Forward and Inverse Modeling of EEG and MEG data

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Overview

- Motivation and background
- Forward modeling
  - Source model
  - Volume conductor model
- Inverse modeling - biophysical models
  - Single and multiple dipole fitting
  - Distributed source models
  - Beamforming methods
- Inverse modeling - independent components
- Summary

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Motivation 1

Strong points of EEG and MEG

- Temporal resolution (~1 ms)
- Characterize individual components of ERP
- Oscillatory activity
- Disentangle dynamics of cortical networks

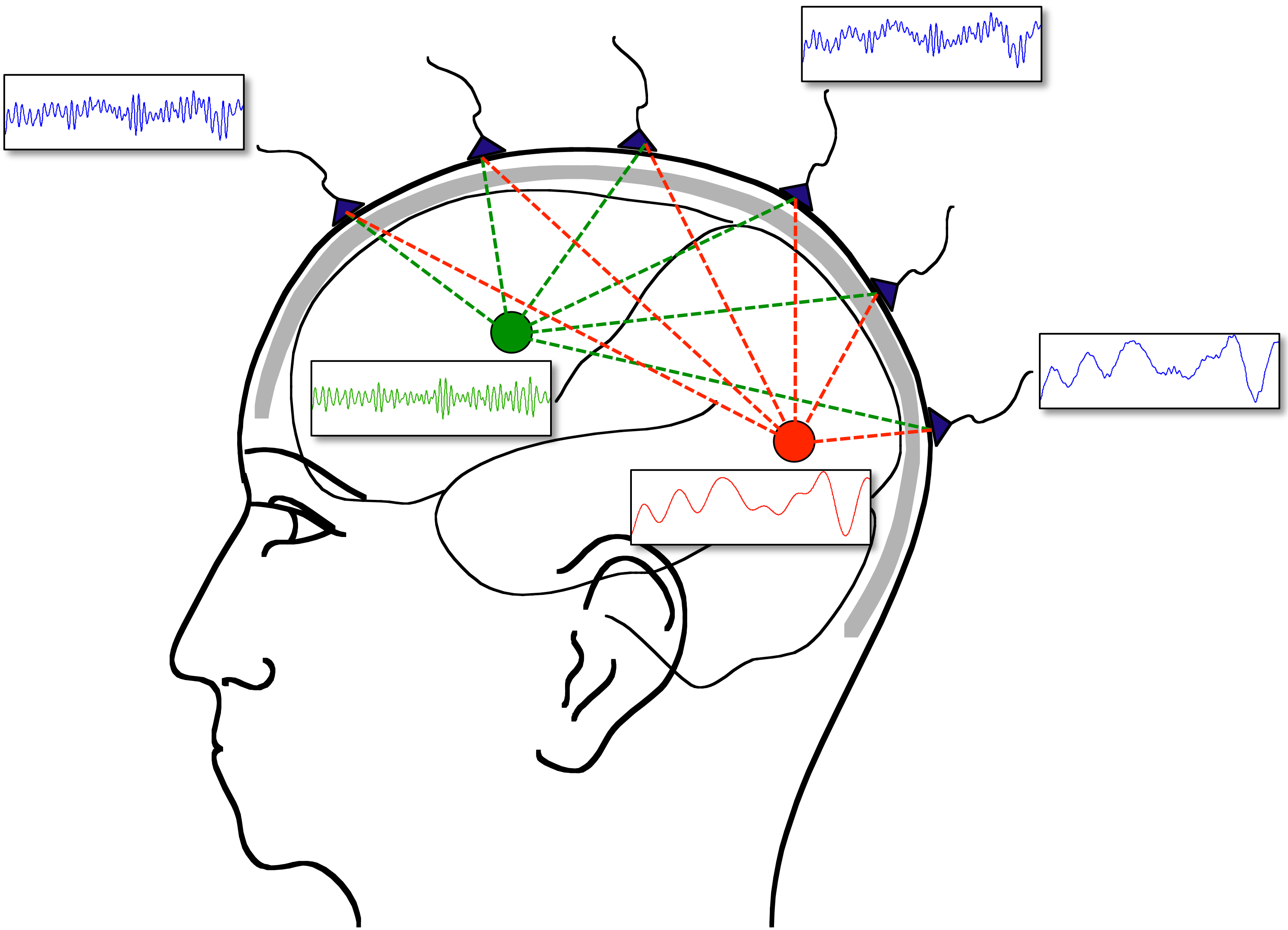
Weak points of EEG and MEG

- Measurement on outside of brain
- Overlap of components
- Low spatial resolution

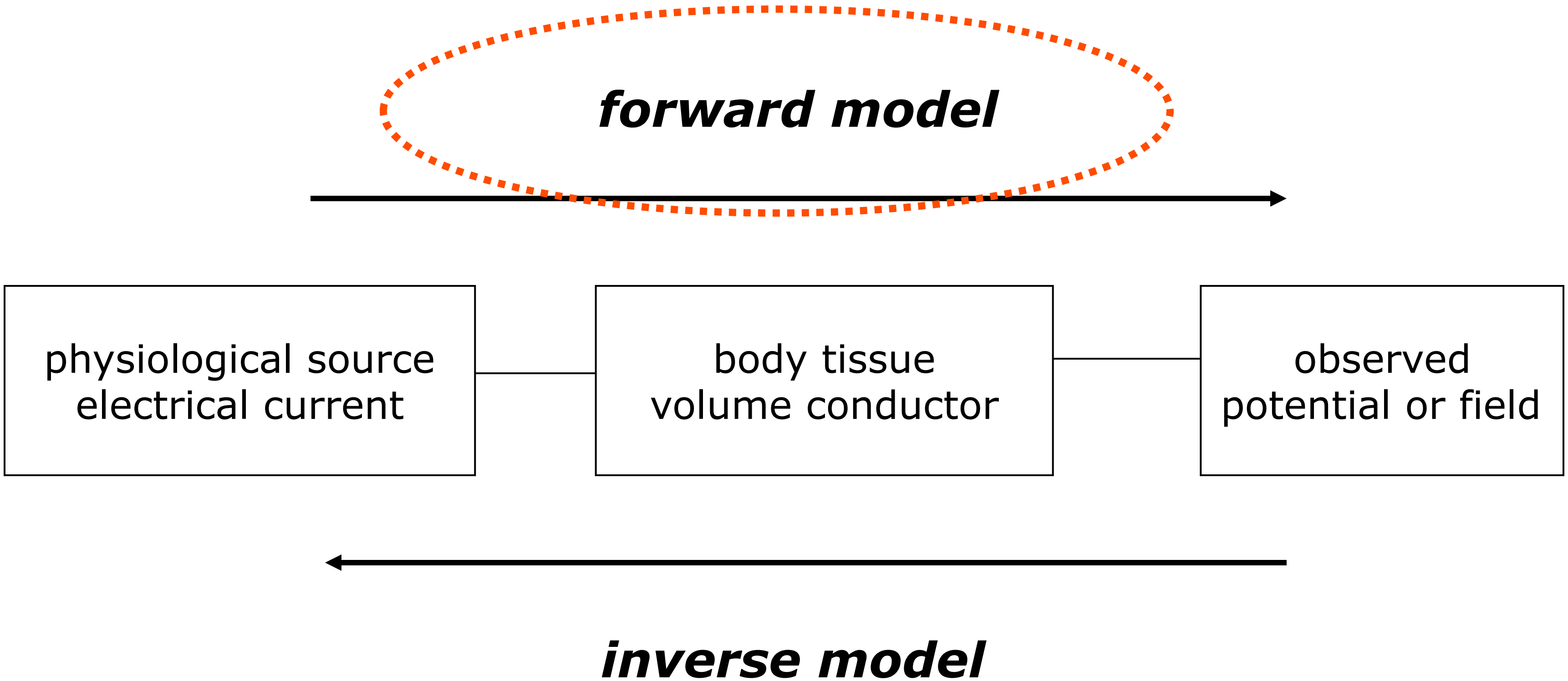
Motivation 2

- If you find a ERP/ERF component, you want to characterize it in physiological terms
  - Time or frequency are the “natural” characteristics
  - “Location” requires interpretation of the scalp topography
- Forward and inverse modeling helps to interpret the topography
- Forward and inverse modeling helps to disentangle overlapping source timeseries

Superposition of source activity



Biophysical source modelling: overview



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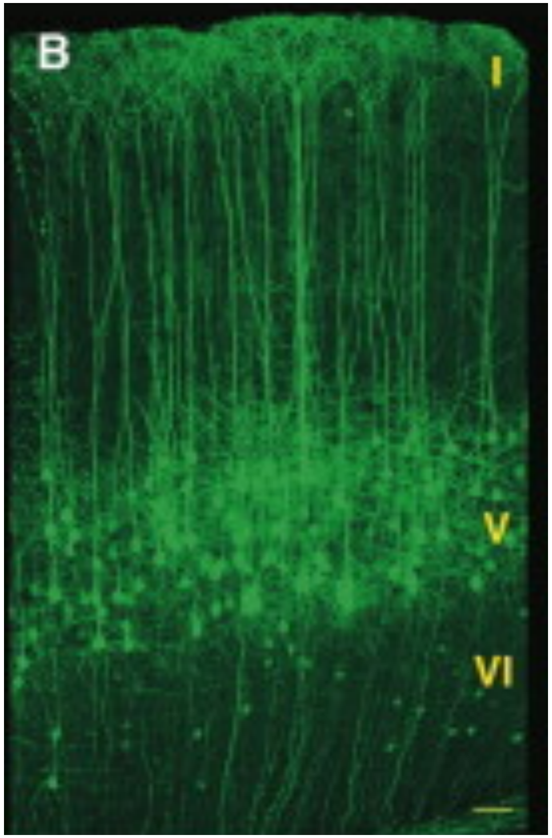
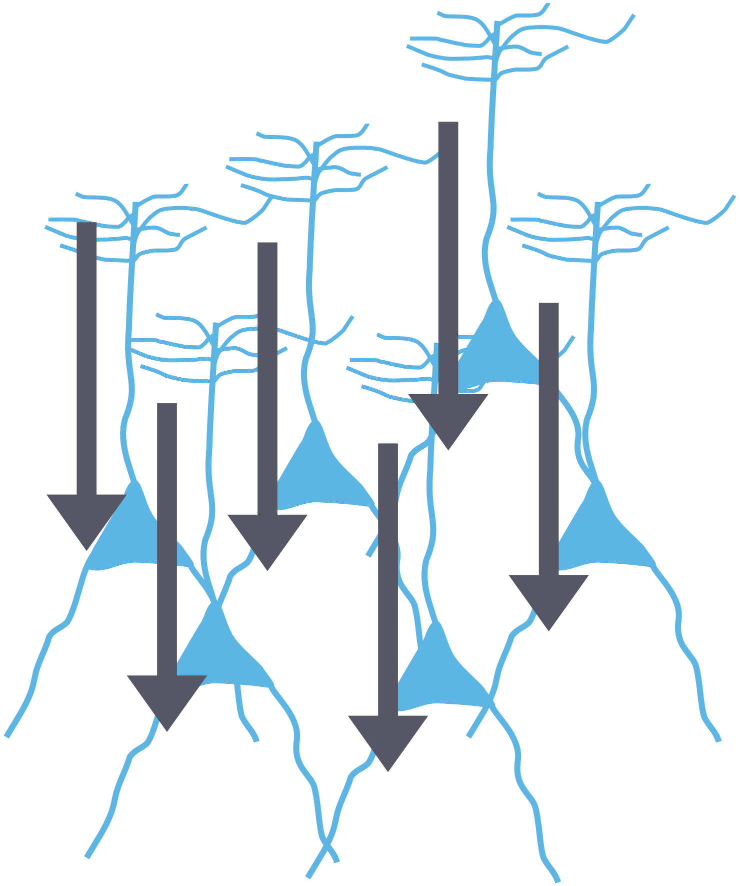
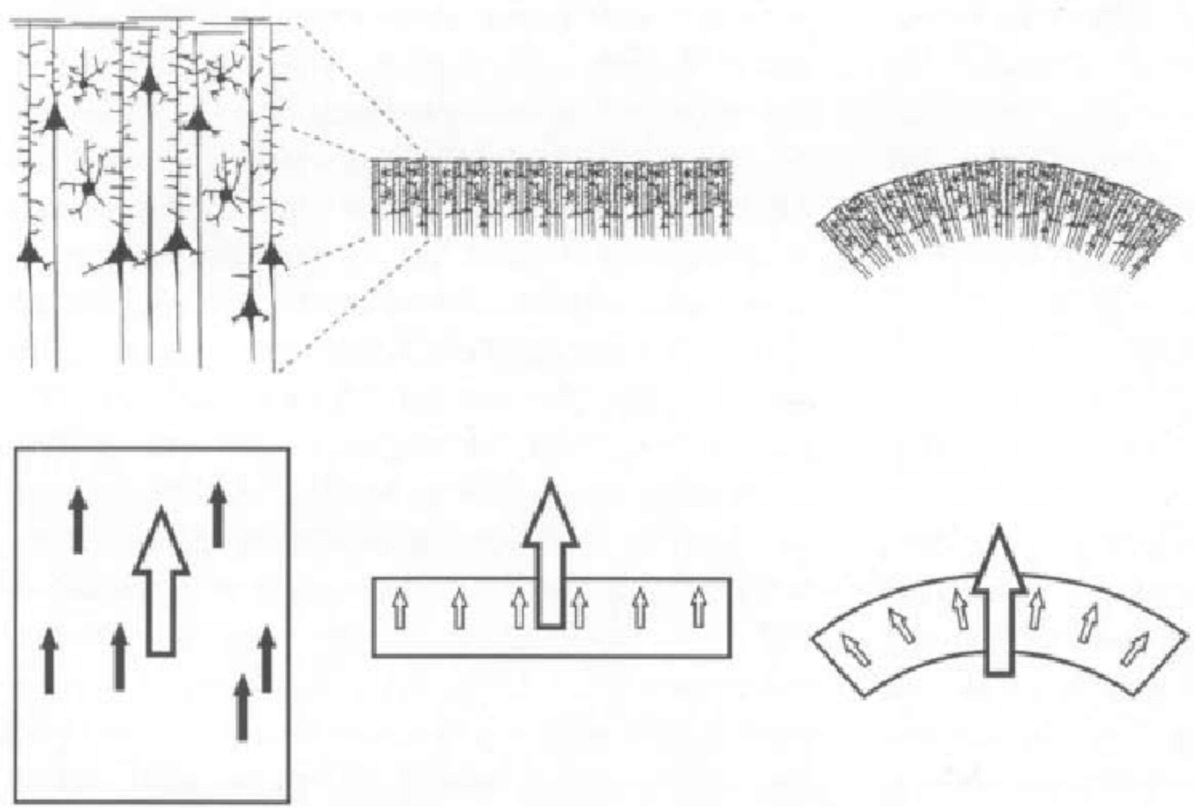
What produces the electric current





Equivalent current dipoles



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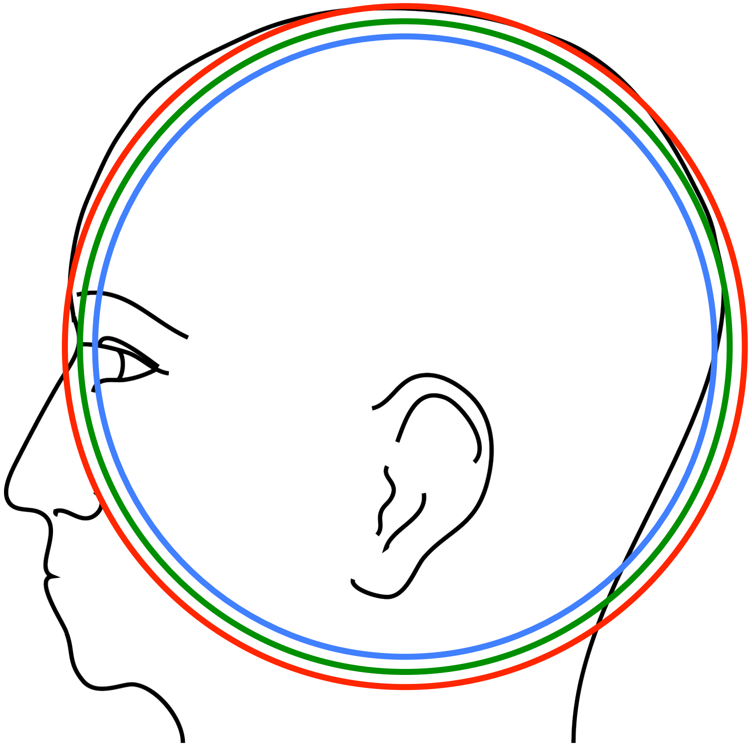
Volume conductor

described electrical
properties of tissue

describes geometrical
model of the head

describes **how** the currents
flow, not where they
originate from

same volume conductor for
EEG as for MEG, but also
for tDCS, tACS, TMS, ...



