# Forward and Inverse Modeling of EEG and MEG data

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# Outline

**Motivation and background**

**Forward modeling**

- Source model
- Volume conductor model

**Inverse modeling**

- Single and multiple dipole fitting
- Distributed source models
- Beamforming methods

**Summary**

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# Motivation

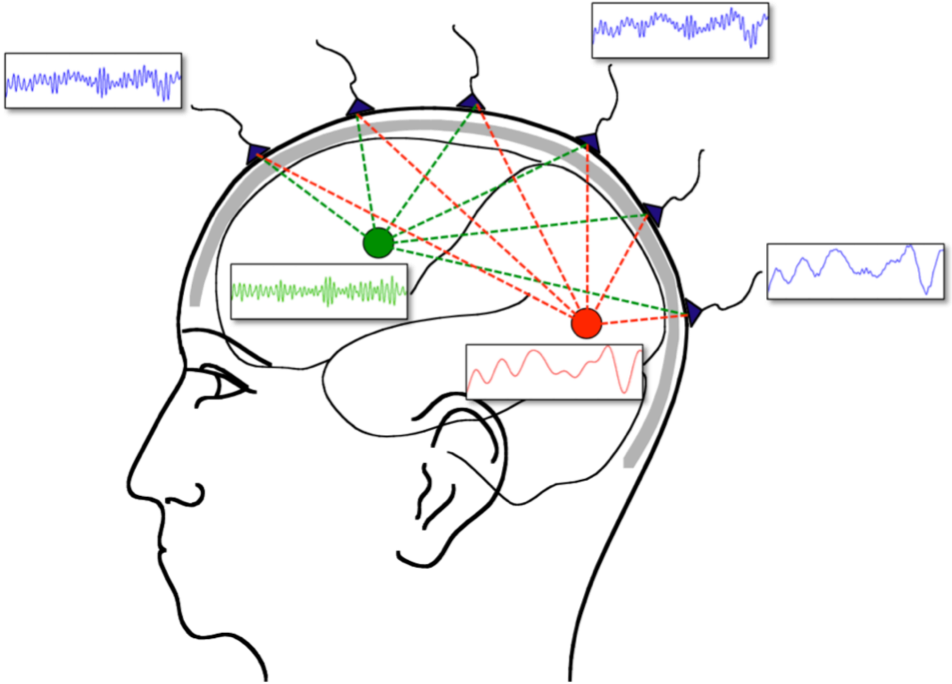
- Strong points of EEG and MEG
  - Temporal resolution (~1 ms)
  - Characterize individual components of ERP/ERF
  - Oscillatory activity
  - Disentangle dynamics of cortical networks

- Weak points of EEG and MEG
  - Measurement on outside of the brain
  - Overlap of components
  - Low spatial resolution

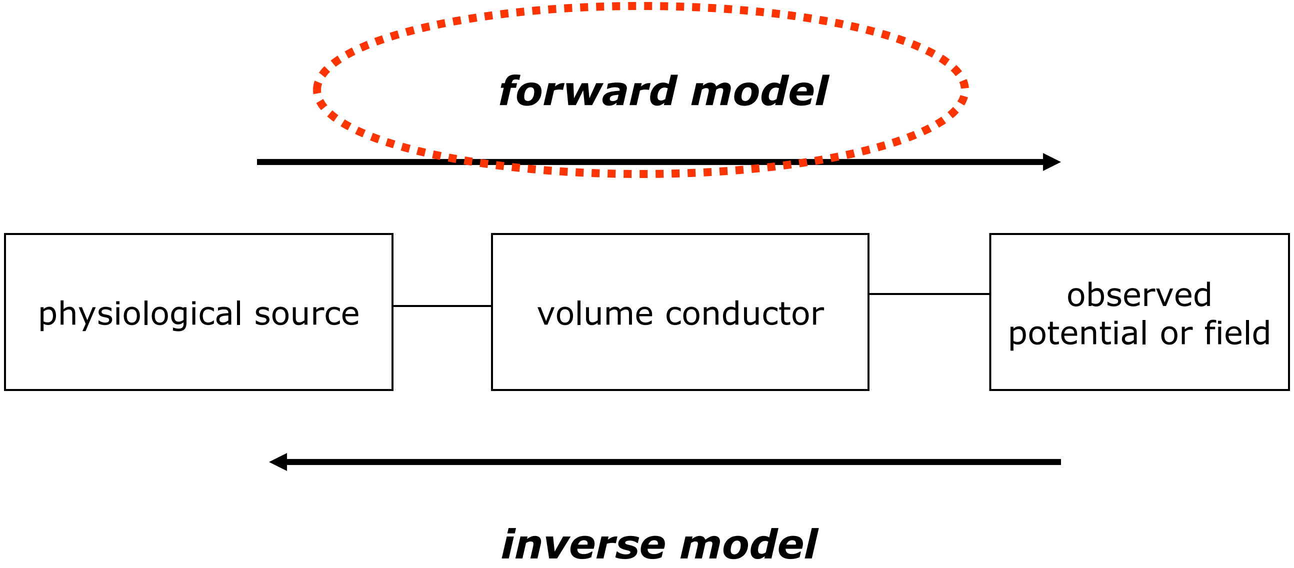
# Motivation

- If you find a ERP/ERF component, you want to characterize it in physiological terms
    - Time or frequency are the “natural” characteristics
    - “Location” requires interpretation of the scalp topography
- Forward and inverse modeling helps to interpret the topography
- Forward and inverse modeling helps to disentangle overlapping source timeseries

# Superposition of source activity



# Biophysical source modelling: overview



***forward model***

physiological source

volume conductor

observed potential or field

***inverse model***

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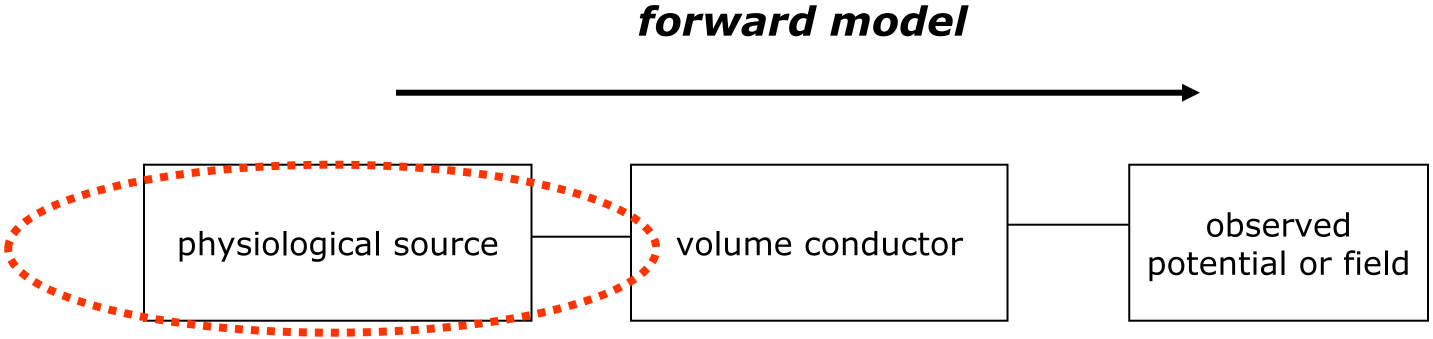

**_forward model_**

physiological source

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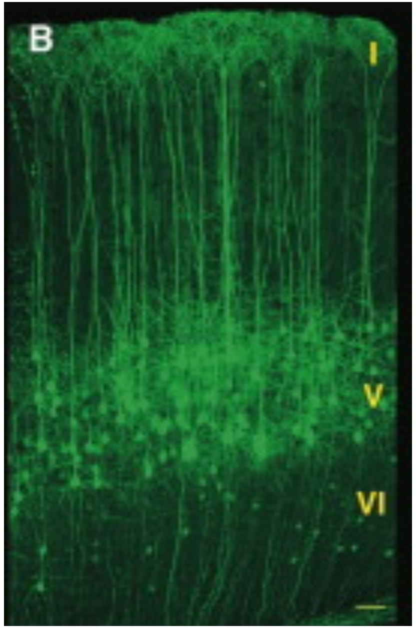
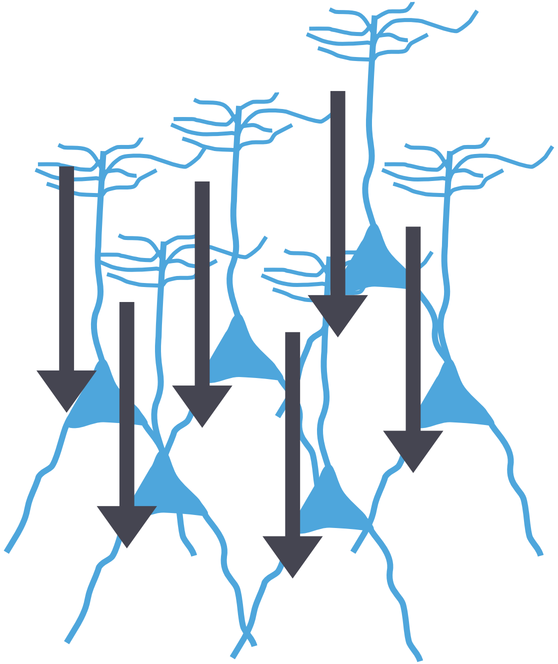
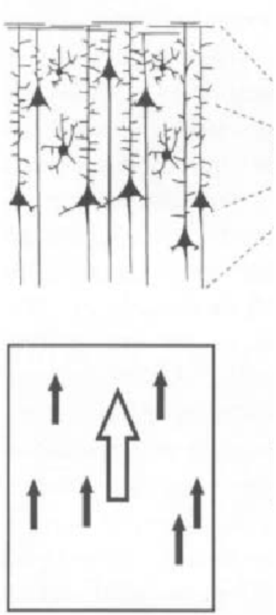
# What produces the electric current





