ae^{i\varphi}$$

What about 2 oscillations?
Let's look at the phase difference

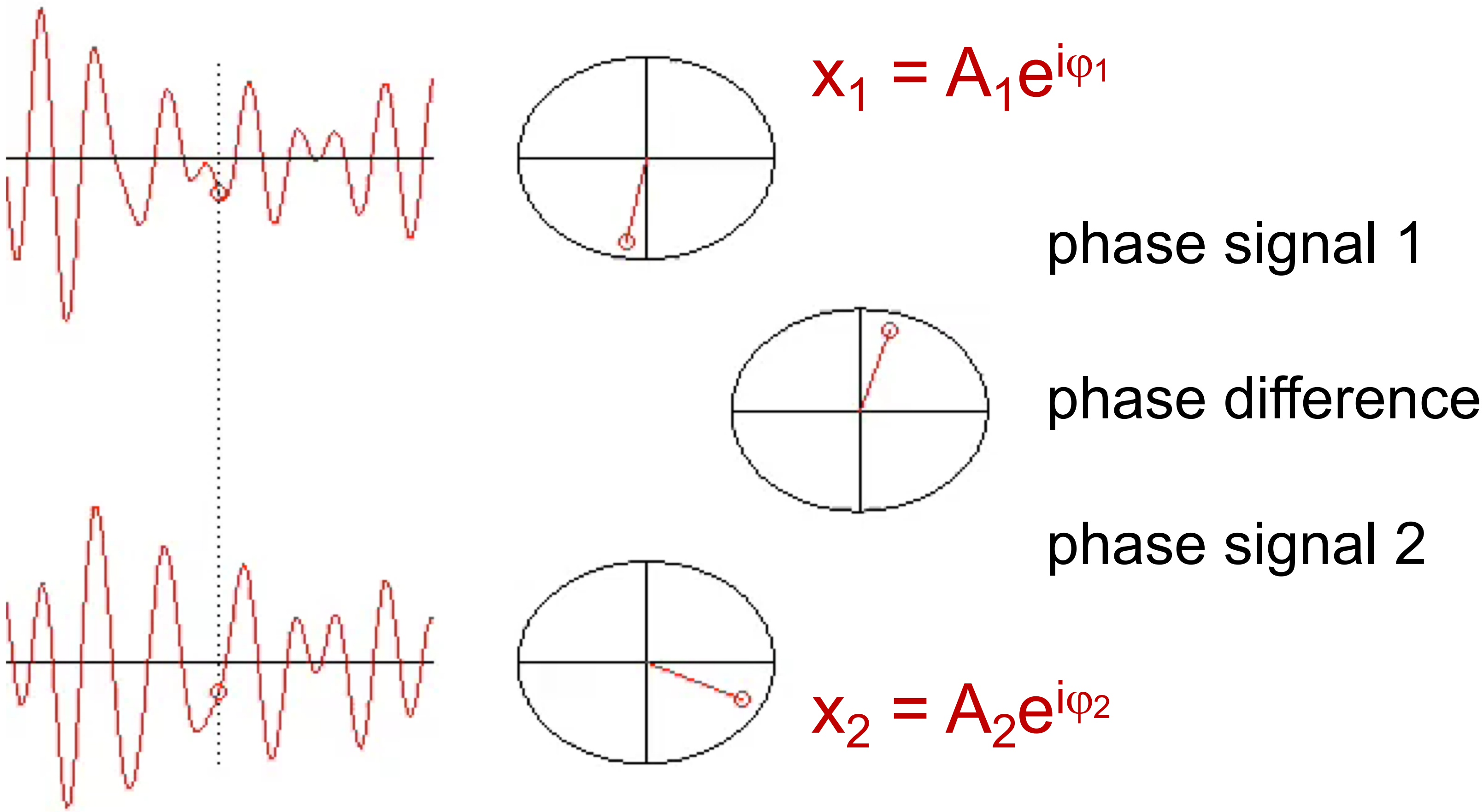
phase signal 1

phase difference

phase signal 2

Phase difference is scattered:
Low synchrony

What about 2 oscillations?
Let's look at the phase difference

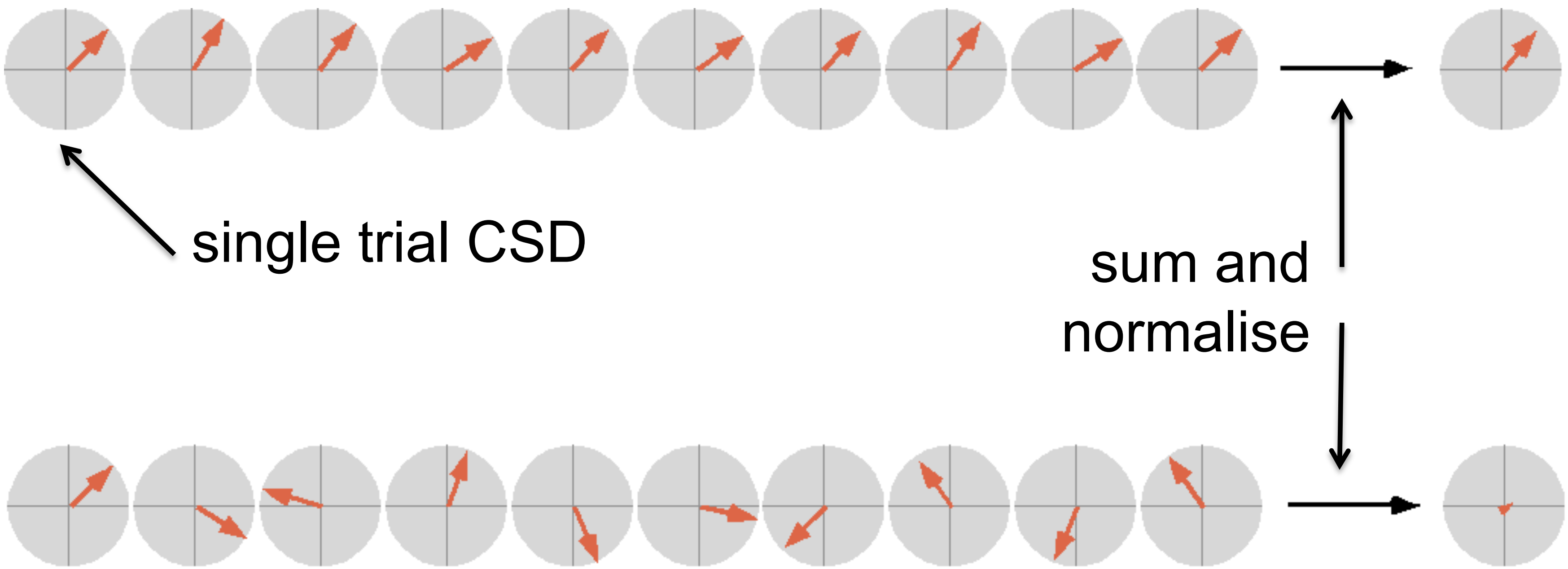


Phase difference is clustered:
High synchrony

Measures of connectivity: coherence (the math view)

Coherence is computed from the *cross-spectral density*, which is obtained by *conjugate multiplication* of the frequency domain representation of the signals

$$x_1 x_2^* = A_1 e^{i\varphi_1} \times A_2 e^{-i\varphi_2} = A_1 A_2 e^{i(\varphi_1 - \varphi_2)}$$