Volume conductor

Computational methods for volume conduction problem that allow for realistic geometries

BEM	*Boundary Element Method*
FEM	*Finite Element Method*
FDM	*Finite Difference Method*

Volume conductor: Boundary Element Method

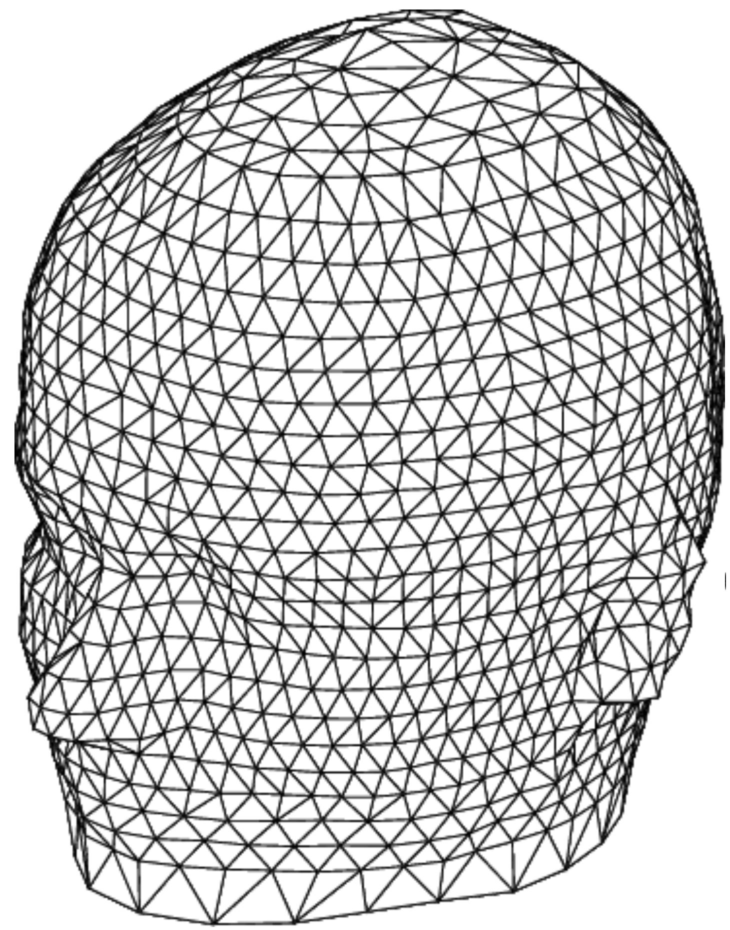
Each compartment is

- homogenous
- isotropic

Important tissues

- skin
- skull
- brain
- (CSF)

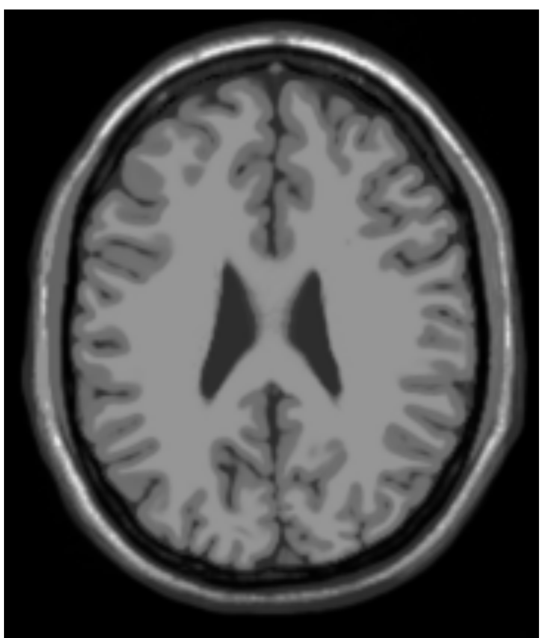
Triangulated surfaces describe boundaries

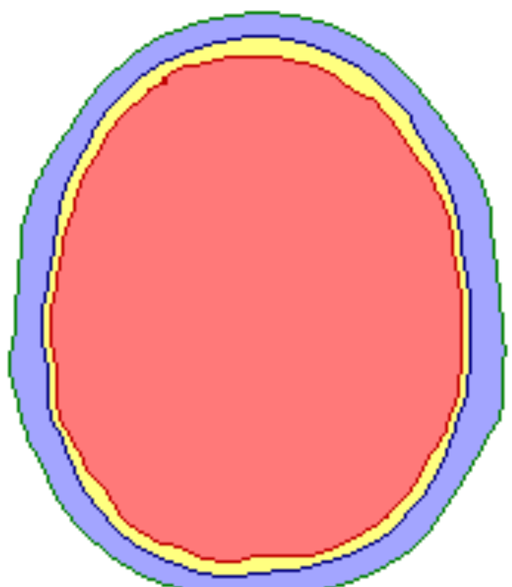


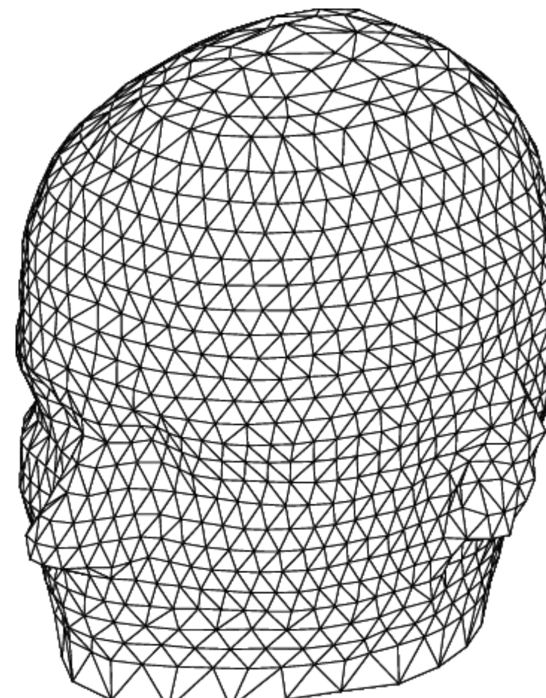
Volume conductor: Boundary Element Method

Construction of geometry

- segmentation in different tissue types
- extract surface description
- downsample to reasonable number of triangles





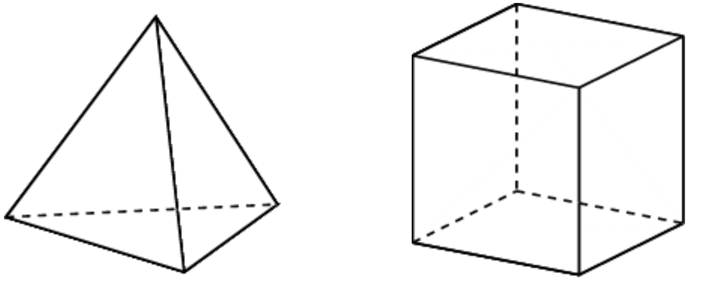


Volume conductor: Boundary Element Method

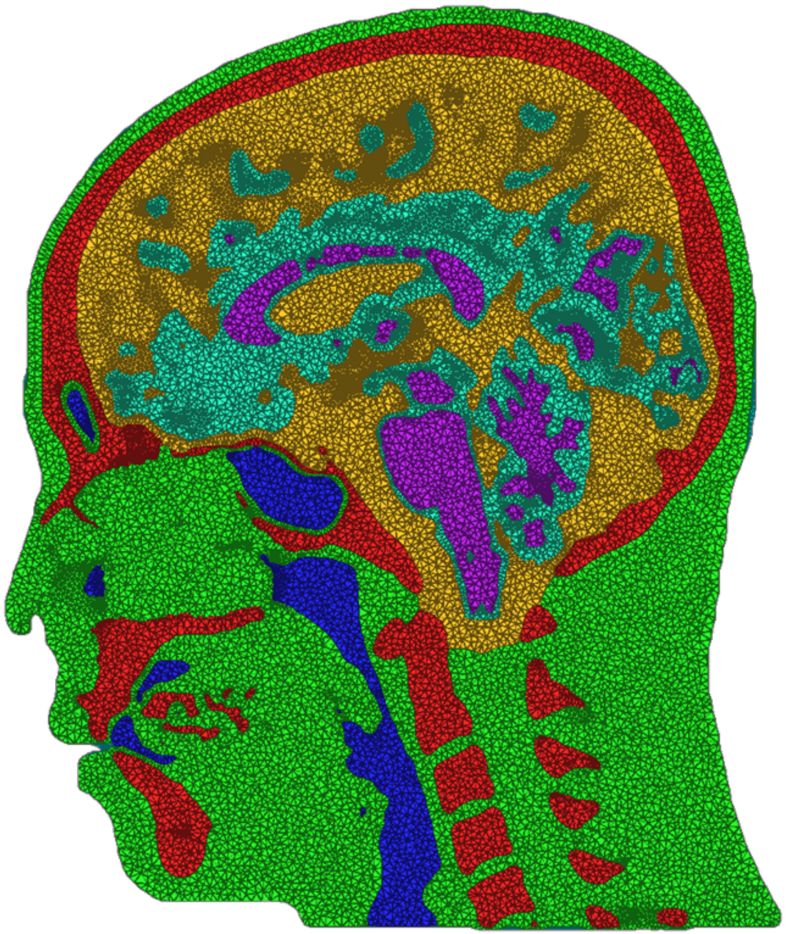
- Construction of geometry
  - segmentation in different tissue types
  - extract surface description
  - downsample to reasonable number of triangles
- Computation of model
  - independent of source model
  - only one lengthy computation
  - fast during application to real data
- Can also include more complex geometrical details
  - ventricles
  - holes in skull

Volume conductor: Finite Element Method

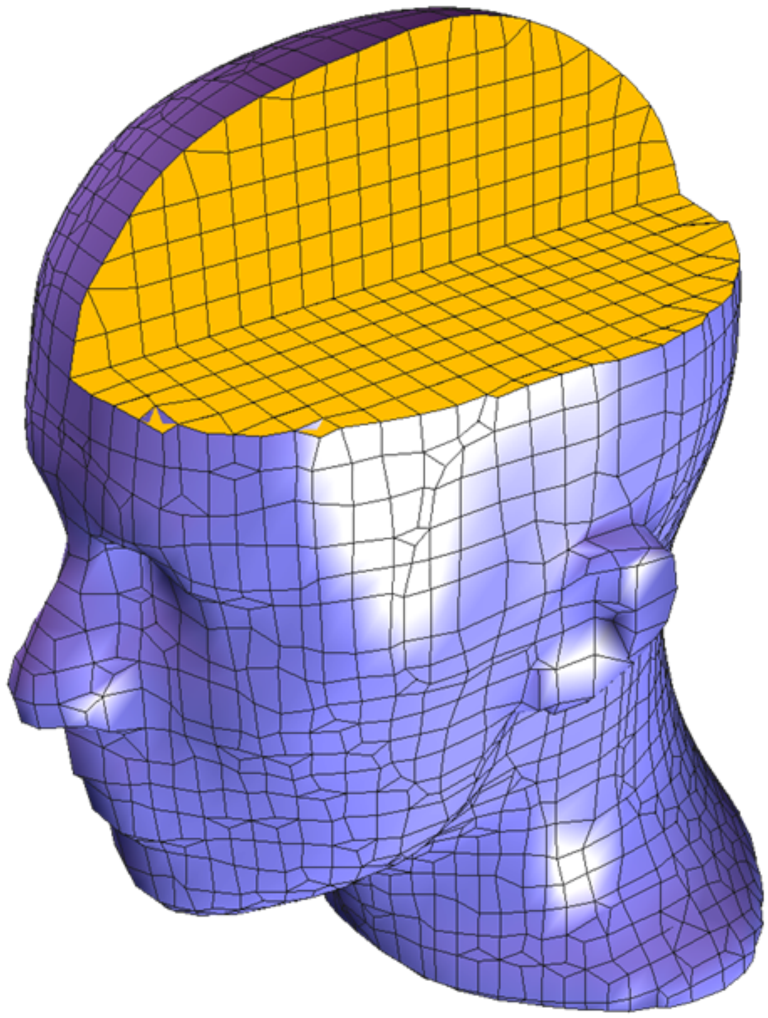
Tesselation of 3D volume in
tetraeders or hexaheders