# Equivalent current dipoles



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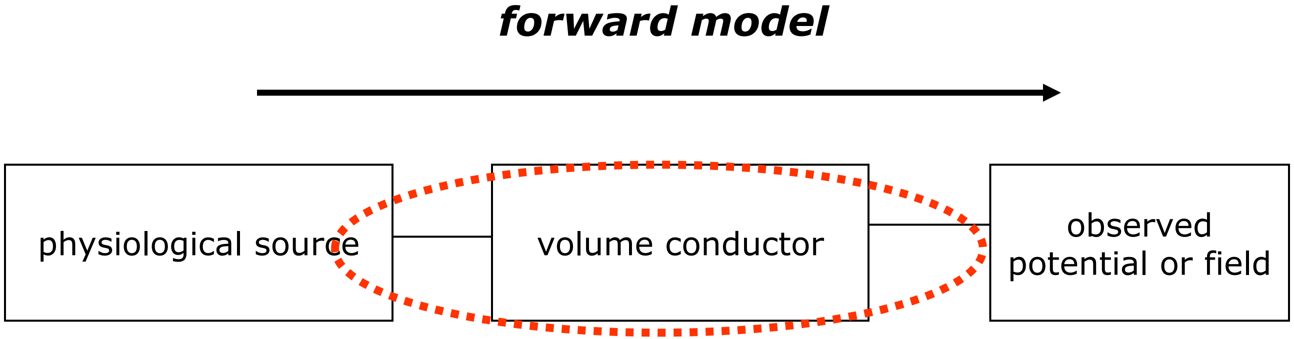

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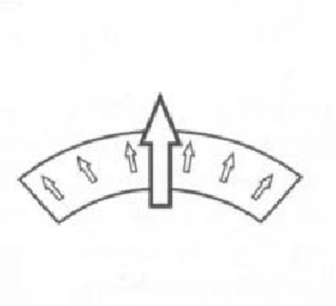
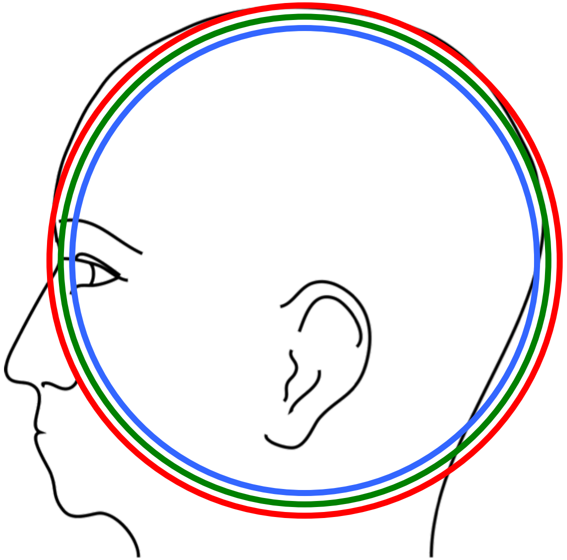


# Volume conductor

- described electrical properties of tissue
- describes geometrical model of the head
- describes **how** the currents flow, not where they originate from
- same volume conductor for EEG as for MEG, but also for tDCS, tACS, TMS, …



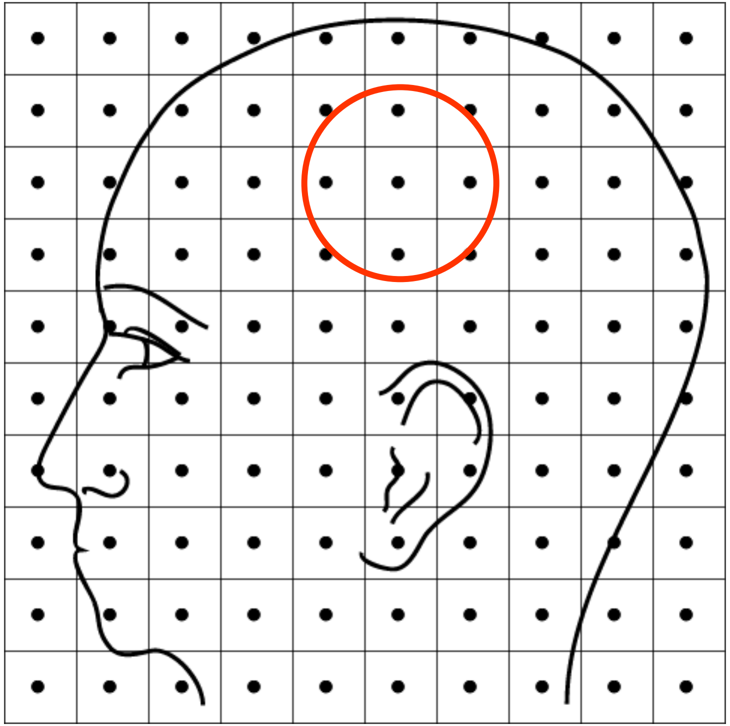
# Volume conductor

- Computational methods for volume conduction problem that allow for realistic geometries

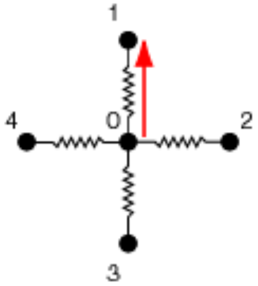
| | |
|---|---|
| FDM | *Finite Difference Method* |
| BEM | *Boundary Element Method* |
| FEM | *Finite Element Method* |

# Volume conductor: Finite Difference Method

- Easy to compute
- Not very useful in practice



# Volume conductor: Finite Difference Method



Kirchhoffs law $I_1 + I_2 + I_3 + I_4 = 0$

Ohm's law $V = I*R$

$\Rightarrow$

$$\Delta V_1/R_1 + \Delta V_2/R_2 + \Delta V_3/R_3 + \Delta V_4/R_4 = 0 \quad \Rightarrow$$

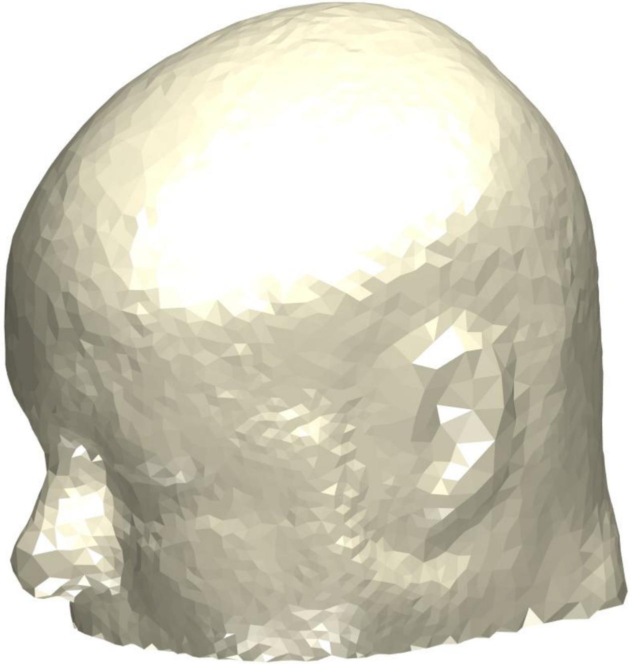
$$(V_1-V_0)/R_1 + (V_2-V_0)/R_2 + (V_3-V_0)/R_3 + (V_4-V_0)/R_4 = 0$$

# Volume conductor: Finite Difference Method

- Unknown potential Vi at each node
- Linear equation for each node
    - approx. 100x100x100 = 1.000.000 linear equations
    - just as many unknown potentials
- Add a source/sink
    - sum of currents is zero for all nodes, except
    - sum of current is I+ for a certain node
    - sum of current is I- for another node
- Solve for unknown potential

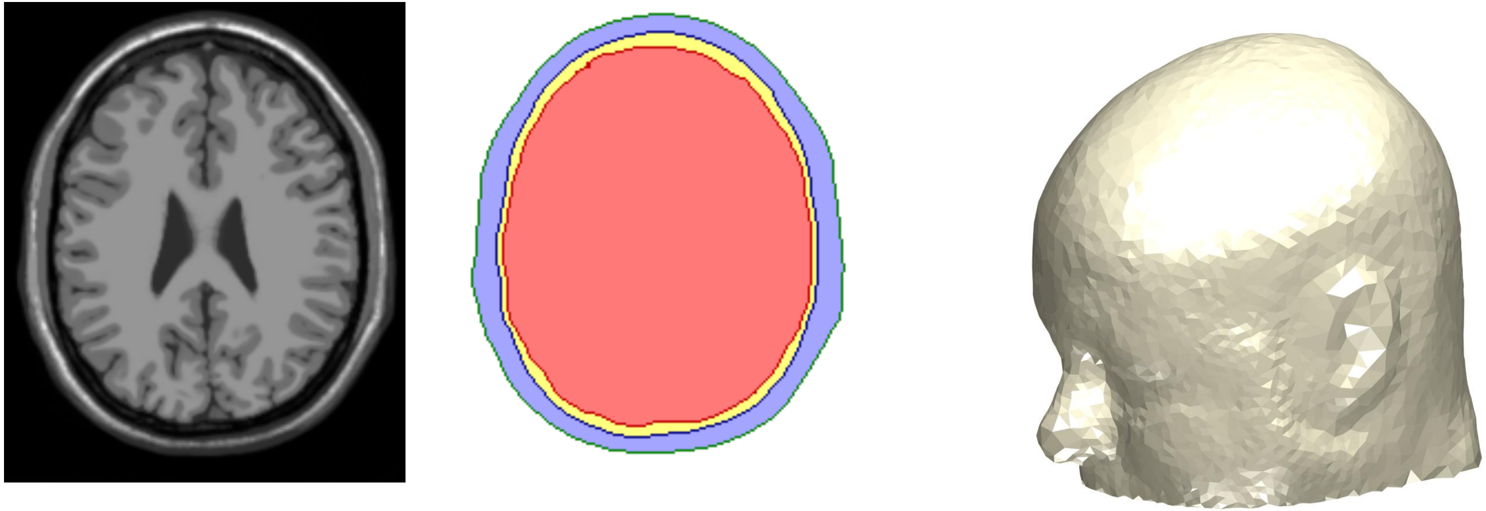
# Volume conductor: Boundary Element Method