Measures of connectivity: coherence & co

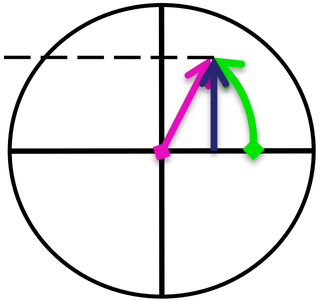
$$\text{Coherence} = \left| \frac{1/N \, \Sigma A_1 A_2 e^{i(\varphi_1 - \varphi_2)}}{\sqrt{(1/N \, \Sigma A_1^2)(1/N \, \Sigma A_2^2)}} \right|$$

$$\text{PLV} = \left| \frac{1/N \, \Sigma 1 \mathrm{x} 1 \mathrm{x} e^{i(\varphi_1 - \varphi_2)}}{\sqrt{(1/N \, \Sigma 1^2)(1/N \, \Sigma 1^2)}} \right| = \left| \frac{\Sigma e^{i(\varphi_1 - \varphi_2)}}{N} \right|$$

Measures of connectivity: coherence & co

$$\text{Coherency} = \frac{1/N \sum A_1 A_2 e^{i(\varphi_1 - \varphi_2)}}{\sqrt{(1/N \sum A_1^2)(1/N \sum A_2^2)}} = C e^{i\Delta\varphi}$$

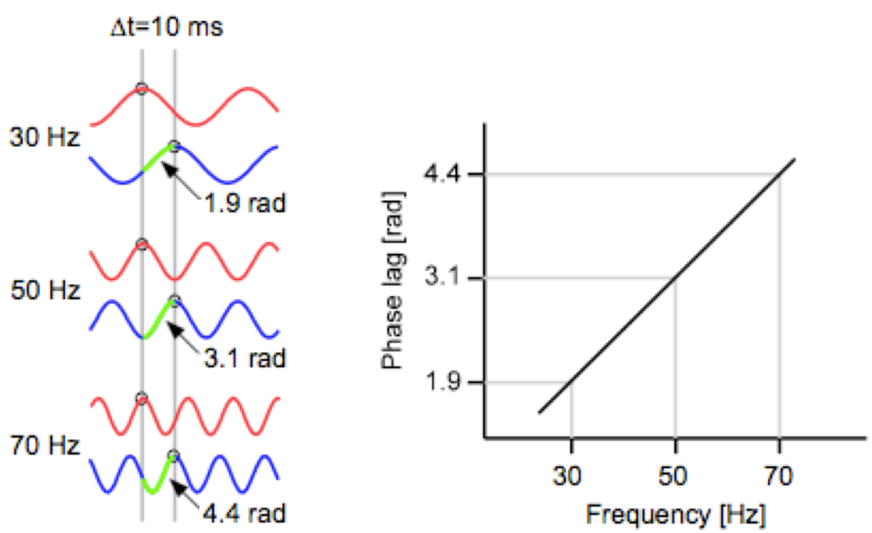
Imaginary part of coherency



Measures of connectivity: coherence & co

$$\text{Coherency} = \frac{1/N\ \Sigma A_1 A_2 e^{i(\varphi_1 - \varphi_2)}}{\sqrt{(1/N\ \Sigma A_1^2)(1/N\ \Sigma A_2^2)}} = Ce^{i\Delta\varphi}$$