Volume conductor: Finite Element Method



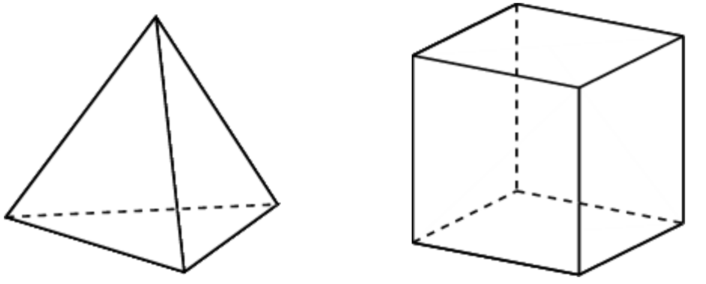
tetraeders



hexaheders

Volume conductor: Finite Element Method

Tesselation of 3D volume in tetraeders or hexaheders



Each element can have its own conductivity

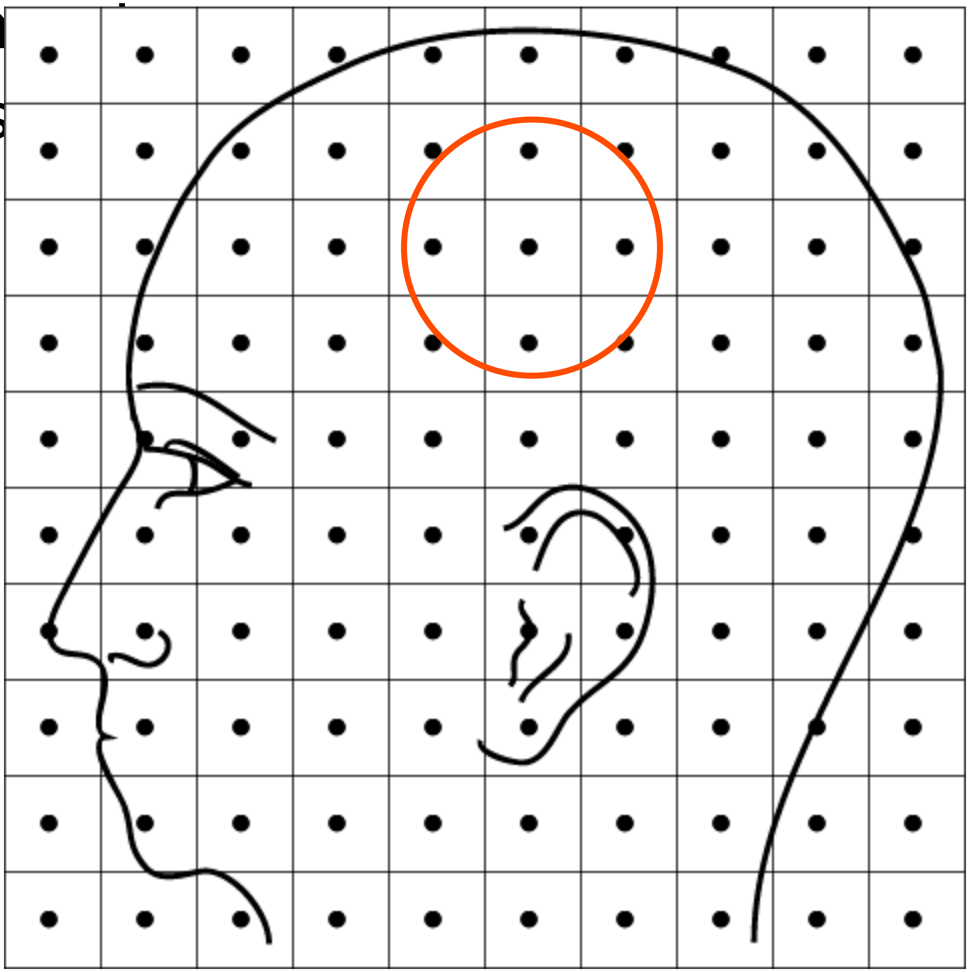
FEM is the most accurate numerical method but computationally quite expensive

Geometrical processing not as simple as BEM

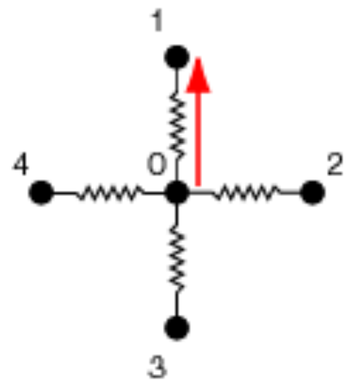
Volume conductor: Finite Difference Method

Easy to con

Not very us



Volume conductor: Finite Difference Method



$$\left.\begin{array}{l} I_1 + I_2 + I_3 + I_4 = 0 \\ V = I*R \end{array}\right\} \Longrightarrow$$

$$\Delta V_1/R_1 + \Delta V_2\,/R_2 + \Delta V_3\,/R_3 + \Delta V_4\,/R_4 = 0 \quad \Longrightarrow$$

$$(V_1\text{-}V_0)/R_1 + (V_2\text{-}V_0)/R_2 + (V_3\text{-}V_0)/R_3 + (V_4\text{-}V_0)/R_4 = 0$$

Volume conductor: Finite Difference Method

Unknown potential Vi at each node

Linear equation for each node

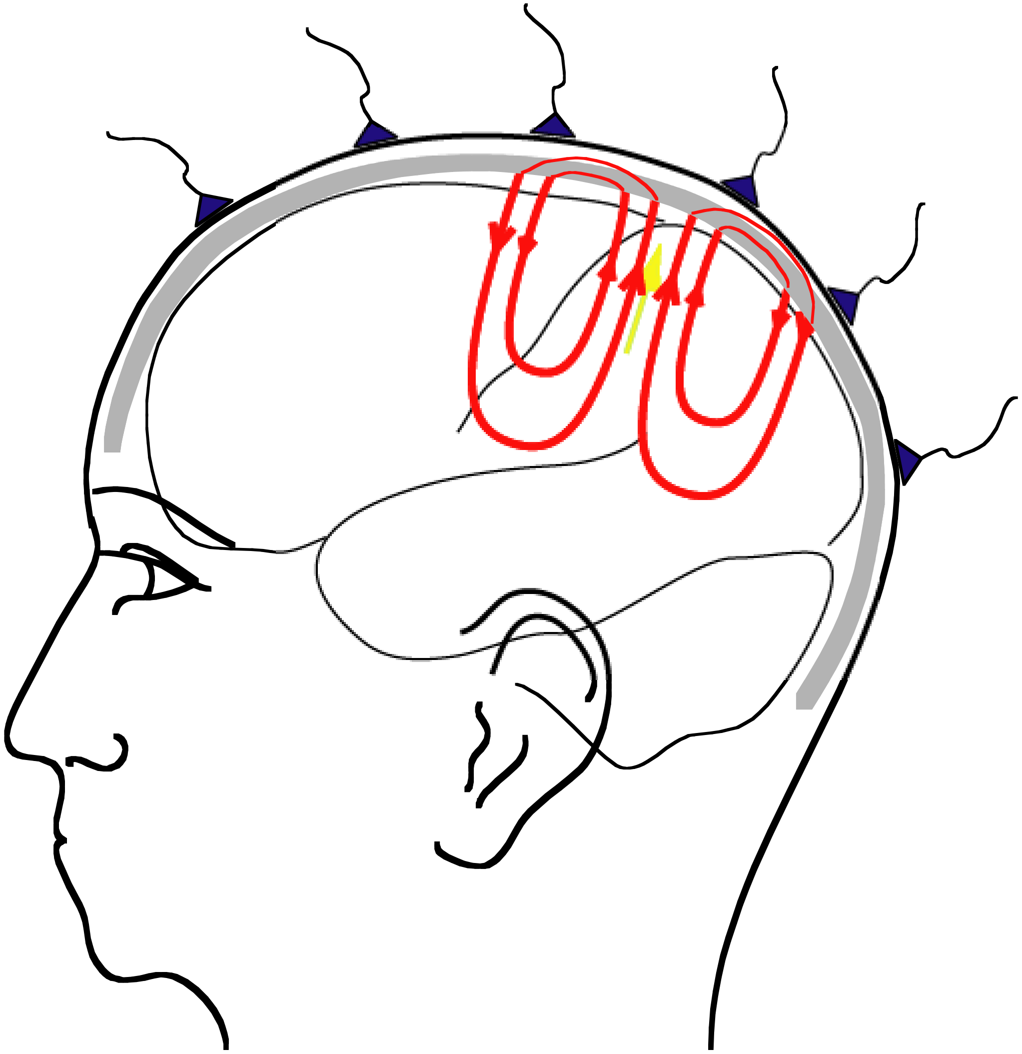
- approx. 100x100x100 = 1.000.000 linear equations
- just as many unknown potentials

Add a source/sink

- sum of currents is zero for all nodes, except
- sum of current is I+ for a certain node
- sum of current is I- for another node

Solve for unknown potential

EEG volume conduction



EEG volume conduction

Potential difference between electrodes corresponds to current flowing through skin

Only tiny fraction of current passes through skull

Therefore the model should describe the skull and skin as accurately as possible

MEG volume conduction

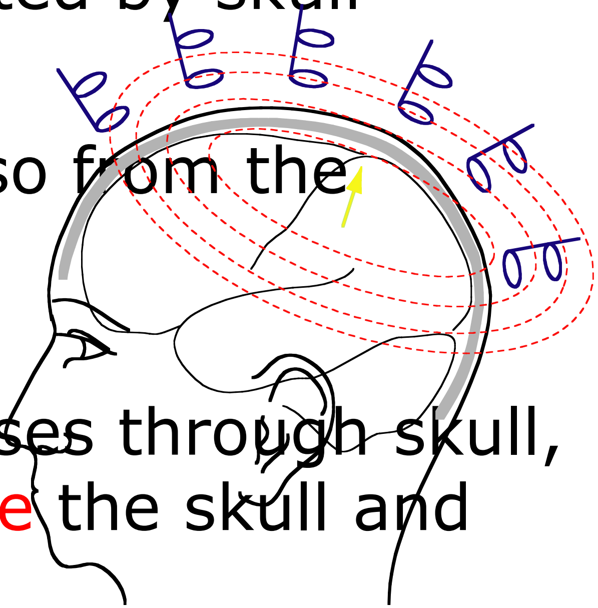
MEG measures magnetic field over the scalp

M istorted by skull

M t also from the

O passes through skull,

gnore the skull and



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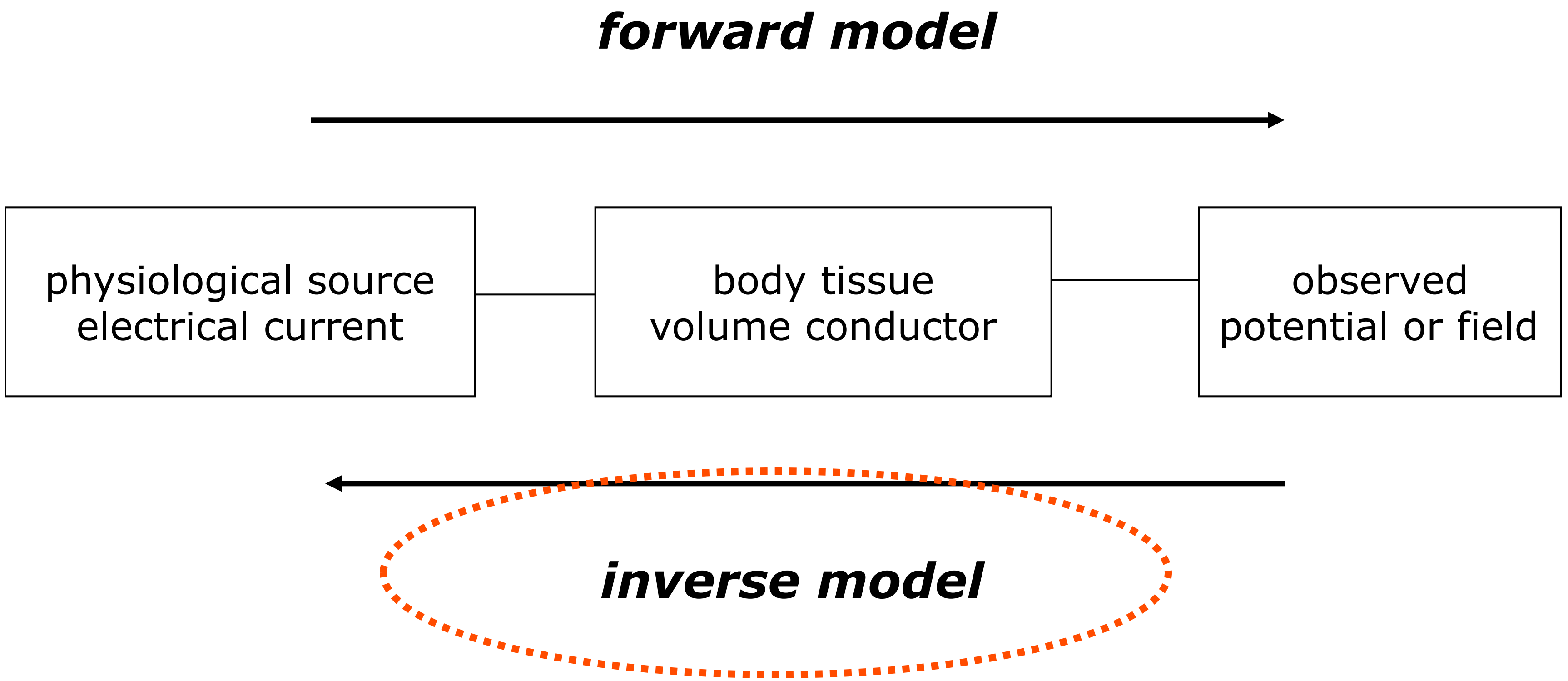
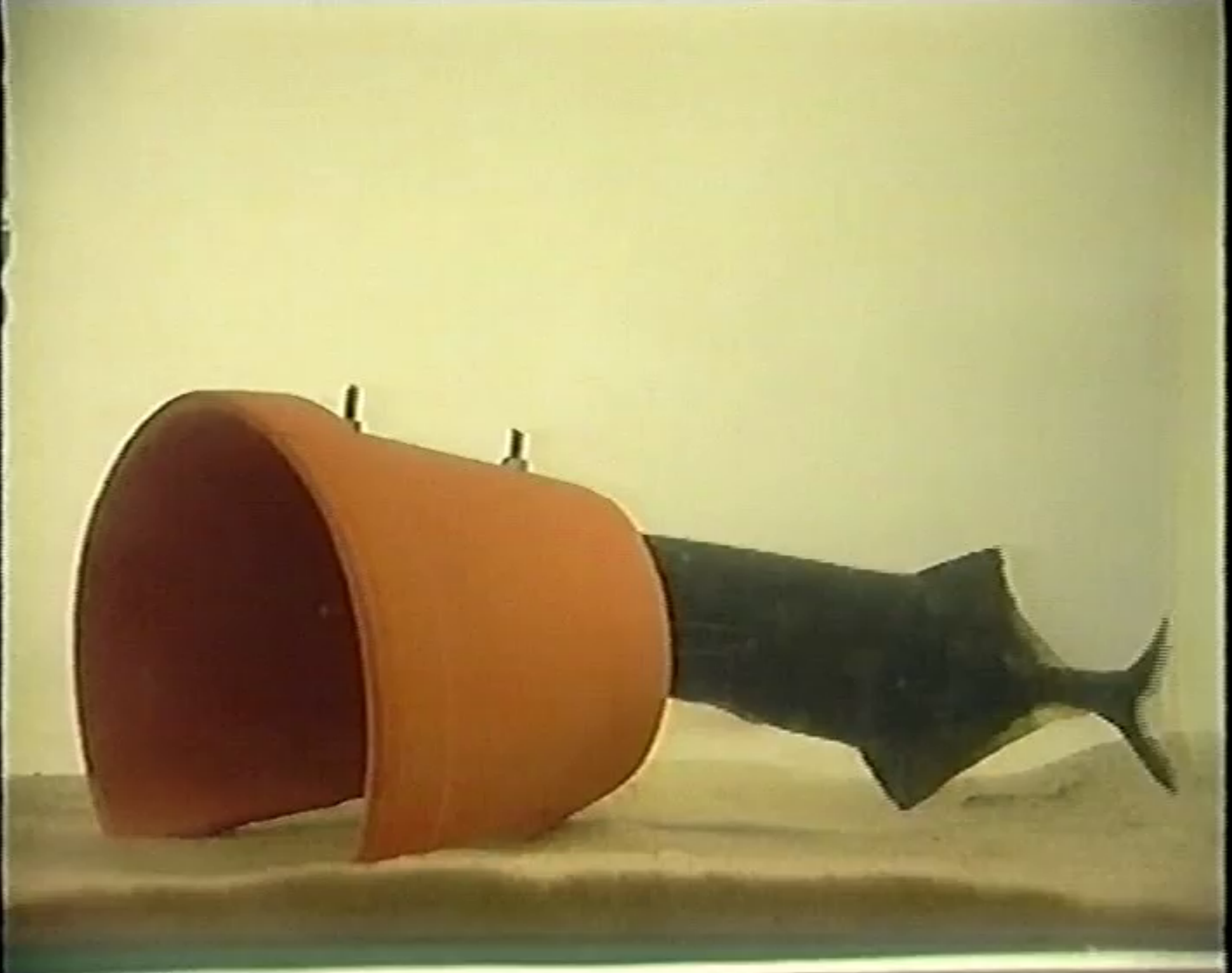