- Important tissues
  - skin
  - skull
  - brain
  - (CSF)
- Each compartment is
  - homogenous
  - isotropic
- Triangulated surfaces describe boundaries



# Volume conductor: Boundary Element Method

- Construction of geometry
  - segmentation in different tissue types
  - extract surface description
  - downsample to reasonable number of triangles

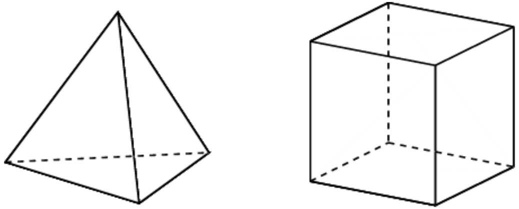


# Volume conductor: Boundary Element Method

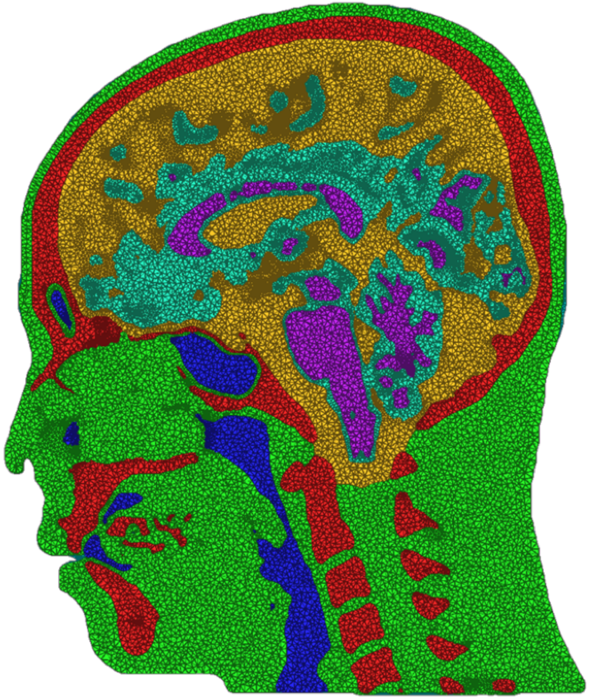
- Construction of geometry
  - segmentation in different tissue types
  - extract surface description
  - downsample to reasonable number of triangles
- Computation of model
  - independent of source model
  - only one lengthy computation
  - fast during application to real data
- Can also include more complex geometrical details
  - ventricles
  - holes in skull
  - dura

# Volume conductor: Finite Element Method

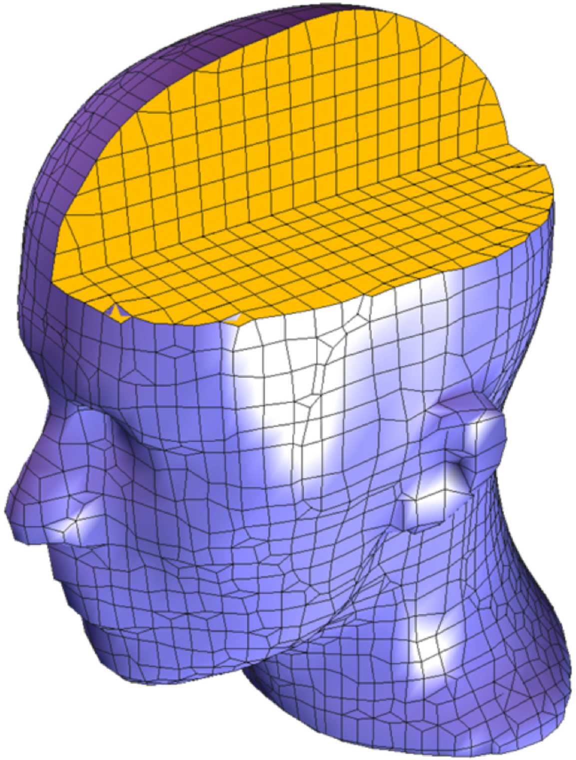
- Tessellation of 3D volume in tetrahedra or hexahedra



# Volume conductor: Finite Element Method

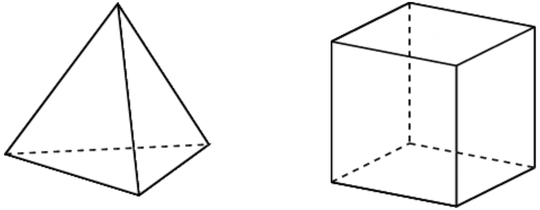


tetraeders



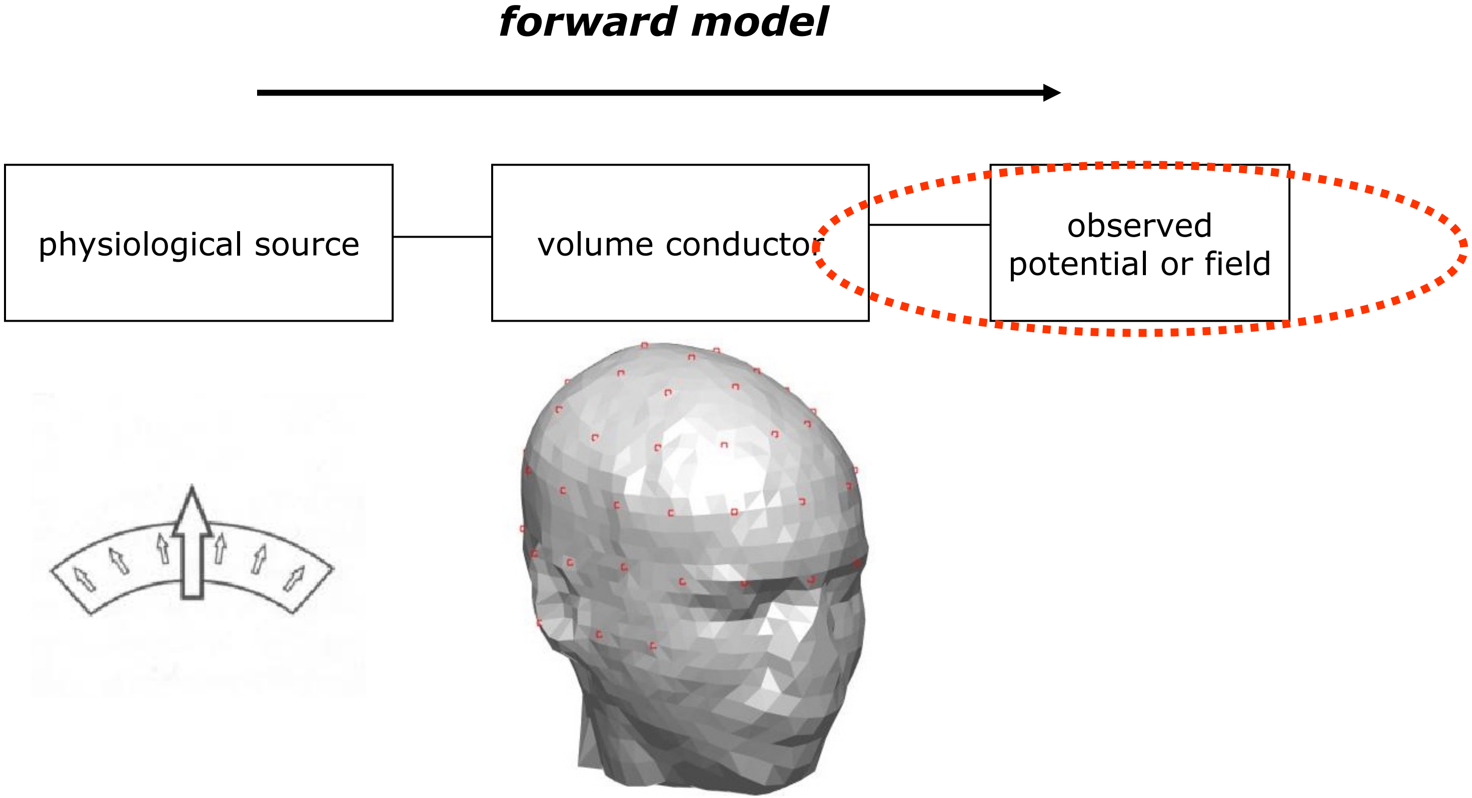
hexaheders

# Volume conductor: Finite Element Method



- Tessellation of 3D volume in tetrahedra or hexahedra
- Each element can have its own conductivity
- FEM is the most accurate numerical method but computationally quite expensive
- Geometrical processing not as simple as BEM