Slope of relative phase spectrum indicates time delay



Coherence and linear prediction

Coherence coefficient ~ cross-correlation coefficient

$|\text{Coherence}|^2$ ~ % variance explained

Coherence coefficient similar to frequency domain regression

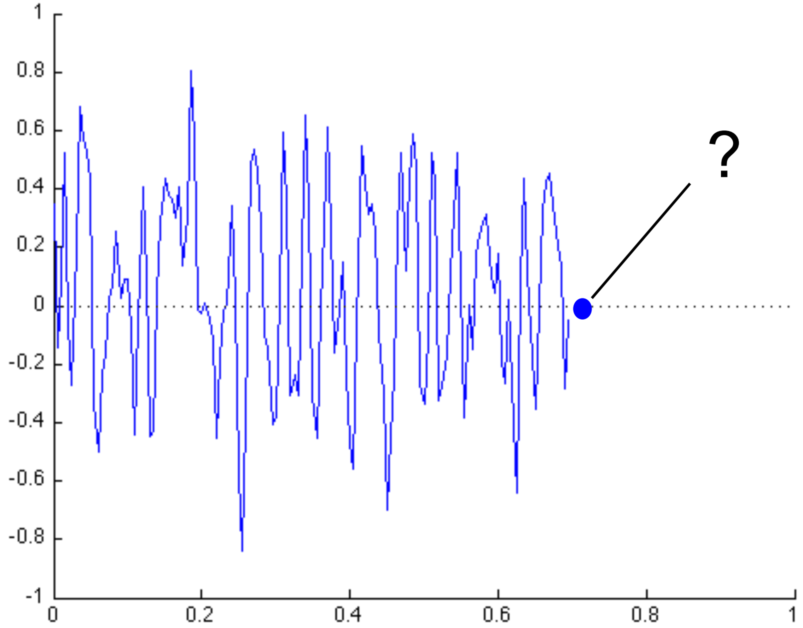
Conceptual difference with regression: independent and dependent variable are interchangeable

Slope of relative phase spectrum indicates the temporal precedence (~ directed influence)

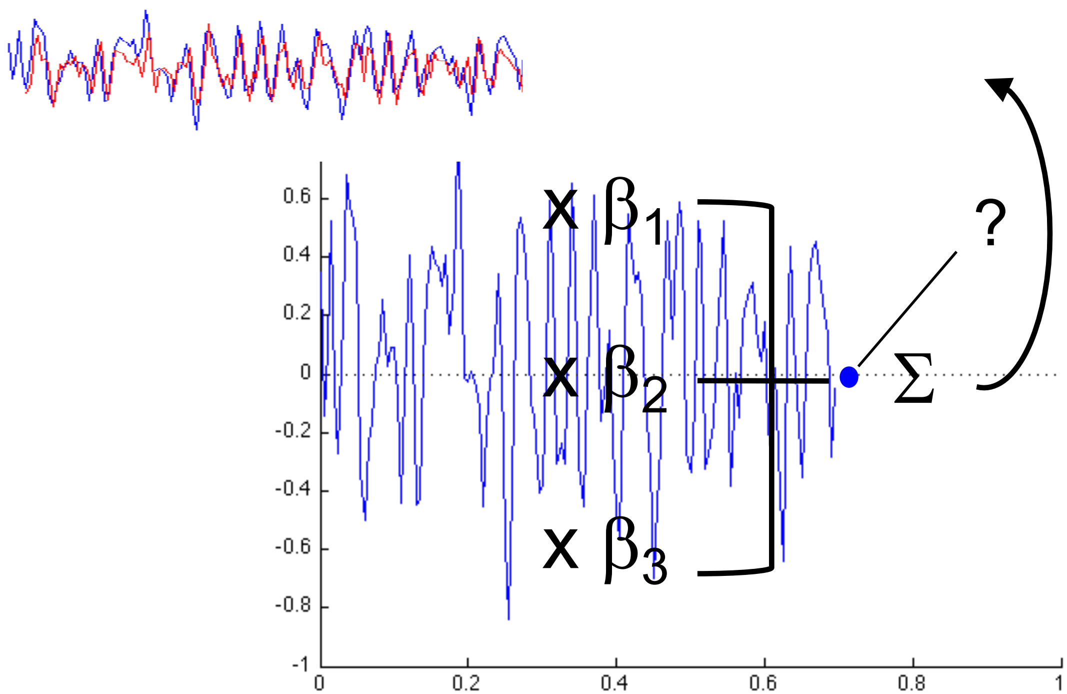
Slope often hard to estimate or close to zero