Inverse localization: demo



Inverse methods

Single and multiple dipole models

Minimize error between model and measured potential/field

Distributed source models

Perfect fit of model to the measured potential/field

Additional constraint on source smoothness, power or amplitude

Spatial filtering

Scan the whole brain with a single dipole and compute the filter output at every location

Beamforming (e.g. LCMV, SAM, DICS)

Multiple Signal Classification (MUSIC)

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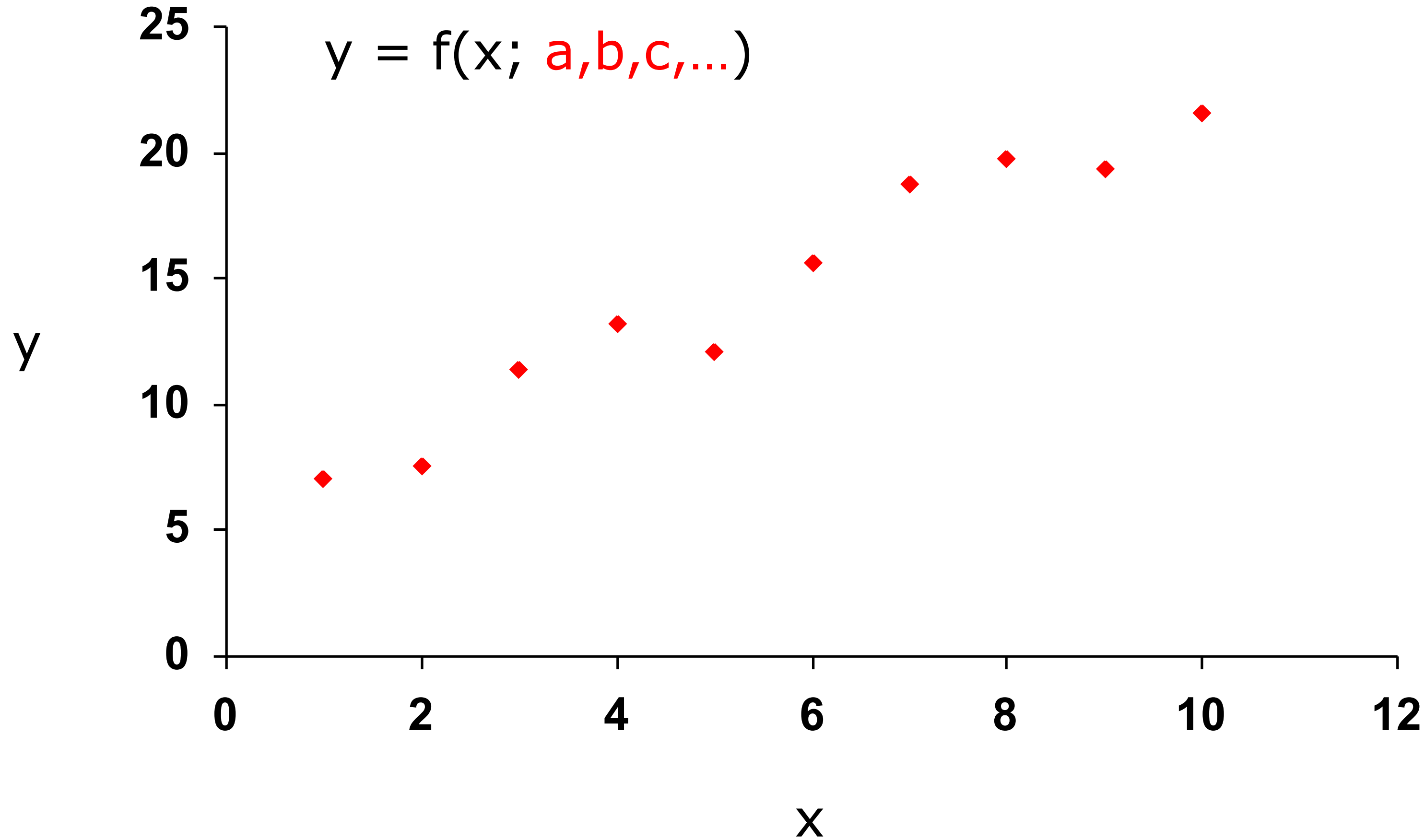
Single or multiple dipole models - Parameter estimation



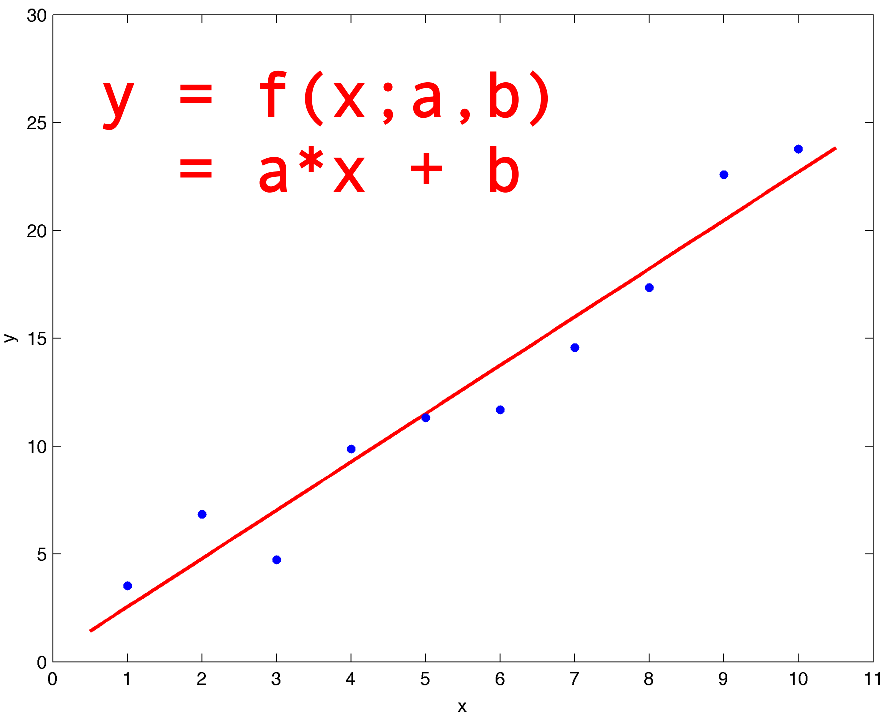
Parameter estimation: dipole parameters

source model with few parameters

- position
- orientation
- strength

compute the model data

minimize difference between actual and model data



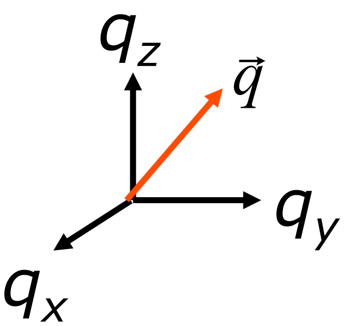
Linear parameters: superposition of sources

three sources with parameters ζ_1, ζ_2 and ζ_3

$$\left.\begin{array}{l} Y(\zeta_1) \\[2ex] Y(\zeta_2) \\[2ex] Y(\zeta_3) \end{array}\right\} \quad Y_{combined} = Y(\zeta_1) + Y(\zeta_2) + Y(\zeta_3)$$

Linear parameters: estimation

$$Y = G_x q_x + G_y q_y + G_z q_z = \begin{bmatrix} G_{x,1} & G_{y,1} & G_{z,1} \\ G_{x,2} & G_{y,2} & G_{z,2} \\ \vdots & \vdots & \vdots \\ G_{x,N} & G_{y,N} & G_{z,N} \end{bmatrix} \cdot \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} = \mathrm{G} \cdot \vec{q}$$



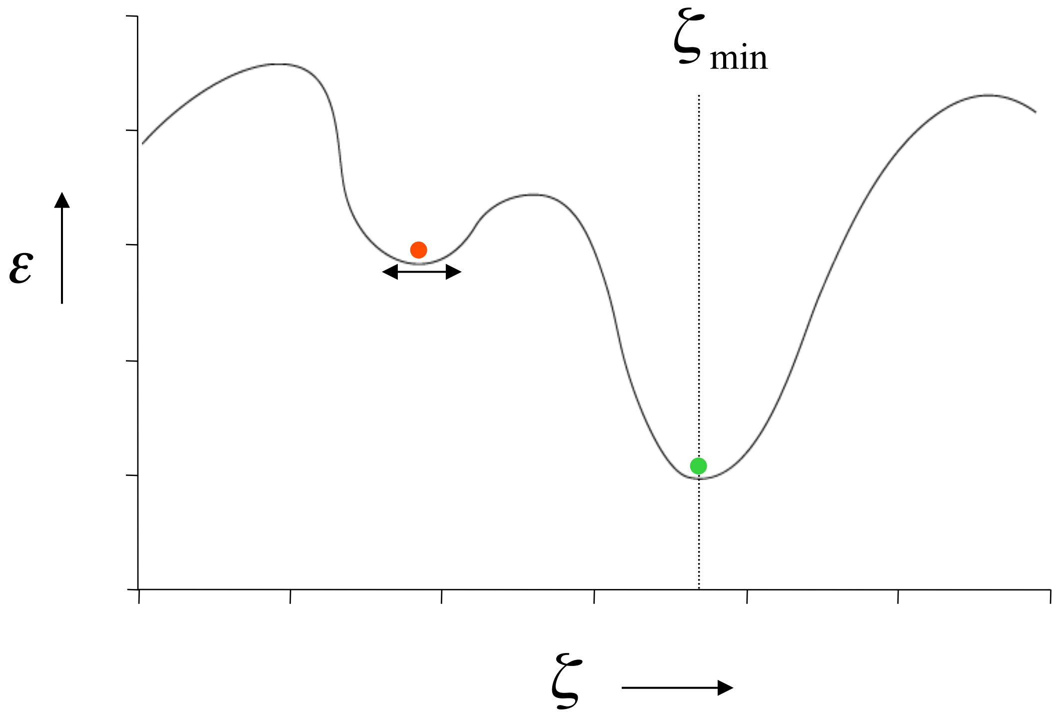
$$\begin{aligned} Y &= \mathrm{G} \cdot \vec{q} \\ &= \mathrm{G}(\zeta) \cdot \vec{q} \end{aligned}$$

$$\vec{q} = \mathrm{G}^{-1} \cdot Y$$

Non-linear parameters

$$\varepsilon rror(\zeta) = \sum_{i=1}^{N} \left(Y_i(\zeta) - V_i\right)^2 \quad \Rightarrow \quad \min_{\zeta}\left(\varepsilon rror(\zeta)\right)$$

$$\zeta = a, b, c, \ldots$$



Non-linear parameters: grid search

One dimension, e.g. location along medial-lateral

100 possible locations

Two dimensions, e.g. med-lat + inf-sup

100x100=10.000

Three dimensions

100x100x100 = 1.000.000 = 10^6

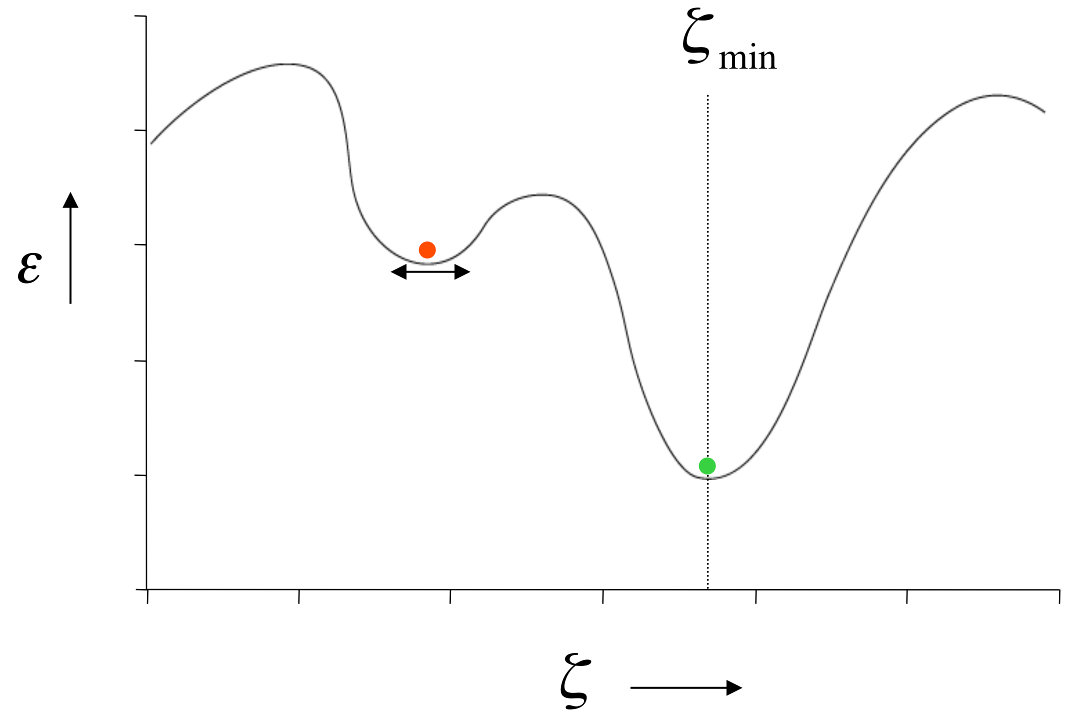
Two dipoles, each with three dimensions

100x100x100x100x100x100 = 10^{12}

Non-linear parameters: gradient descent optimization

$$\varepsilon rror(\zeta) = \sum_{i=1}^{N} \left(Y_i(\zeta) - V_i\right)^2 \quad \Rightarrow \quad \min_{\zeta}\left(\varepsilon rror(\zeta)\right)$$