# Biophysical source modelling: overview



***forward model***

physiological source

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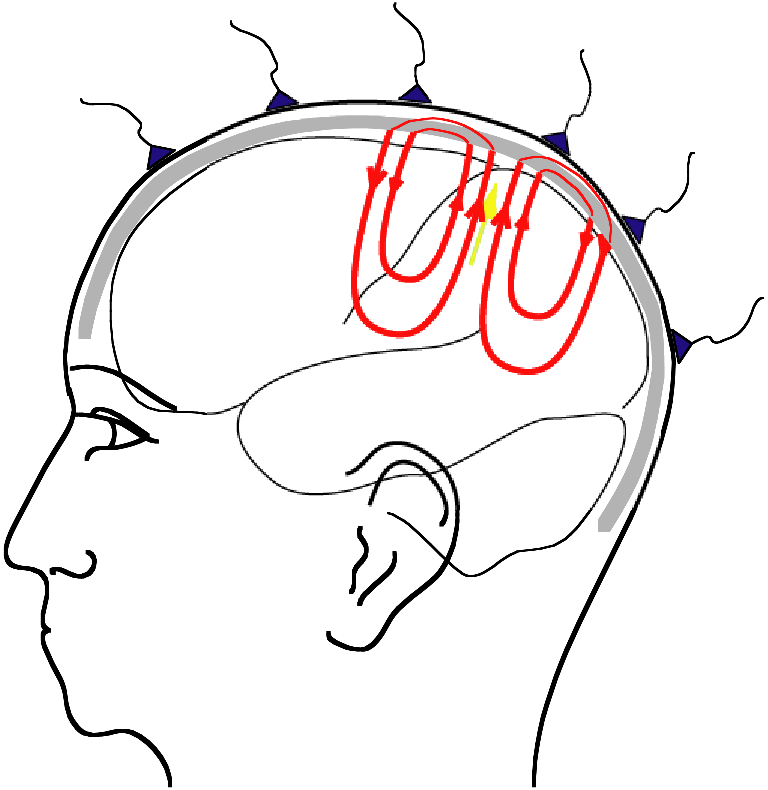
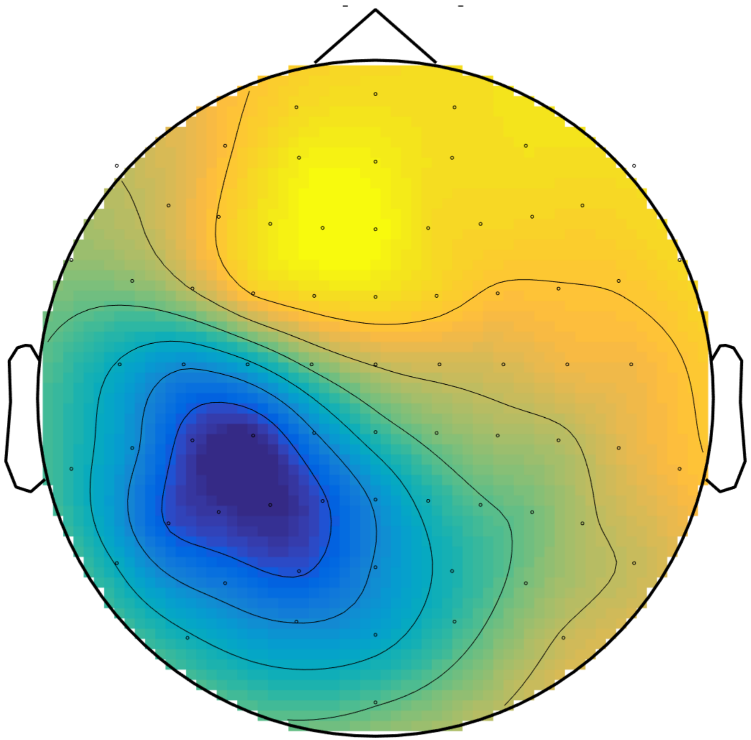
observed potential or field

# EEG volume conduction



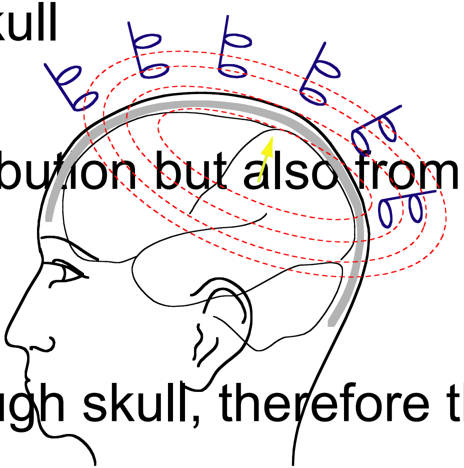


# EEG volume conduction

- Potential difference between electrodes corresponds to current flowing through skin
- Only tiny fraction of current passes through skull
- Therefore the model should describe the skull and skin as accurately as possible

# MEG volume conduction

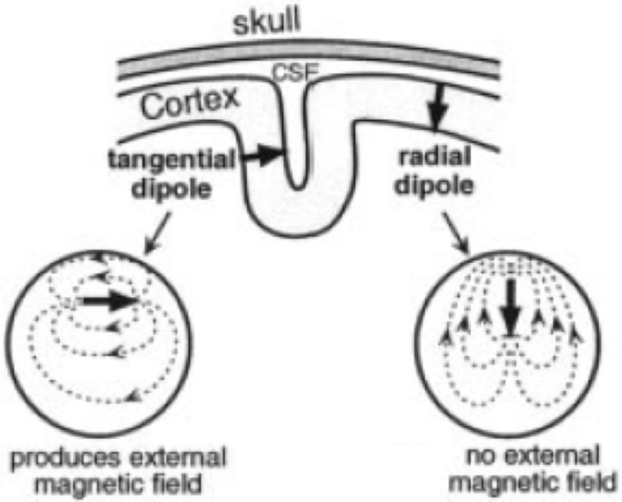
- MEG measures magnetic field over the scalp
- Magnetic … y skull
- Magnetic … distribution but also from the volume c…
- Only tiny … through skull, therefore the model can ignor…



# MEG volume conduction compared to EEG

- EEG is measurement on scalp
  - potential difference due to volume currents

- MEG field not affected by head
  - magnetic field due to primary current (source)
  - magnetic field due to secondary (volume) currents



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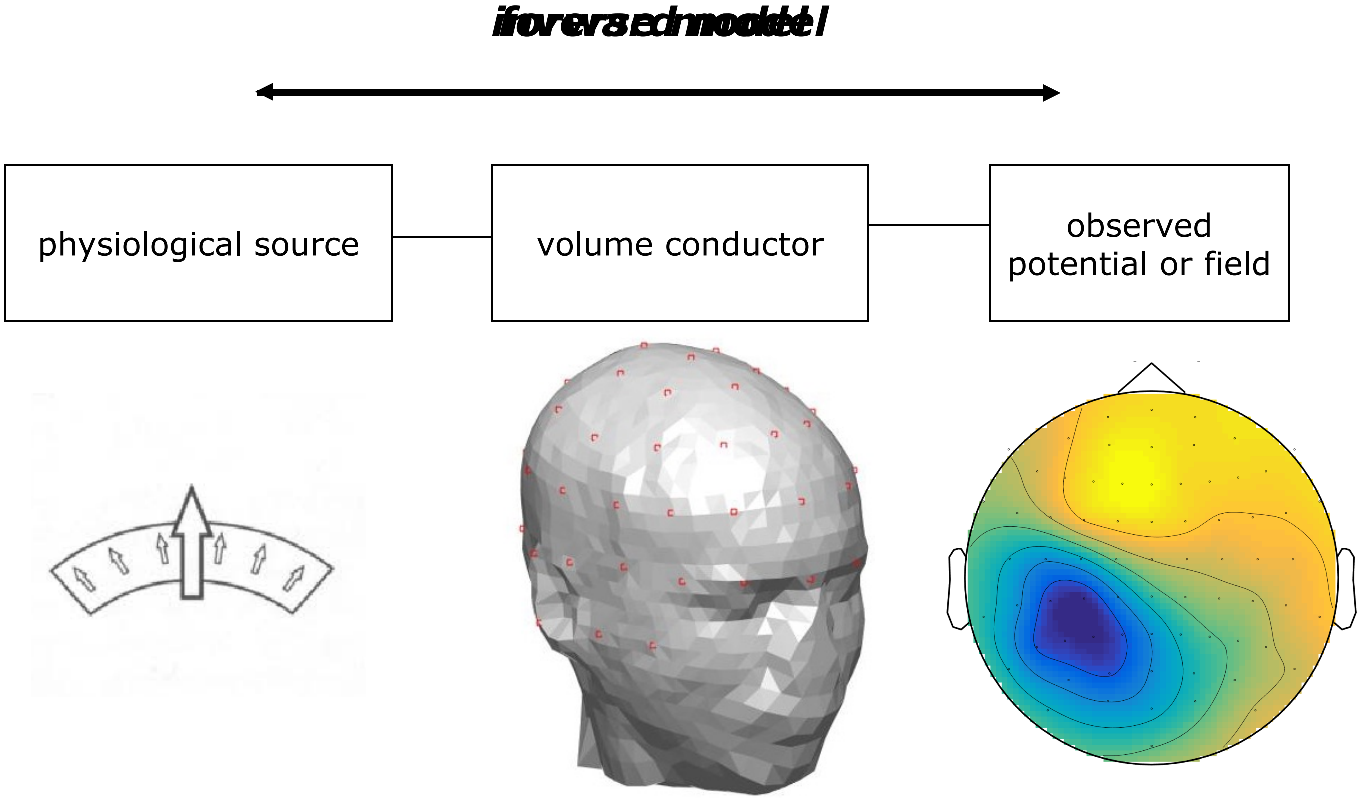