Linear prediction and directed interaction: the concept of Granger causality

Linear prediction and directed interaction: the concept of Granger causality



Linear prediction: autoregressive models



$$X(t) = \sum \beta_\tau X(t-\tau) + \eta$$

Two signals: bivariate autoregressive models

$$X(t) = \sum \beta_{\tau 1} X(t-\tau) + \eta_1$$

$$Y(t) = \sum \beta_{\tau 2} Y(t-\tau) + \eta_2$$

$$X(t) = \sum \beta_{\tau 11} X(t-\tau) + \sum \beta_{\tau 21} Y(t-\tau) + \varepsilon_1$$

$$Y(t) = \sum \beta_{\tau 12} X(t-\tau) + \sum \beta_{\tau 22} Y(t-\tau) + \varepsilon_2$$

Granger causality: compare the residuals

$$X(t) = \sum \beta_{\tau 1} X(t-\tau) + \eta_1$$

$$Y(t) = \sum \beta_{\tau 2} Y(t-\tau) + \eta_2$$

$$X(t) = \sum \beta_{\tau 11} X(t-\tau) + \sum \beta_{\tau 21} Y(t-\tau) + \varepsilon_1$$

$$Y(t) = \sum \beta_{\tau 12} X(t-\tau) + \sum \beta_{\tau 22} Y(t-\tau) + \varepsilon_2$$

$$F_{Y \rightarrow X} = \ln\left(\frac{var(\eta_1)}{var(\varepsilon_1)}\right)$$

$$F_{X \rightarrow Y} = \ln\left(\frac{var(\eta_2)}{var(\varepsilon_2)}\right)$$

Analogy between Granger and 'plain' regression

$X(t) = \sum \beta_{\tau 1} X(t-\tau) + \eta_1$

$Y(t) = \sum \beta_{\tau 2} Y(t-\tau) + \eta_2$

$X(t) = \sum \beta_{\tau 11} X(t-\tau) + \sum \beta_{\tau 21} Y(t-\tau) + \varepsilon_1$

$Y(t) = \sum \beta_{\tau 12} X(t-\tau) + \sum \beta_{\tau 22} Y(t-\tau) + \varepsilon_2$

$\text{data} = \sum \beta_\kappa X_\kappa + \eta$

$\text{data} = \sum \beta'_\kappa X_\kappa + \beta'_{\kappa+1} X_{\kappa+1} + \varepsilon$

$$F_{Y \rightarrow X} = \ln\left(\frac{\mathrm{var}(\eta_1)}{\mathrm{var}(\varepsilon_1)}\right)$$