$$\zeta = a, b, c, \ldots$$



Single or multiple dipole models - Strategies

Single dipole:

scan the whole brain, followed by iterative optimization

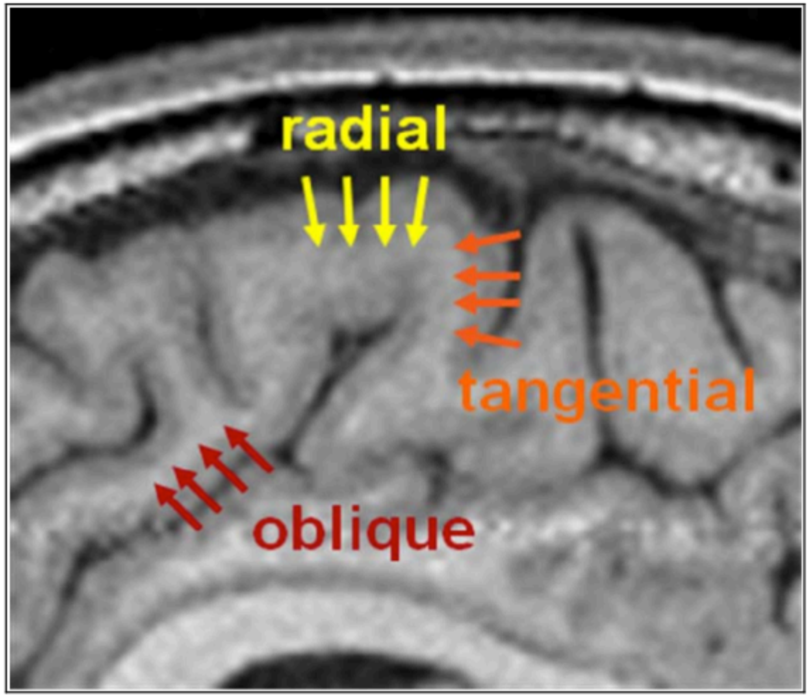
Two dipoles:

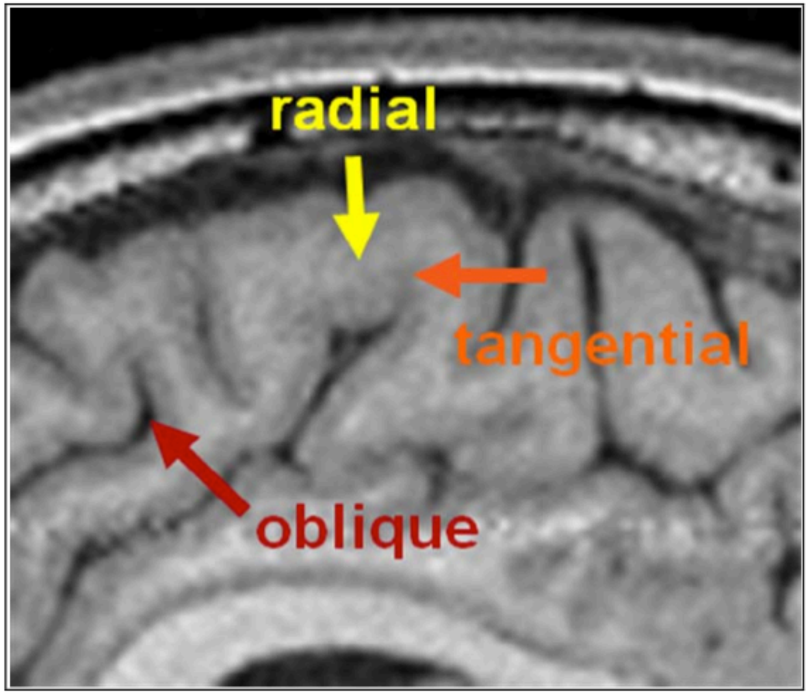
scan with symmetric pair, use that as starting point for iterative optimization

More dipoles:

sequential dipole fitting

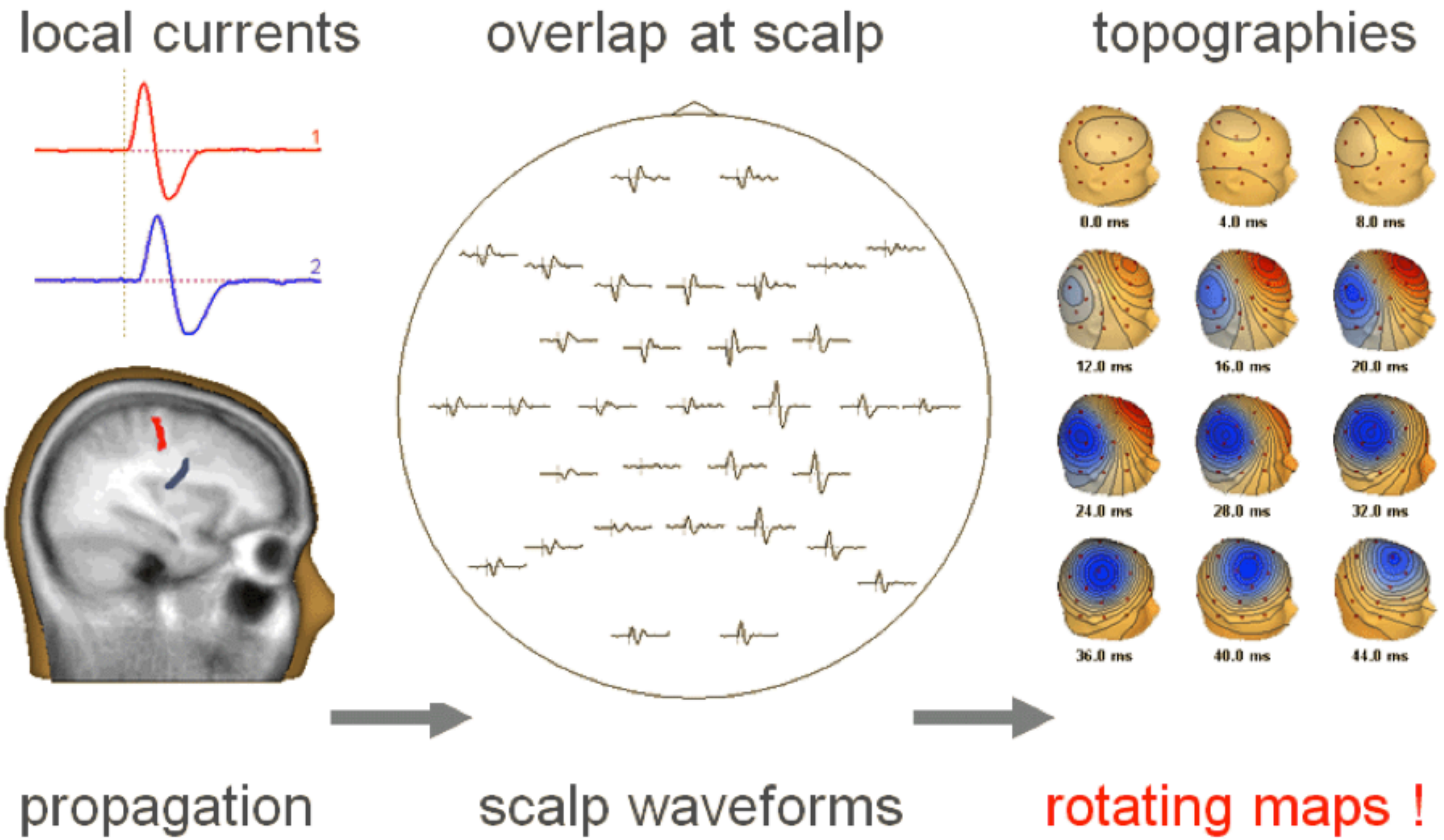
Sequential dipole fitting





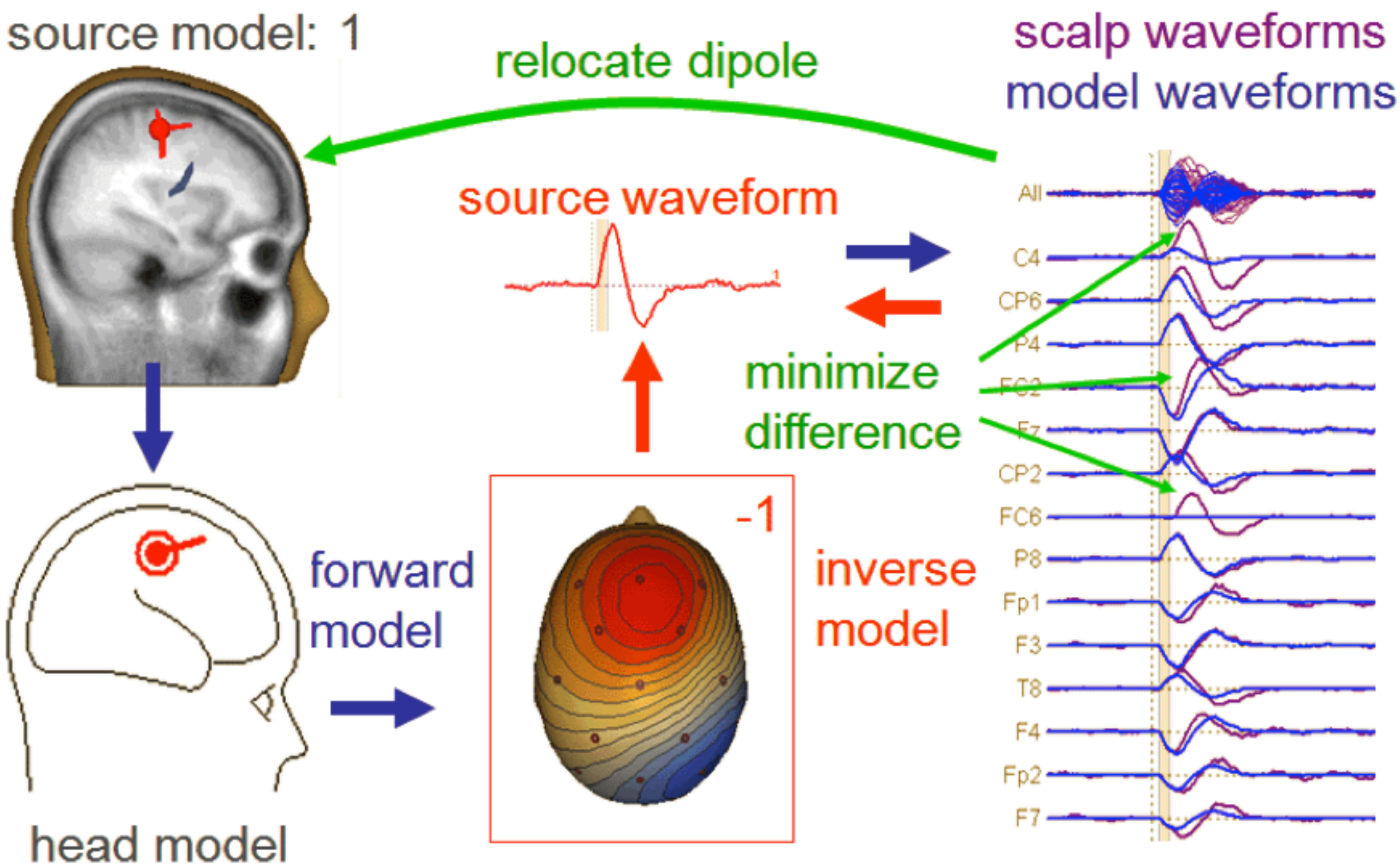
BESA manual

Sequential dipole fitting



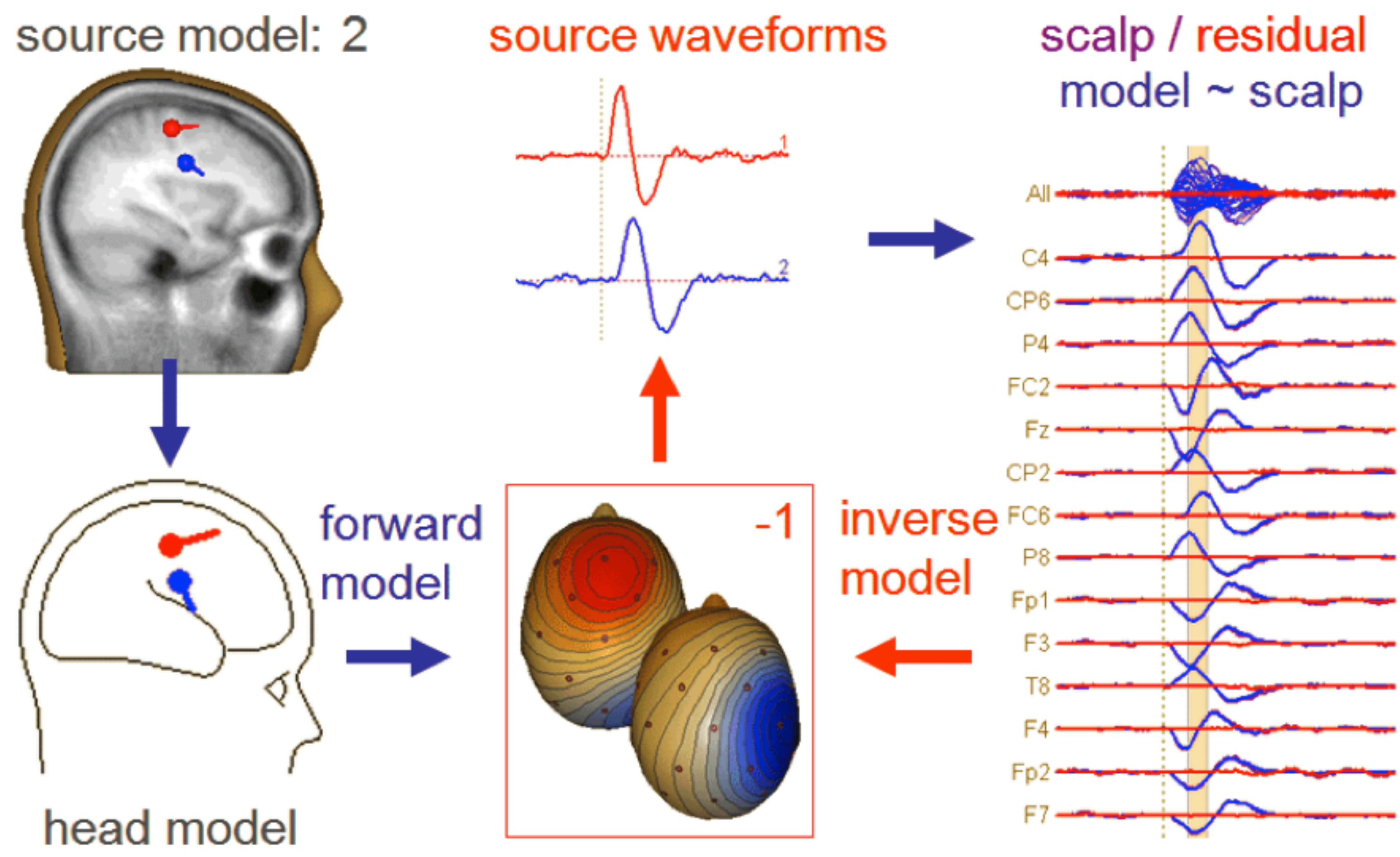
BESA manual

Sequential dipole fitting



BESA manual

Sequential dipole fitting



BESA manual

Spread of cortical activity

- Assume that activity starts “small”
  - explain earliest ERP component with single equivalent current dipole
- Assume later activity to be more widespread
  - add ECDs to explain later ERP components
  - estimate position of new dipoles
  - re-estimate the activity of all dipoles