*forward model*

*inverse model*

physiological source

volume conductor

observed
potential or field

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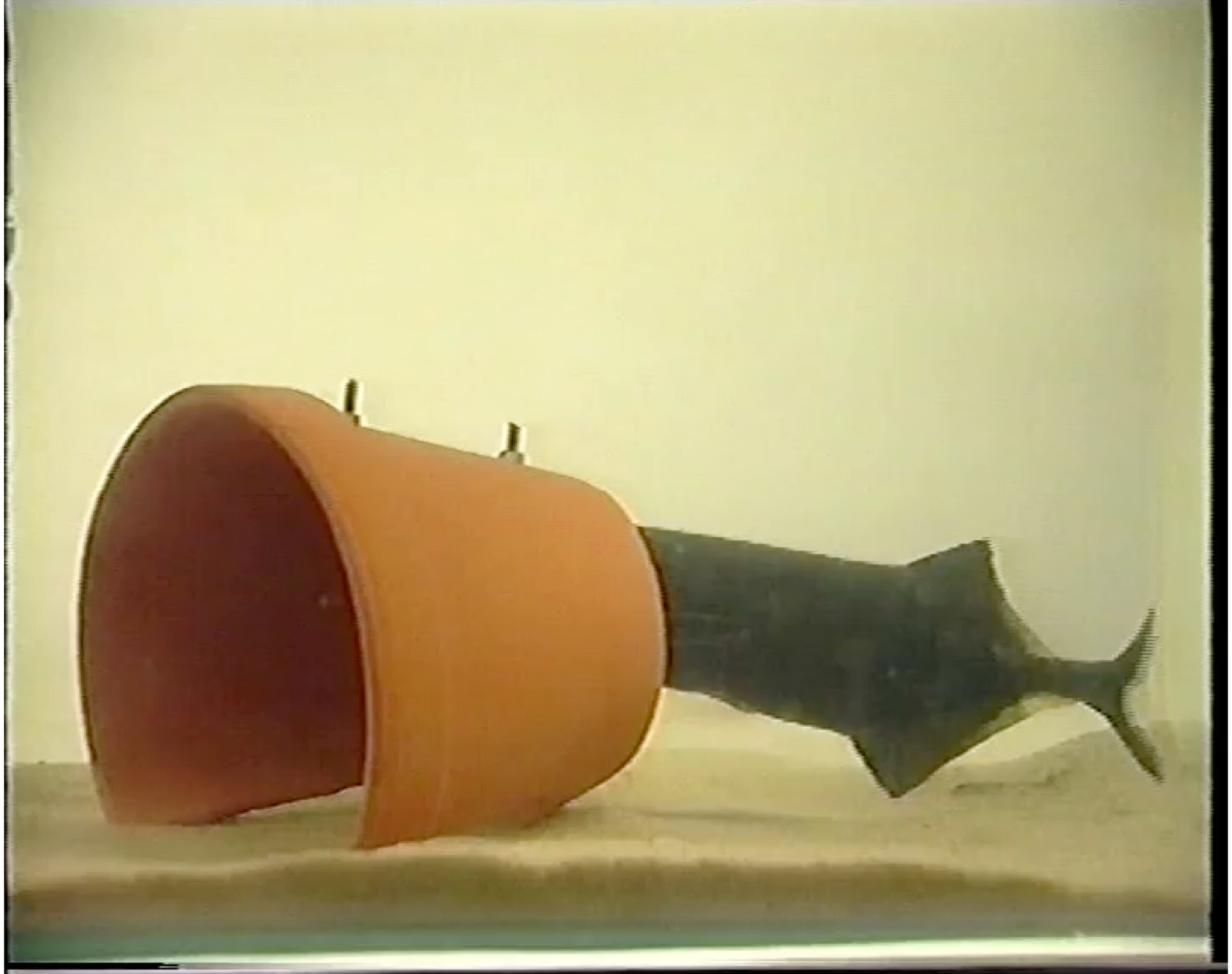
## **Inverse modeling**

- Single and multiple dipole fitting
- Distributed source models
- Beamforming methods

## Summary



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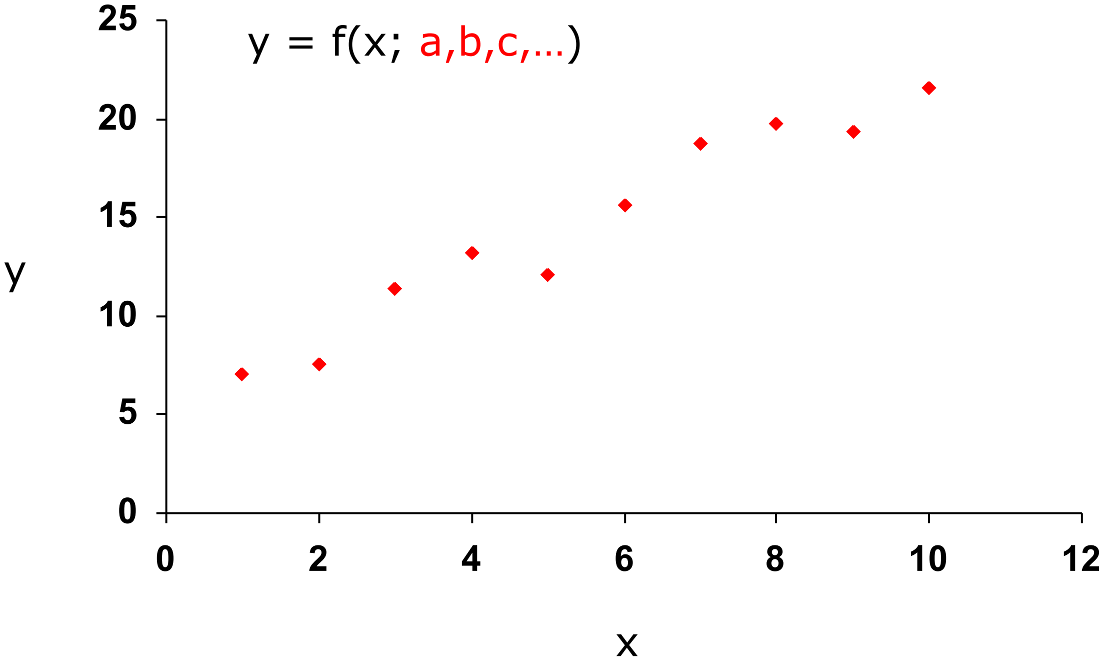
# Inverse methods

- Single and multiple dipole models
    - Few patches of cortical activity
- Distributed source models
    - Distributed activity over the whole cortex
- Spatial filtering
    - Activity of different sources is uncorrelated to each other and to the noise

# Single or multiple dipole models – Parameter estimation

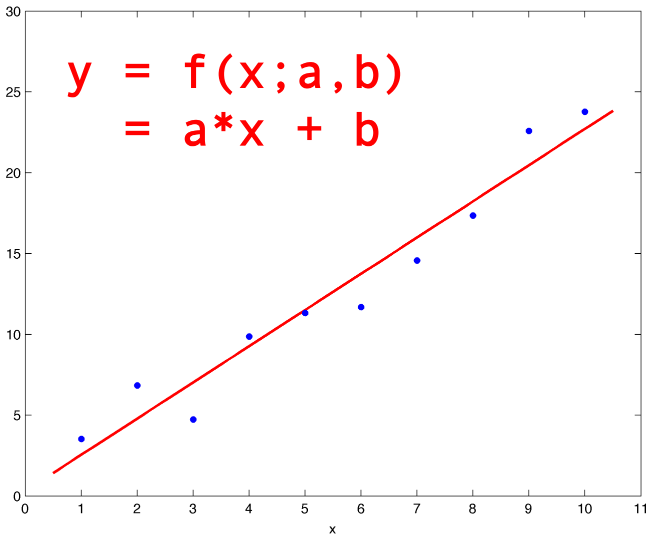


# Parameter estimation: dipole parameters

source model with few parameters

- position
- orientation
- strength

minimize difference between actual and model data



$$y = f(x;a,b) = a*x + b$$

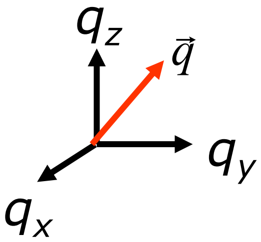
# Linear parameters: superposition of sources

- three sources with parameters $\zeta_1$, $\zeta_2$ and $\zeta_3$

$$\left.\begin{array}{l} Y(\zeta_1) \\ Y(\zeta_2) \\ Y(\zeta_3) \end{array}\right\} \quad Y_{combined} = Y(\zeta_1) + Y(\zeta_2) + Y(\zeta_3)$$

# Linear parameters: estimation

$$Y = G_x q_x + G_y q_y + G_z q_z = \begin{bmatrix} G_{x,1} & G_{y,1} & G_{z,1} \\ G_{x,2} & G_{y,2} & G_{z,2} \\ \vdots & \vdots & \vdots \\ G_{x,N} & G_{y,N} & G_{z,N} \end{bmatrix} \cdot \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} = \mathrm{G} \cdot \vec{q}$$



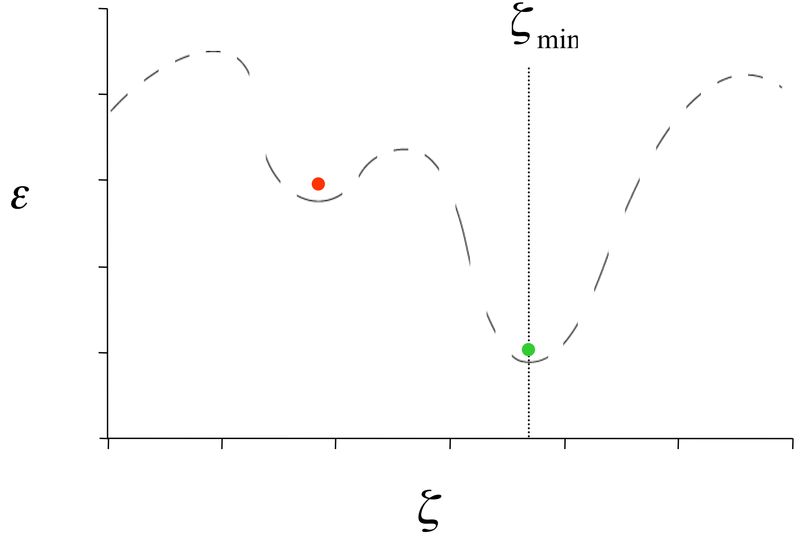
$$\begin{aligned} Y &= \mathrm{G} \cdot \vec{q} \\ &= \mathrm{G}(\zeta) \cdot \vec{q} \end{aligned}$$

$$\vec{q} = \mathrm{G}^{-1} \cdot Y$$

# Non-linear parameters

$$\varepsilon rror(\zeta) = \sum_{i=1}^{N} \left(Y_i(\zeta) - V_i\right)^2 \quad \Rightarrow \quad \min_{\zeta}\left(\varepsilon rror(\zeta)\right)$$

$$\zeta = a, b, c, \ldots$$



# Non-linear parameters: grid search

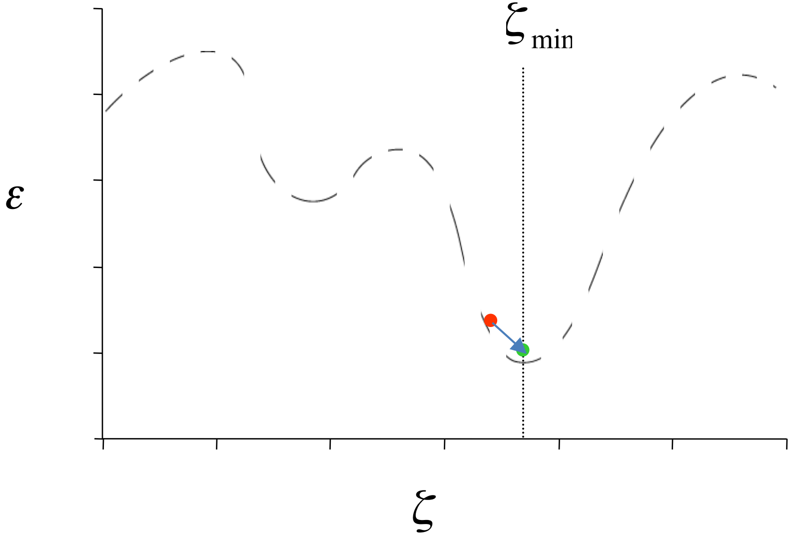
- One dimension, e.g. location along medial-lateral
    - 100 possible locations
- Two dimensions, e.g. med-lat + inf-sup
    - 100x100=10.000
- Three dimensions
    - 100x100x100 = 1.000.000 = $10^6$