$$F \sim \frac{\mathrm{var}(\eta)}{\mathrm{var}(\varepsilon)}$$

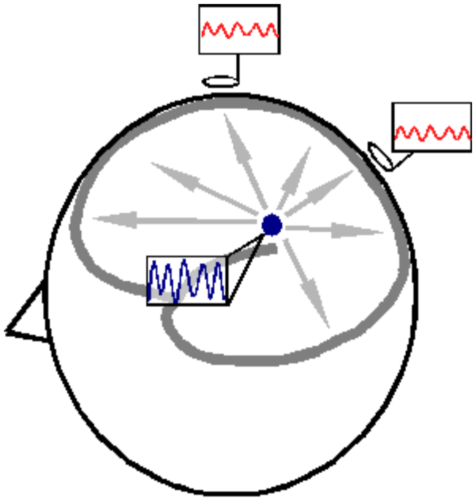
...only the inference is different

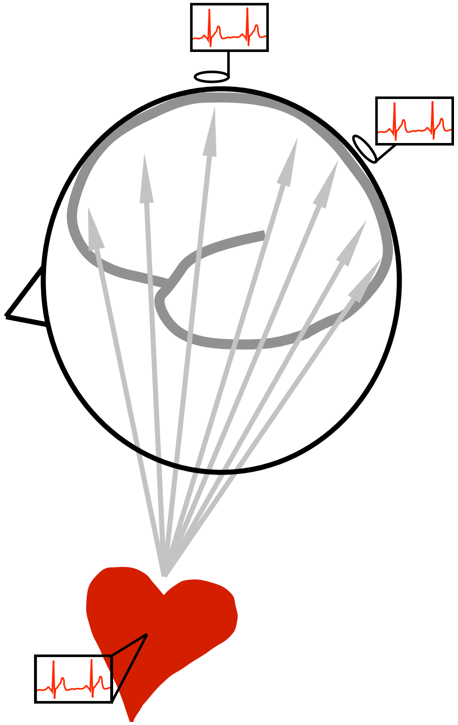
Interpretational issues

Interpretational issues

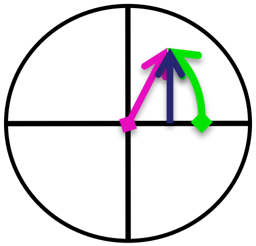
- Many connectivity metrics are ‘biased’
- Bias is often sample size dependent
- Common pick up / field spread
  - other sources in the brain
  - other physiological sources
  - especially problematic if those sources have some “internal synchronization” themselves
- Differences in signal (or noise) between experimental conditions
  - better SNR -> more reliable estimate of the phase
  - more reliable phase -> more consistent phase difference

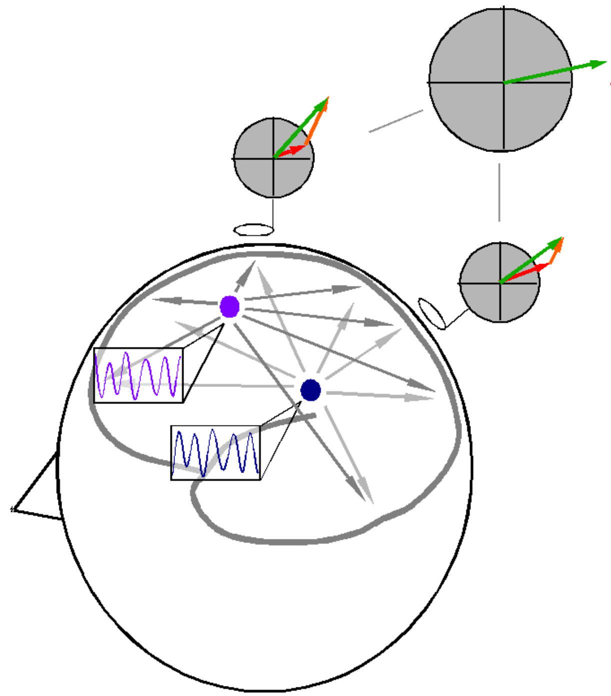
Practial issues: Electromagnetic field spread