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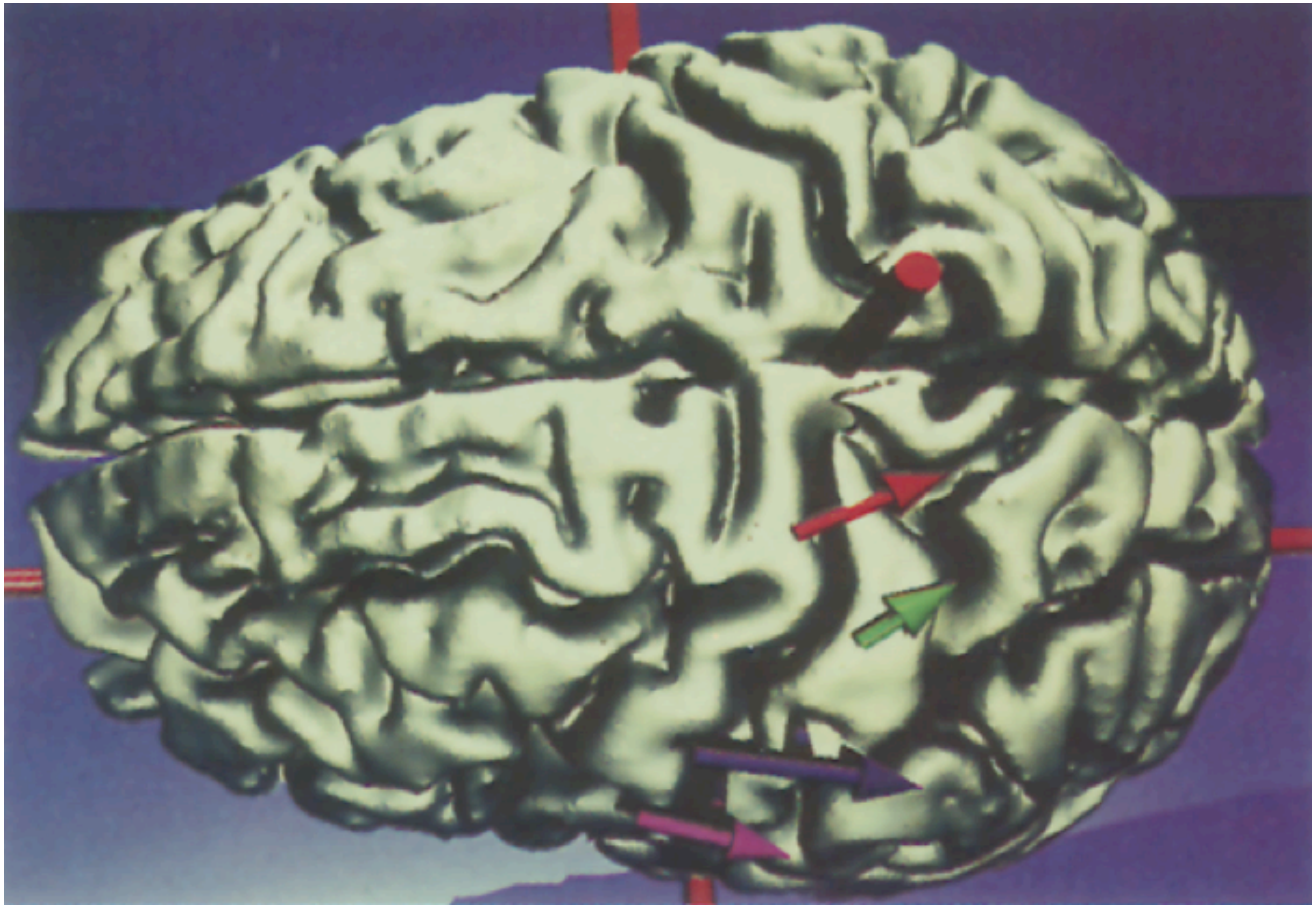
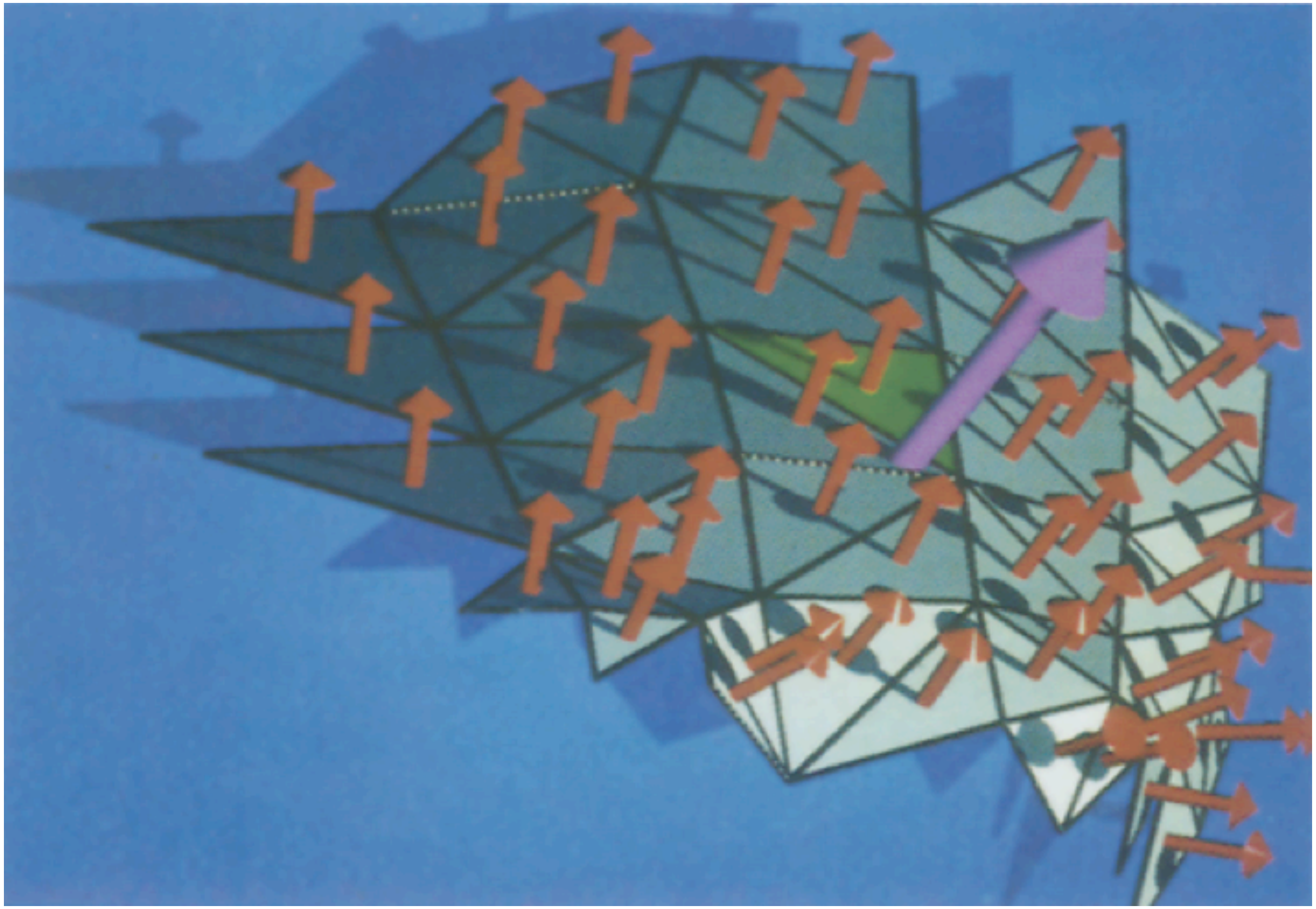
Distributed source model

- Position of the source is not estimated as such
  - Pre-defined grid (3D volume or on cortical sheet)
- Strength is estimated
  - In principle easy to solve, however…
  - More “unknowns” (parameters) than “knowns” (measurements)
  - Infinite number of solutions can explain the data perfectly
  - Additional constraints required
  - Linear estimation problem

Distributed source model



Distributed source model



Distributed source model: linear estimation

$$Y = G_1 q_1 + G_2 q_2 + \ldots = \begin{bmatrix} G_{1,1} & G_{2,1} & \cdots \\ G_{1,2} & G_{2,2} & \cdots \\ \vdots & \vdots & \ddots \\ G_{1,N} & G_{2,N} & \cdots \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ \vdots \end{bmatrix} = \mathrm{G} \cdot \vec{q}$$

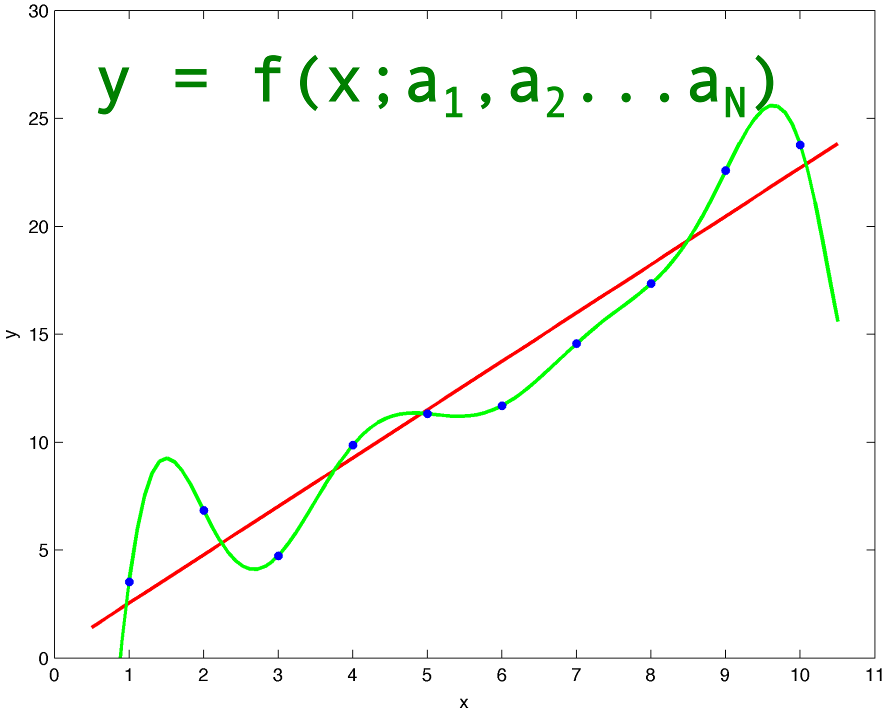
$$\vec{q} = \mathrm{G}^{-1} \cdot Y$$

Distributed source model: linear estimation

distributed source model with **many dipoles** throughout the whole brain

estimate the strength of all dipoles

data and noise can be perfectly explained



Distributed source model: regularization

$$Y = G \cdot q + Noise$$

$$\min_q \{\| V - G \cdot q \|^2\} = 0 \;\; !!$$

Regularized linear estimation:

$$\rightarrow \min_q \{\underbrace{\| V - G \cdot q \|^2}_{\text{mismatch with data}} + \lambda \cdot \underbrace{\| D \cdot q \|^2}_{\text{mismatch with prior assumptions}}\}$$

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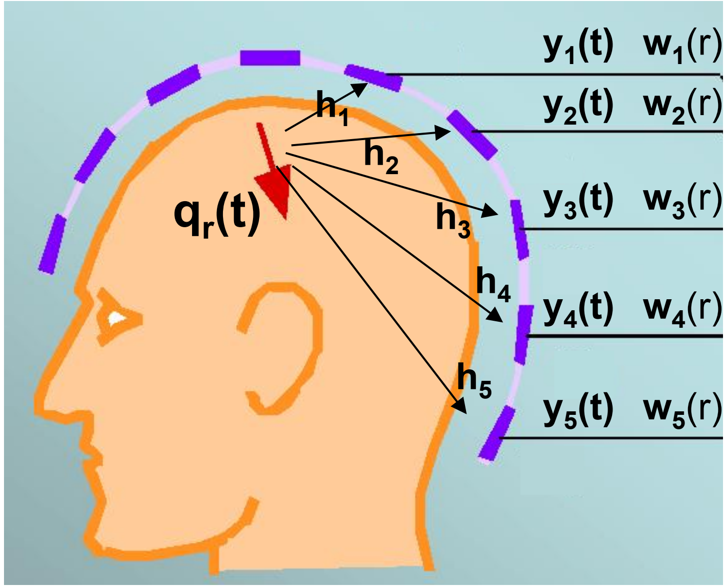
Spatial filtering with beamforming

- Position of the source is not estimated as such
- Manipulate filter properties, not source properties
  - No explicit assumptions about source constraints (implicit: single dipole)
  - Assumption that sources that contribute to the data should be uncorrelated

Beamformer: the question

What is the activity of a source **q**, at a location **r**, given the data **y**?