- Two dipoles, each with three dimensions
    - 100x100x100x100x100x100 = $10^{12}$

# Non-linear parameters: Gradient descent method

$$\varepsilon rror(\zeta) = \sum_{i=1}^{N} \left(Y_i(\zeta) - V_i\right)^2 \quad \Rightarrow \quad \min_{\zeta}\left(\varepsilon rror(\zeta)\right)$$

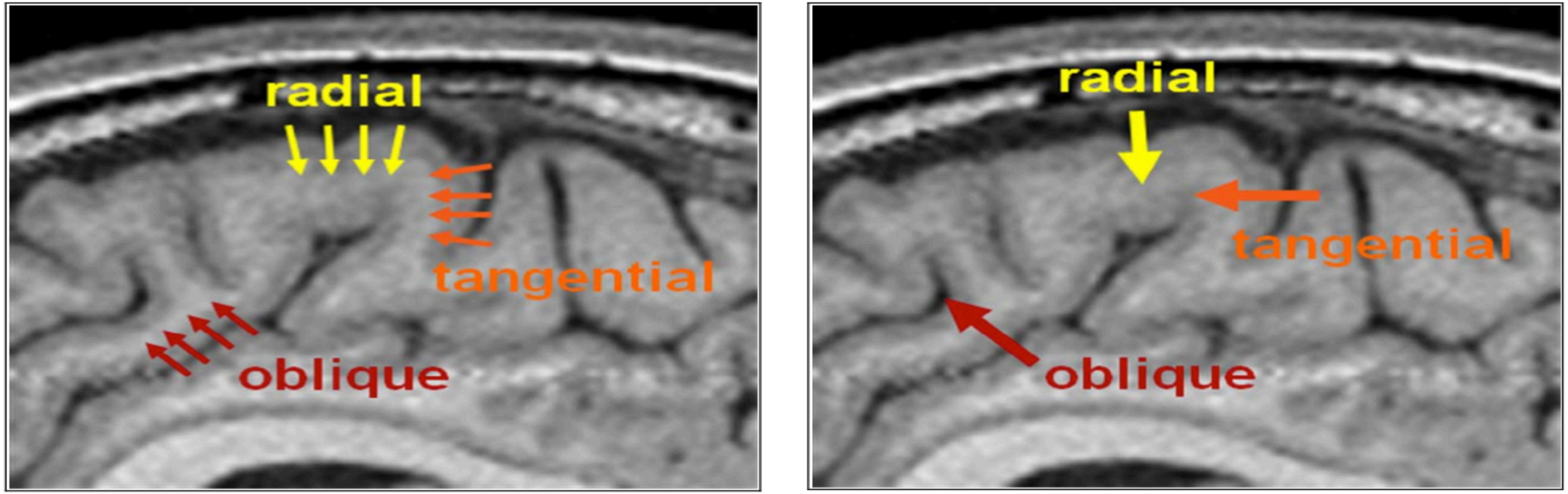
$$\zeta = a, b, c, \ldots$$



# Single or multiple dipole models - Strategies

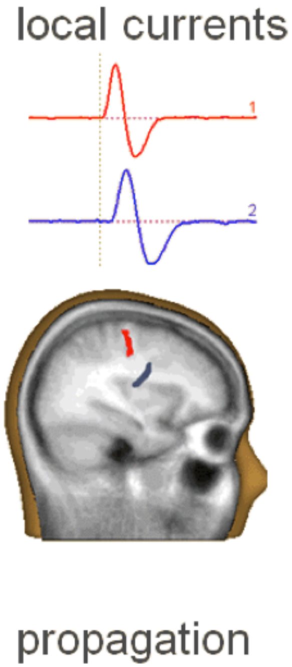
- Single dipole:
  - scan the whole brain, followed by iterative optimization
- Two dipoles:
  - scan with symmetric pair, use that as starting point for iterative optimization
- More dipoles:
  - sequential dipole fitting

# Sequential dipole fitting



BESA manual

# Sequential dipole fitting



BESA manual

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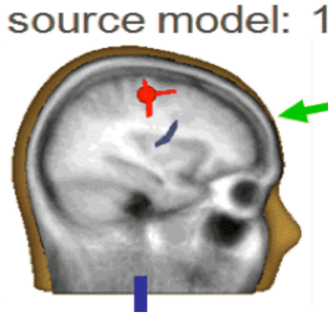

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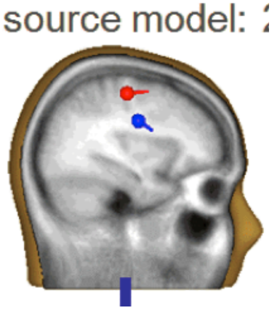
# Sequential dipole fitting



BESA manual

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## Forward modeling

Source model
Volume conductor model

## Inverse modeling

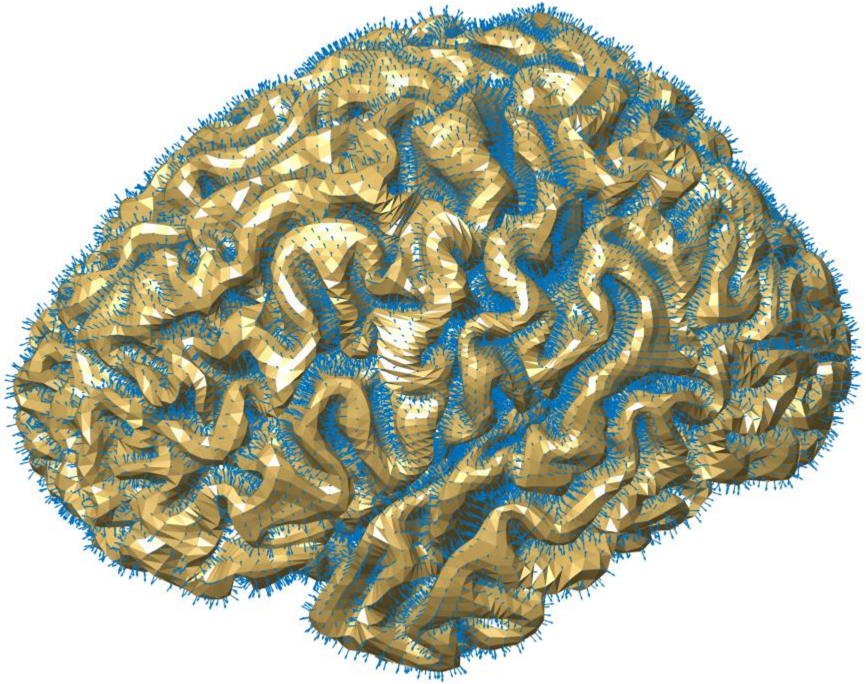
Single and multiple dipole fitting
Distributed source models
Beamforming methods

## Summary

# Distributed source model

- Position of the source is not estimated as such
  - Pre-defined grid (3D volume or on cortical sheet)
- Strength is estimated
  - In principle easy to solve, however…
  - More "unknowns" (parameters) than "knowns" (measurements)
  - Infinite number of solutions can explain the data perfectly
  - Additional constraints required
  - Linear estimation problem