Practical issues: imaginary part of coherency

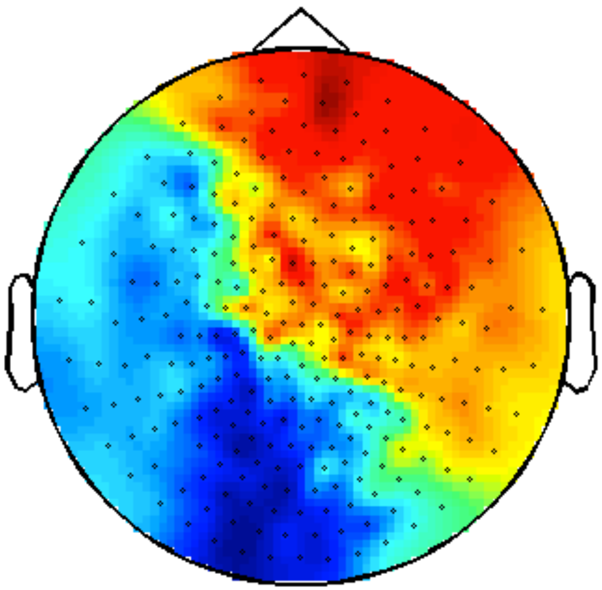


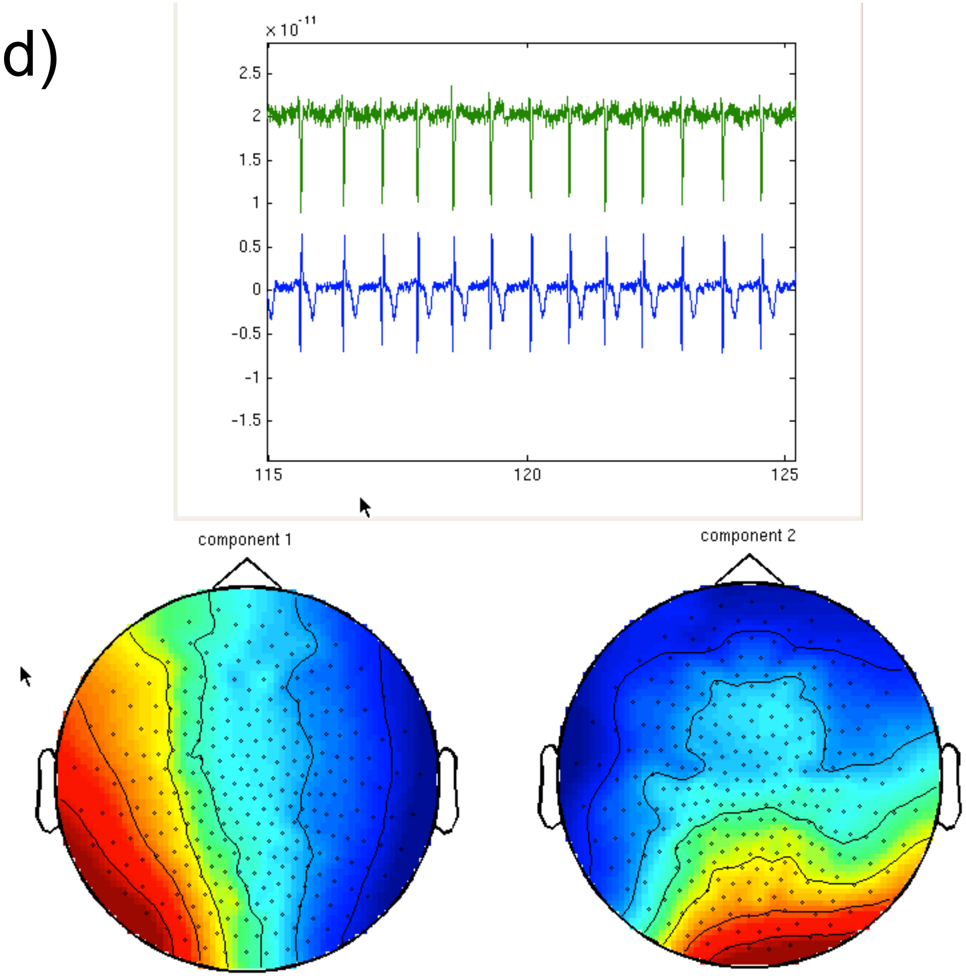


Im(coherency) ≠ **0**

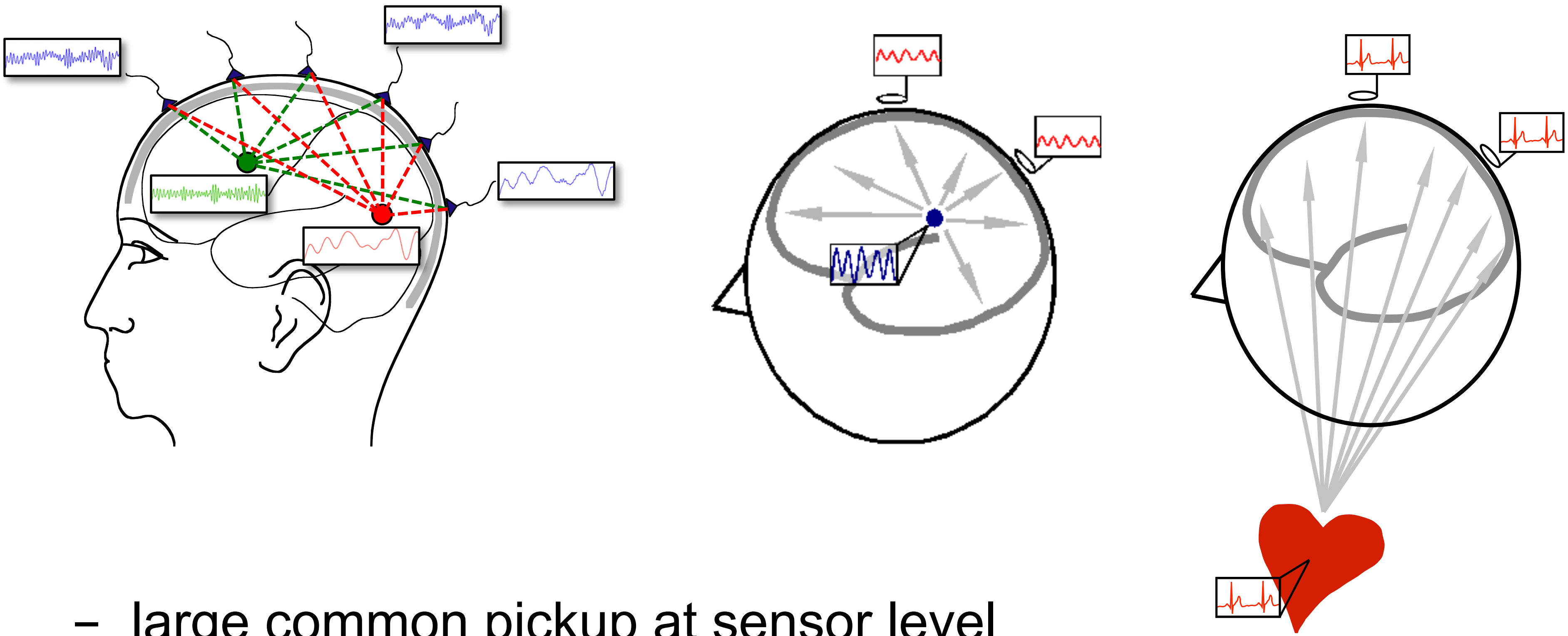
MEG connectivity: pitfalls with assumptions

WPLI suggests fronto-occipital directed interaction (alpha band)



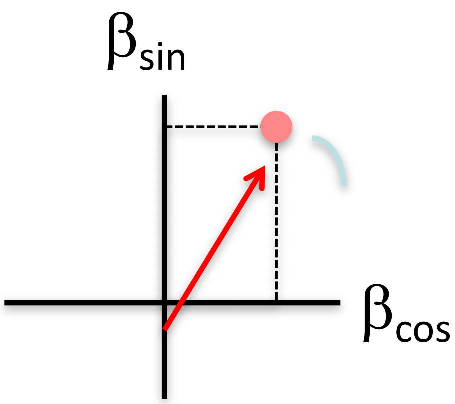


Common pick up



- large common pickup at sensor level
- not all interfering sources are 1-dimensional
- no common pickup if you have a perfect source model
- some common pickup if source model is not perfect

Practial issues: Power and phase are confounded



Fourier Phase estimates depend on S/N ratio

More power -> more accurate phase estimates

Better phase estimates -> higher connectivity

Concluding remarks

Connectivity analysis is cool

Many measures on the market

Many of the confounds are not easy to deal with

Interpretation of results should therefore be done with care