We estimate **q** with a spatial filter **w**



$$\hat{q}_r(t) = \mathbf{w}(r)^T \mathbf{y}(t)$$

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Estimating source timecourse activity

$$Y = G_1X_1 + G_2X_2 + ... + G_nX_n + noise$$

Estimating source timecourse activity using dipole fitting

$$Y = G_1X_1 + G_2X_2 + \ldots + G_nX_n + \text{noise}$$

n is typically small

$$(Y - G_1X_1 - \qquad\qquad = \text{residual}$$

measured data model data

$X' = W\,Y$, where $W = G^T (G\,G^T)^{-1}$

Estimating source timecourse activity using distributed source models

$$Y = G_1X_1 + G_2X_2 + \ldots + G_nX_n + \text{noise}$$

n is typically large (> # channels)

$$Y = (G_1X_1 + G_2X_2 + \ldots + G_nX_n) + \text{noise}$$

$$Y = G\,X + \text{noise}$$

$X' = W\,Y$,   where W ensures $\min_X \{\| Y - G \cdot X \|^2 + \lambda \cdot \| X \|^2\}$

Estimating source timecourse activity using spatial filtering

$$Y = G_1X_1 + G_2X_2 + \ldots + G_nX_n + \text{noise}$$

any number of n

$$Y = (G_1X_1 + G_2X_2 + \ldots) + G_nX_n + (\text{noise})$$

$$X'_n = W_n Y, \quad \text{where} \quad W^T = [G_n^T C_Y^{-1} G_n]^{-1} G_n^T C_Y^{-1}$$