# Distributed source model



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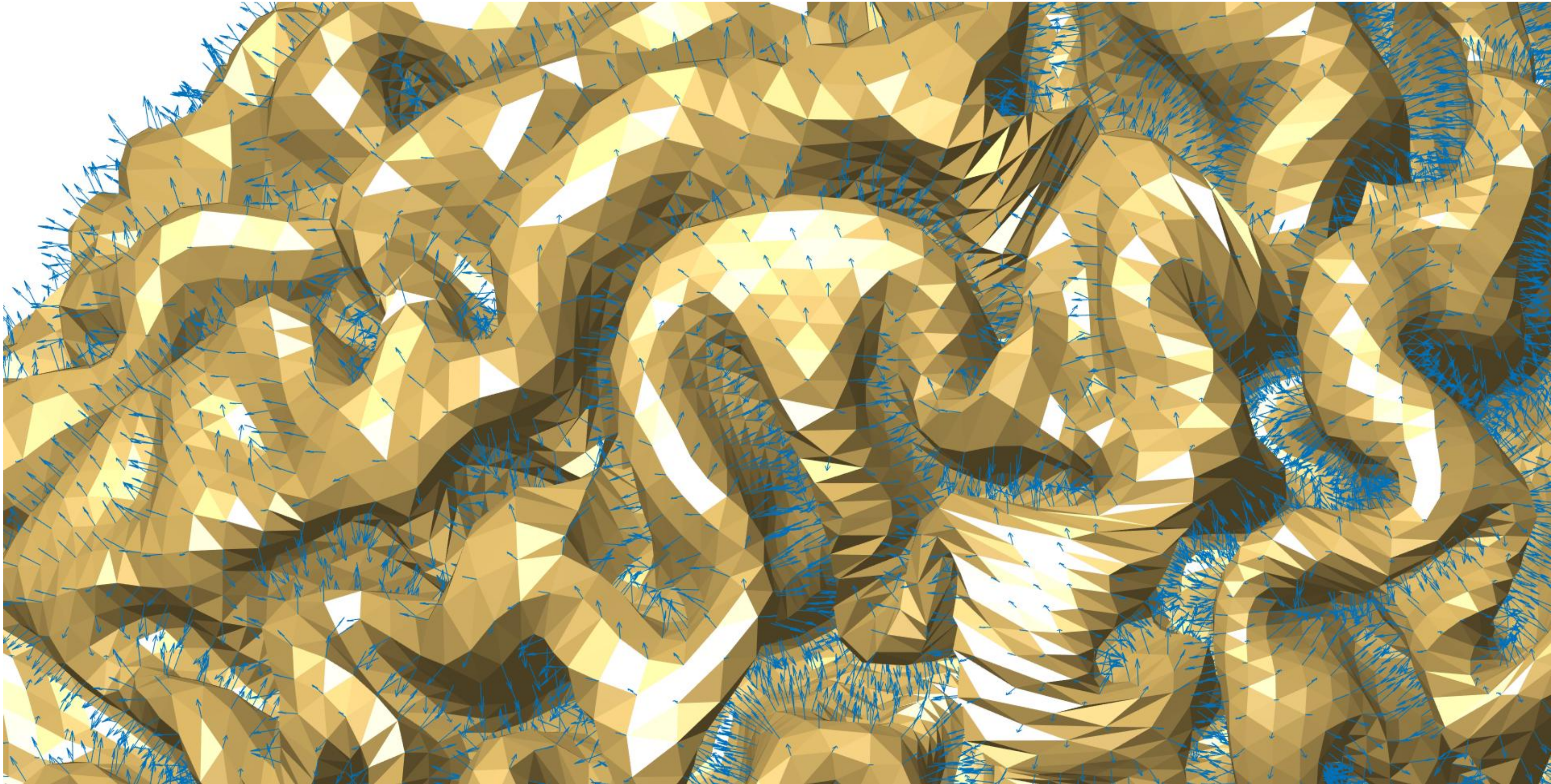


# Distributed source model: linear estimation

$$Y = G_1 q_1 + G_2 q_2 + \ldots = \begin{bmatrix} G_{1,1} & G_{2,1} & \cdots \\ G_{1,2} & G_{2,2} & \cdots \\ \vdots & \vdots & \ddots \\ G_{1,N} & G_{2,N} & \cdots \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ \vdots \end{bmatrix} = \mathrm{G} \cdot \vec{q}$$

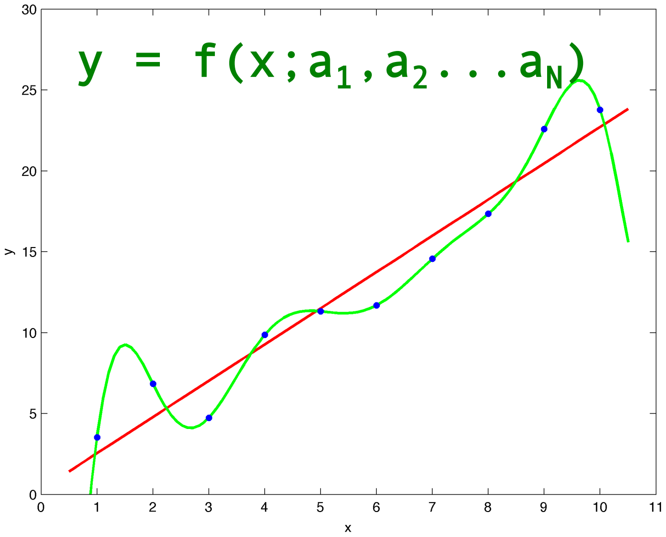
$$\vec{q} = \mathrm{G}^{-1} \cdot Y$$

# Distributed source model: linear estimation

distributed source model
with **many dipoles**
throughout the whole brain

estimate the strength of all
dipoles

data and noise can be
perfectly explained



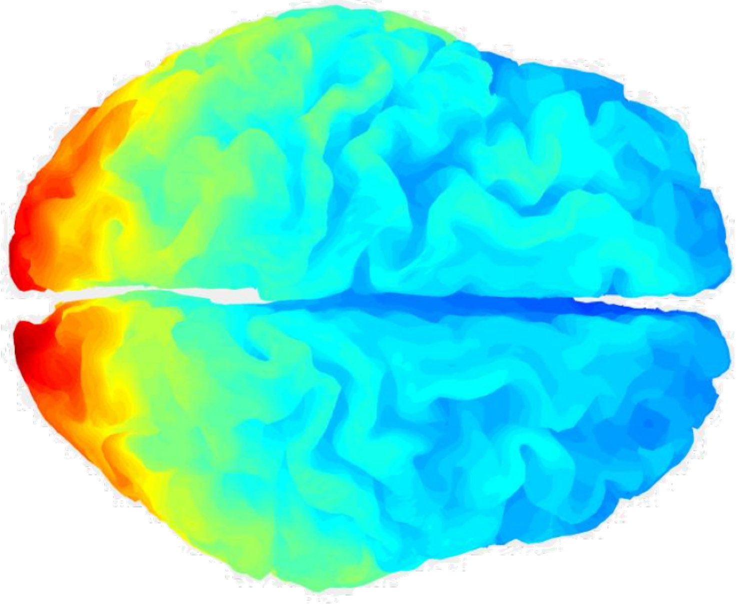
# Distributed source model: regularization

$$V = G \cdot q + Noise$$

$$\min_q \{\| V - G \cdot q \|^2\} = 0 \;\; !!$$

Regularized linear estimation:

$$\rightarrow \min_q \{\underbrace{\| V - G \cdot q \|^2}_{\text{mismatch with data}} + \lambda \cdot \underbrace{\| D \cdot q \|^2}_{\text{mismatch with prior assumptions}}\}$$



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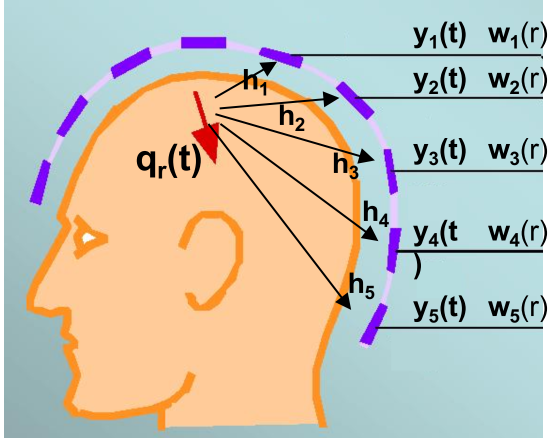
# Spatial filtering with beamforming

- Position of the source is not estimated as such
- Manipulate filter properties, not source properties
  - No explicit assumptions about source constraints (implicit: single dipole)
  - Assumption that sources that contribute to the data should be uncorrelated

# Beamformer: the question

- What is the activity of a source **q**, at a location **r**, given the data **y**?
- We estimate **q** with a spatial filter **w**



$$\hat{q}_r(t) = \mathbf{w}(r)^T \mathbf{y}(t)$$

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# Summmary 1

- Forward modelling
  - Required for the interpretation of scalp topographies
  - Different methods with varying accuracy
- Inverse modelling
  - Estimate source parameters from data
- Assumptions on source locations
  - Single or multiple point-like source
  - Distributed source
  - Spatial filtering

# Summary 2

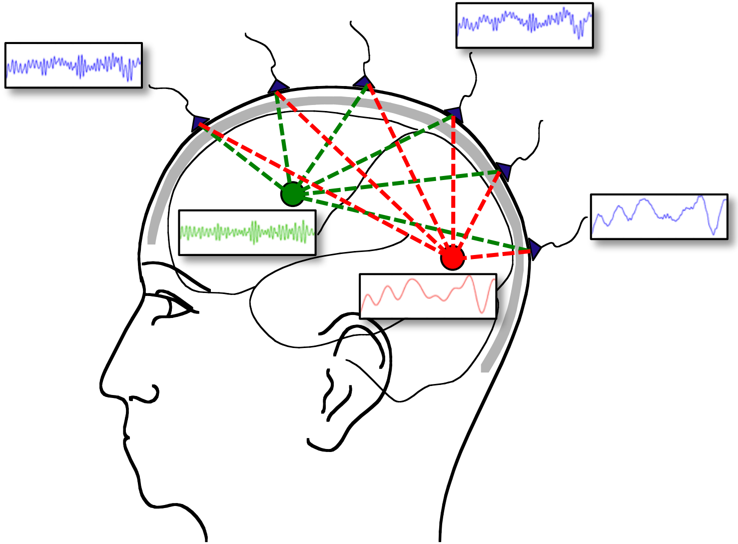
- Source analysis is not only about the “where” but also about untangling the “what” and “when”

spatial distribution of activity over the head
-> source reconstruction

timecourse of activity
-> ERP

spectral characteristics
-> power spectrum

temporal changes in power
-> time-frequency response (TFR)



# Forward and Inverse Modeling of EEG and MEG data

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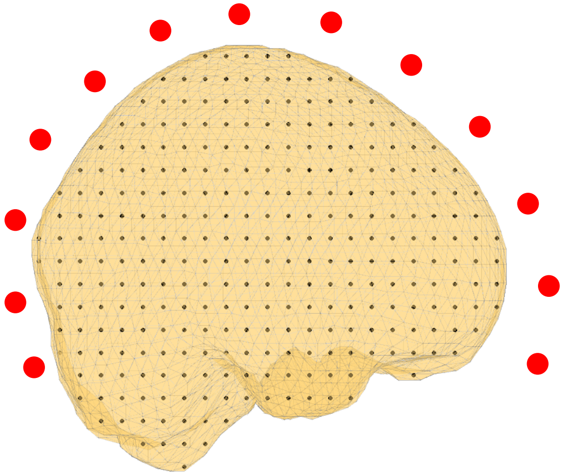
Donders Institute, Radboud University, Nijmegen, NL