Estimating source timecourse activity

$$Y = G_1X_1 + G_2X_2 + \ldots + G_nX_n + \text{noise}$$

few sources

distributed sources

one at a time

$$X'(t) = W\,Y(t)$$

dipole fitting

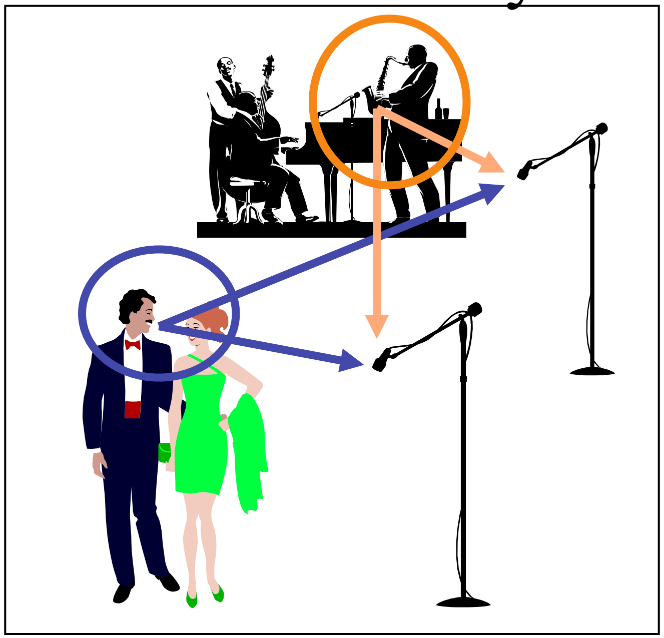
minimum norm estimate

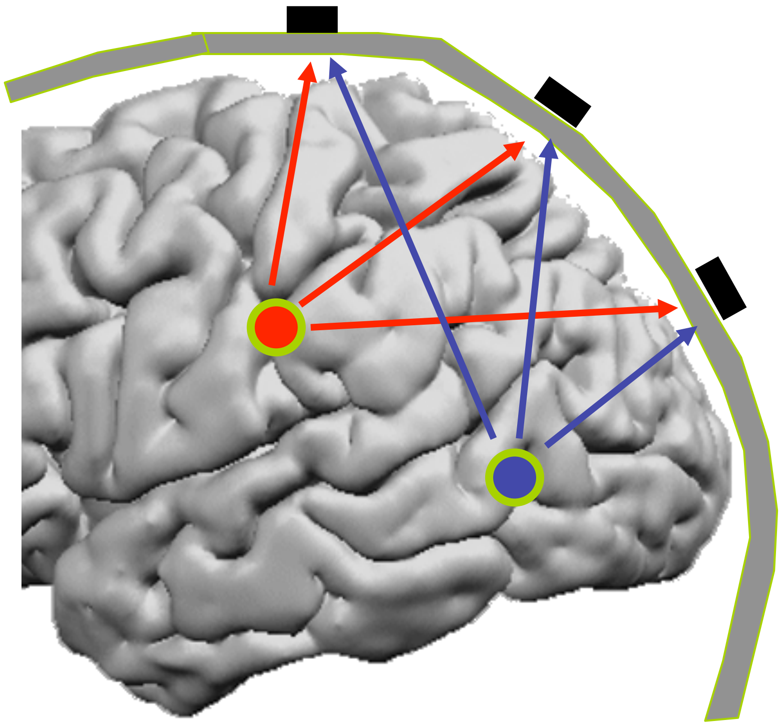
beamforming

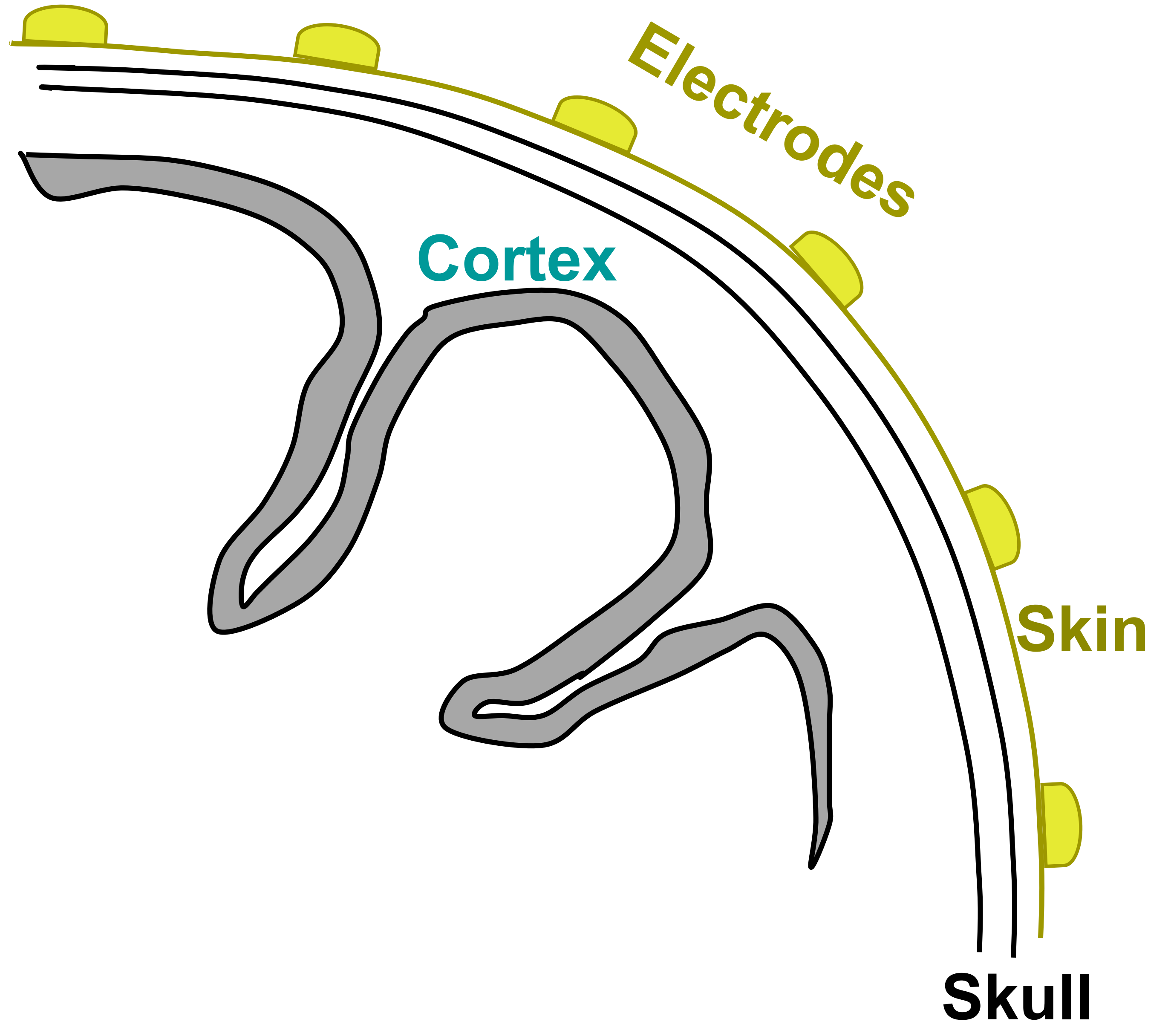
Independent component analysis

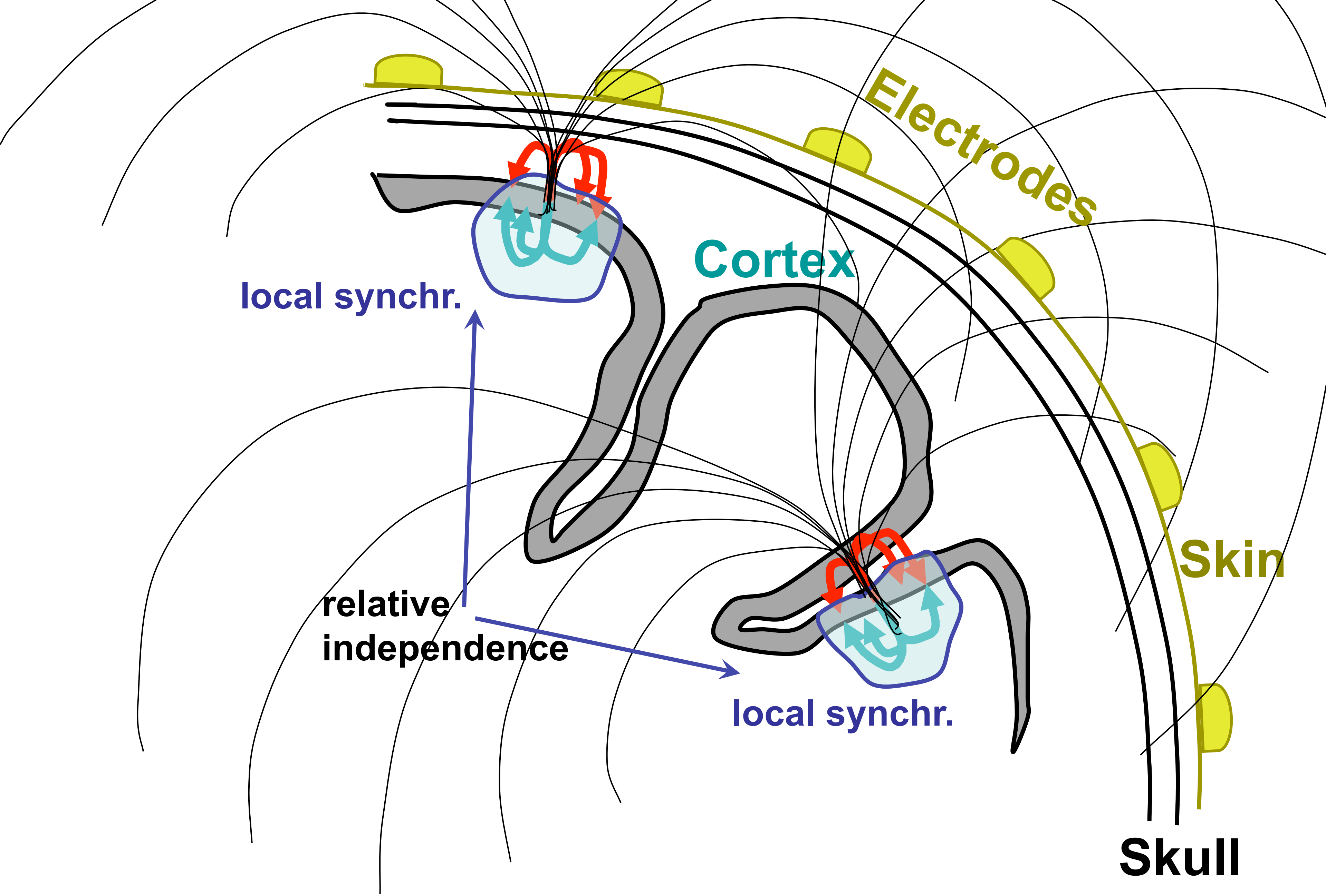
Mixture of Brain source activity

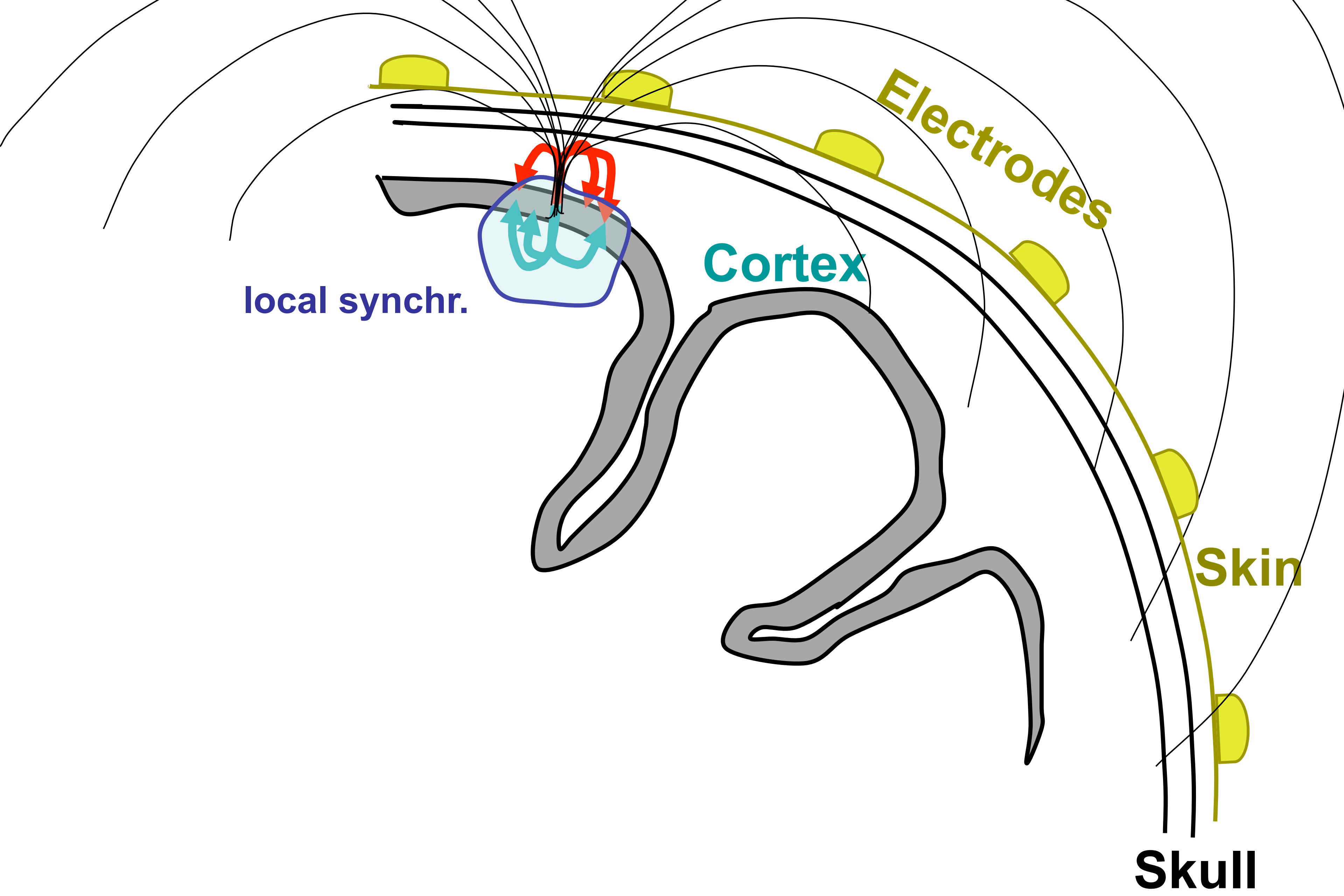
Cocktail Party





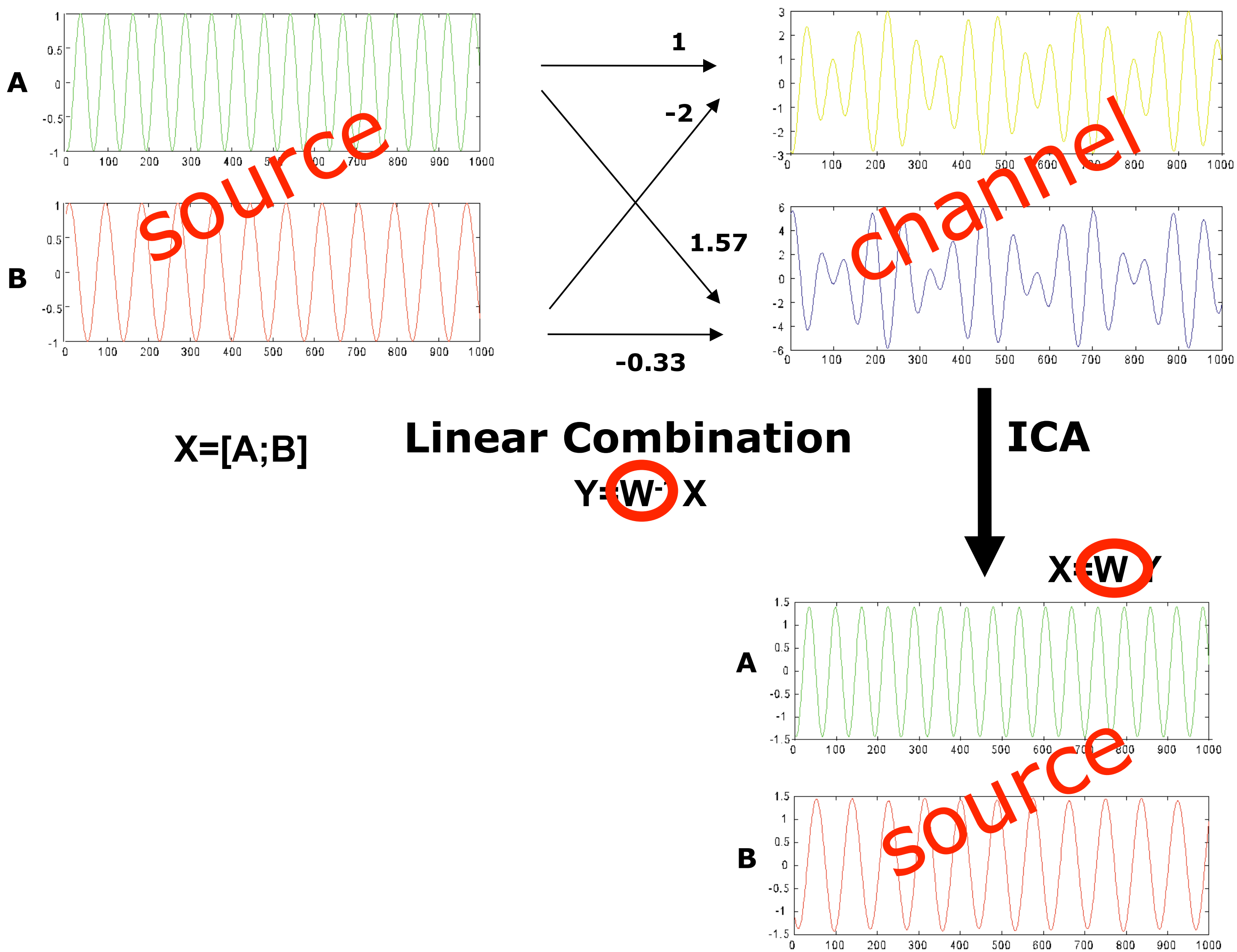

Electrodes
Cortex
Skin
Skull


Electrodes
Cortex
local synchr.
Skin
relative
independence
local synchr.
Skull



Electrodes
Cortex
local synchr.
Skin
Skull

Independent component analysis



Estimating source timecourse activity using independent component analysis

$$Y = G_1X_1 + G_2X_2 + \ldots + G_nX_n + \text{noise}$$

n typically the same as the number of channels

$$Y = G\,(X\ \ + \text{noise})$$

includes line-noise, EOG, ECG and other noise that is visible on all channels

$X' = W\,Y$, where W maximizes the independence of X'

rows of W^{-1} correspond to $G_1, G_2, \ldots$

Estimating source timecourse activity

$$Y = G_1X_1 + G_2X_2 + \ldots + G_nX_n + \text{noise}$$

few sources
distributed sources
one at a time
all brain (and artifact) sources

$$X'(t) = W\,Y(t)$$

dipole fitting
minimum norm estimate
beamforming
independent component analysis

Source modelling of independent components

- Components have (maximal) independent timecourses
- Unmixing of timeseries has already been taken care of
- Assumption: components correspond to compact spatial patches (or bilateral patches)
- Use simple biophysical dipole models to model the spatial component topographies
- It can be challenging to match ICA sources over subjects

Overview

- Motivation and background
- Forward modeling
  - Source model
  - Volume conductor model
  - EEG versus MEG
- Inverse modeling - biophysical models
  - Single and multiple dipole fitting
  - Distributed source models
  - Spatial filtering
- Inverse modeling - independent components
- **Summary**

Summary 1

Forward modelling

- Required for the interpretation of scalp topographies
- Different methods with varying accuracy

Inverse modelling

- Estimate source location and timecourse from data

Assumptions on source locations

- Single or multiple point-like source
- Distributed source

Assumptions on source timecourse

- Uncorrelated (and dipolar)
- Independent

Summary 2

- Independent component analysis
  - separates topography and timecourse
  - no biophysical assumptions yet
- Inverse methods to interpret topography
  - Single or multiple point-like source
  - Distributed source

Summary 3

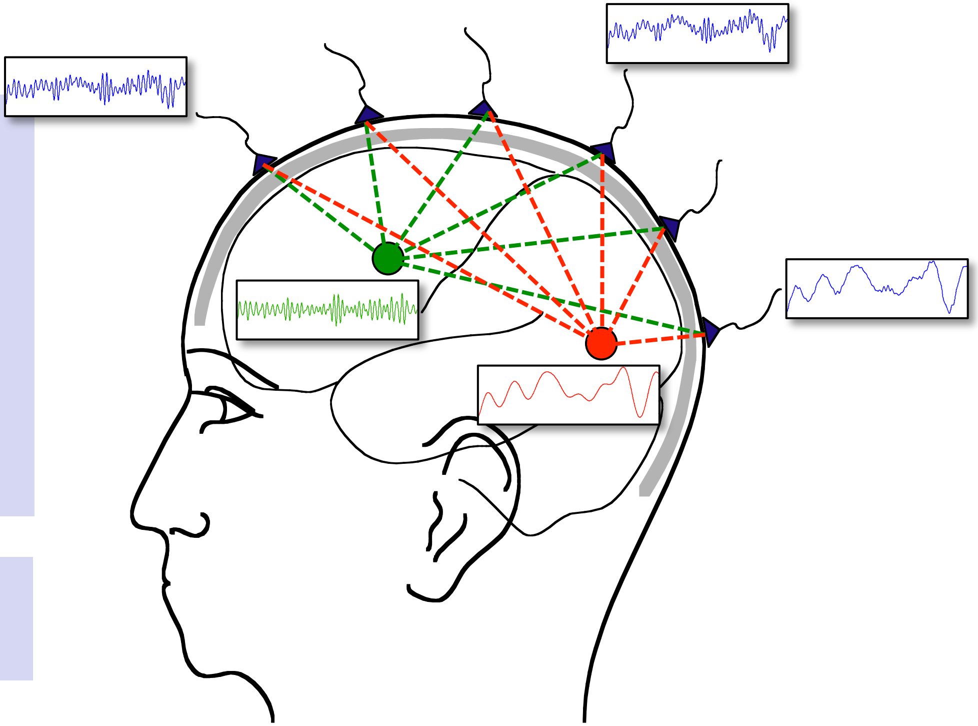
Source analysis is not only about the "where" but also about untangling the "what" and "when"

timecourse of activity
-> ERP

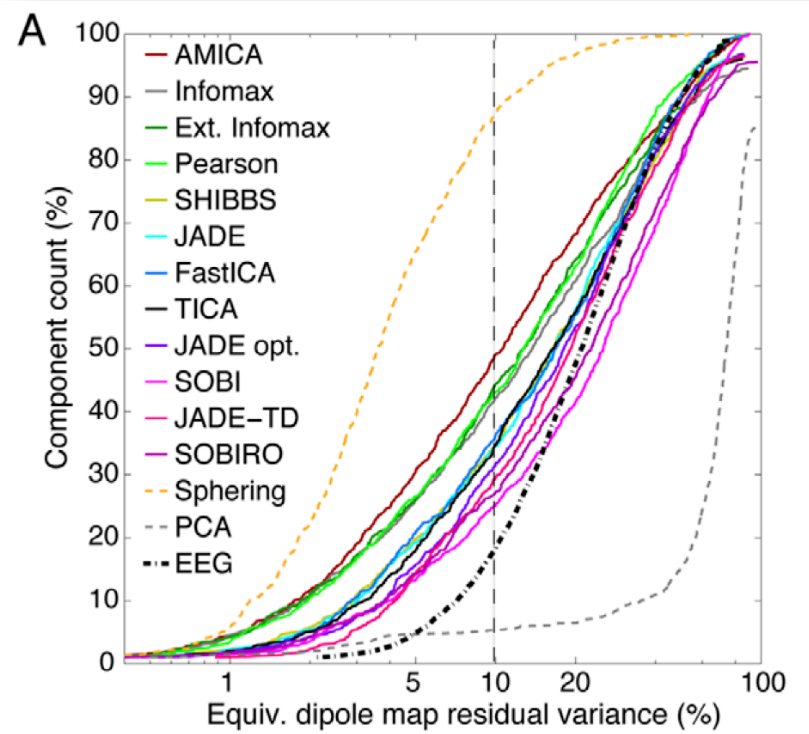
spectral characteristics
-> power spectrum

temporal changes in power
-> time-frequency response (TFR)

spatial distribution of activity over the head
-> source reconstruction

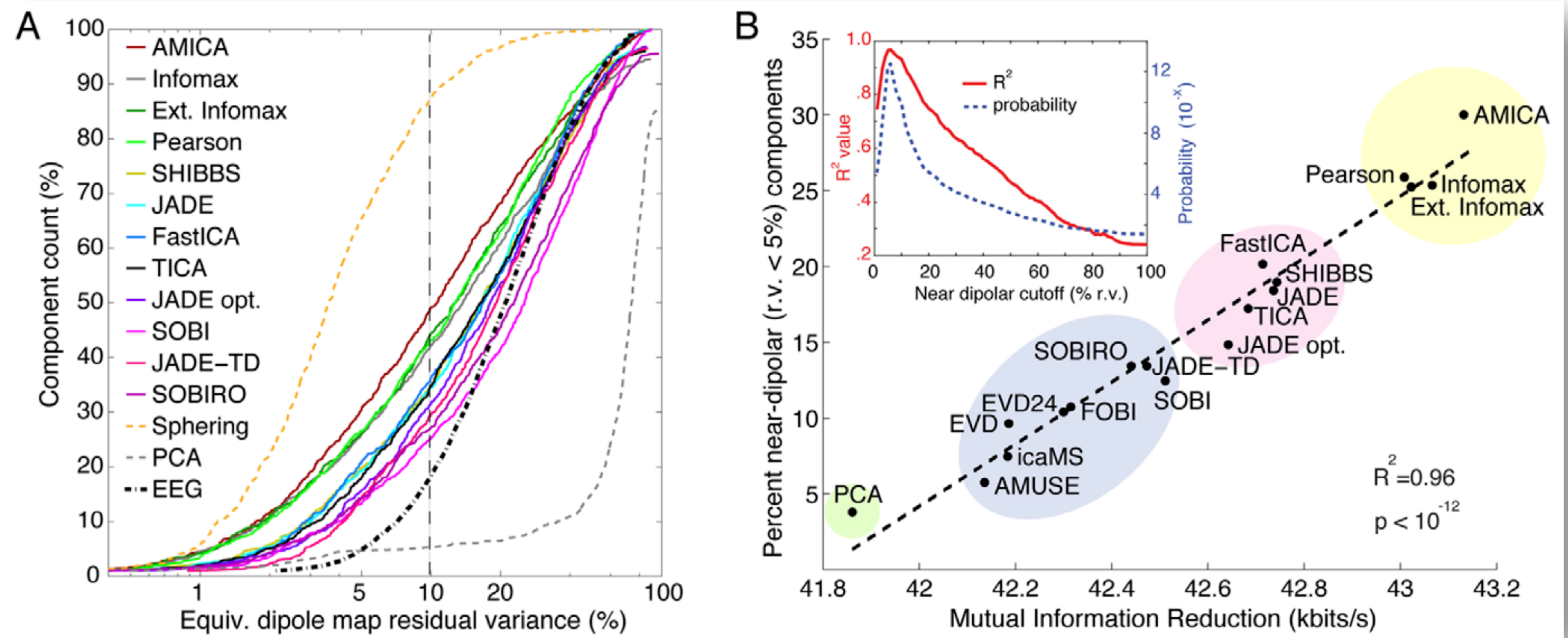


Independent components are dipolar



Delorme et al. *Independent EEG sources are dipolar*. PLoS One. 2012.

Independent components are dipolar



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