# Forward and Inverse Modeling of EEG and MEG data
# Back-up

# Single dipole solution with dipole fit



$X_1$ = 100
$X_2$ = 10
$X_3$ = 15
$X_4$ = 20
$X_5$ = 1
…
$X_n$ = 3

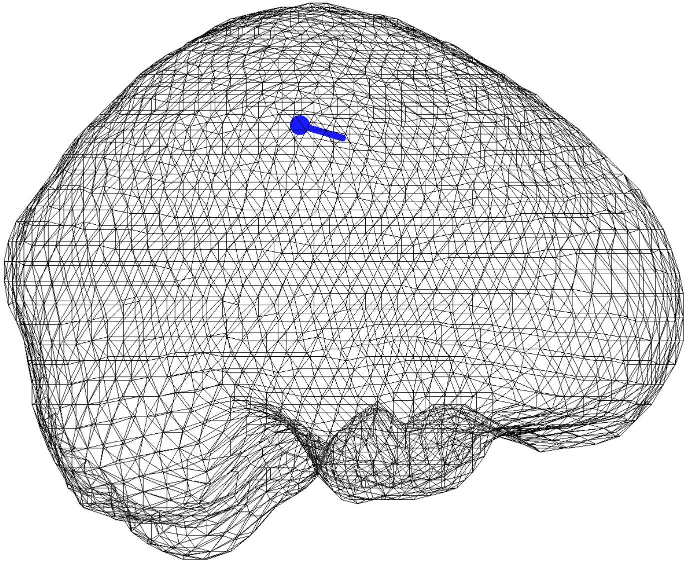
→

Further improvement
with
gradient descent

# Single dipole solution with dipole fit

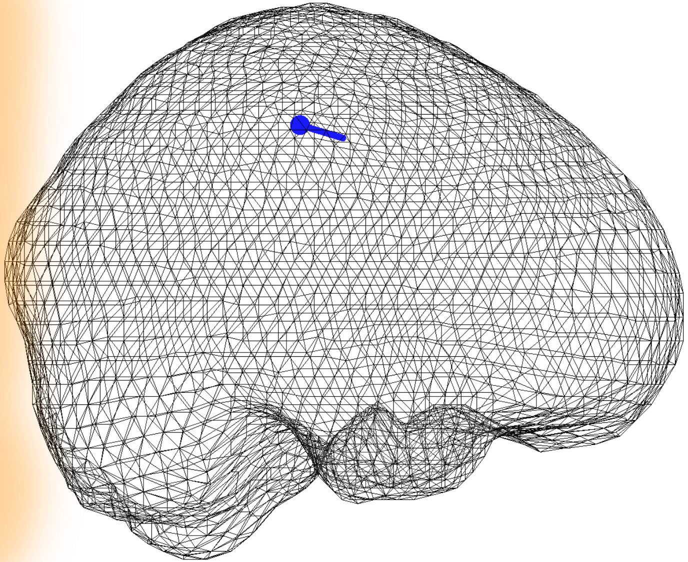
ft_dipolefitting

This uses the functional and anatomical data to perform the dipole fit



# Single dipole solution with dipole fit

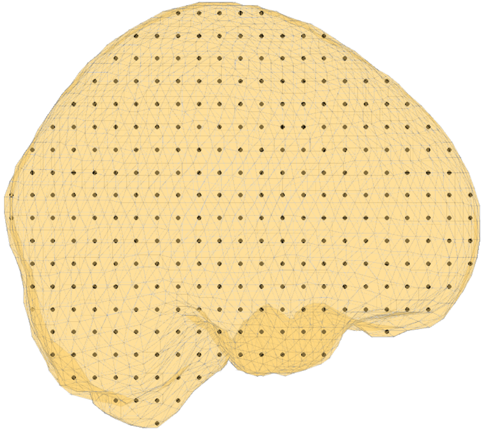
```
dipfit_bem.dip

ans =

     pos: [13.958237048680118 34.388465910583285 97.809684095
     mom: [3x1 double]
     pot: [74x1 double]
      rv: 0.034549469532012
    unit: 'mm'
```



residual variance (rv) lies between 0 and

# Distributed sources

- We are not looking for a single dipole location, but a distributed source

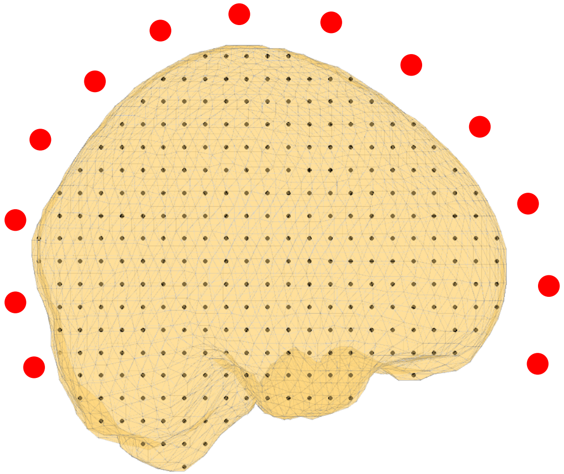


# Distributed sources

*Y* is the measured data
A is our model
x is the distributed source model
λ is the regularisation parameter
D is a operator

$$\min_x\{\underbrace{||Y-Ax||_2}_{\text{mismatch with data}} + \underbrace{\lambda||Dx||_2}_{\text{mismatch with prior assumptions}}\}$$

# Distributed sources



$$\left.\begin{array}{l} X_1 = 100 \\ X_2 = 10 \\ X_3 = 15 \\ X_4 = 20 \\ X_5 = 1 \\ \ldots \\ X_n = 3 \end{array}\right\}$$

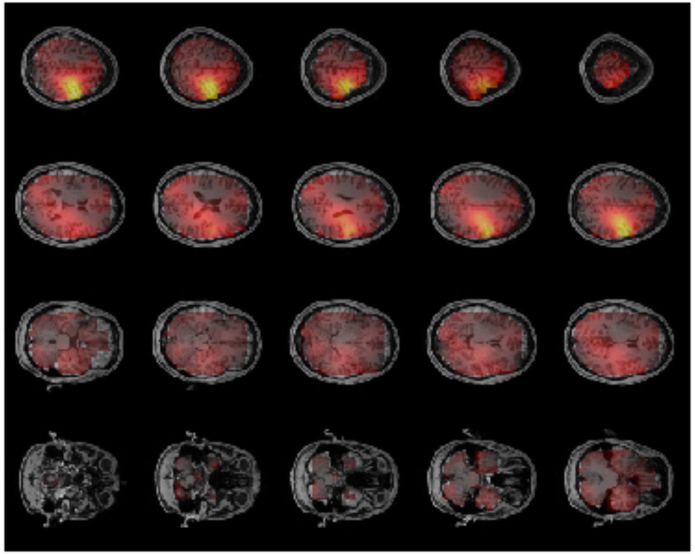
All at the same time

+

regularization

# Single dipole solution with dipole fit

Cfg.mne. …
ft_dipolefitting(cfg,…)
This uses the functional and anatomical data to perform mne



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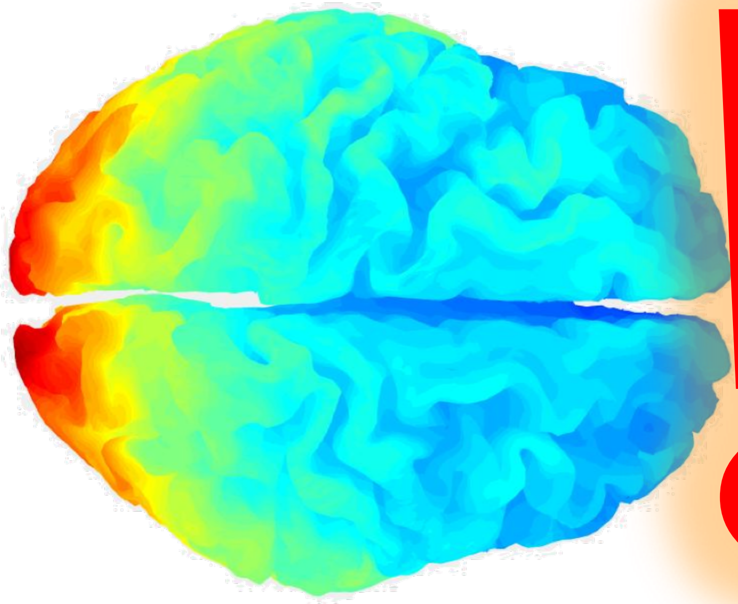

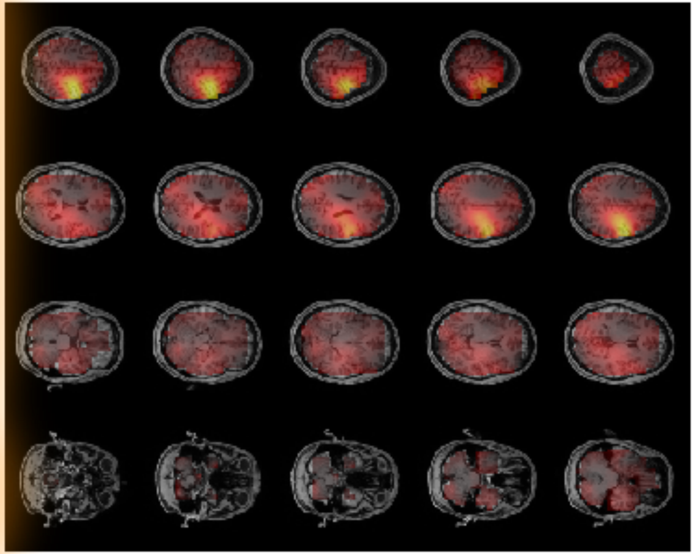

# Summary

## Single and multiple dipole models

Minimize error between model and measured potential

## Distributed source models

Perfect fit of model to the measured potential
Additional constraint on source smoothness, power or amplitude

## Forward model

Influences the inverse solution
Take care of parameters

# Forward and Inverse Modeling of EEG and MEG data

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# Superposition of source activity