Donders Institute
for Brain, Cognition and Behaviour

FieldTrip

Forward and inverse modelling

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Overview

- Motivation and background
- Forward modeling
  - Source model
  - Volume conductor model
    - Analytical (spherical model)
    - Numerical (realistic model)
  - Comparison EEG and MEG
- Inverse modeling
  - Single and multiple dipole fitting
  - Distributed source models
  - Spatial filtering

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Motivation and background

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Motivation 1

Strong points of EEG and MEG

- Temporal resolution (~1 ms)
- Characterize individual components of ERP
- Oscillatory activity
- Disentangle dynamics of cortical networks

Weak points of EEG and MEG

- Measurement on outside of brain
- Overlap of components
- Low spatial resolution

Motivation 2

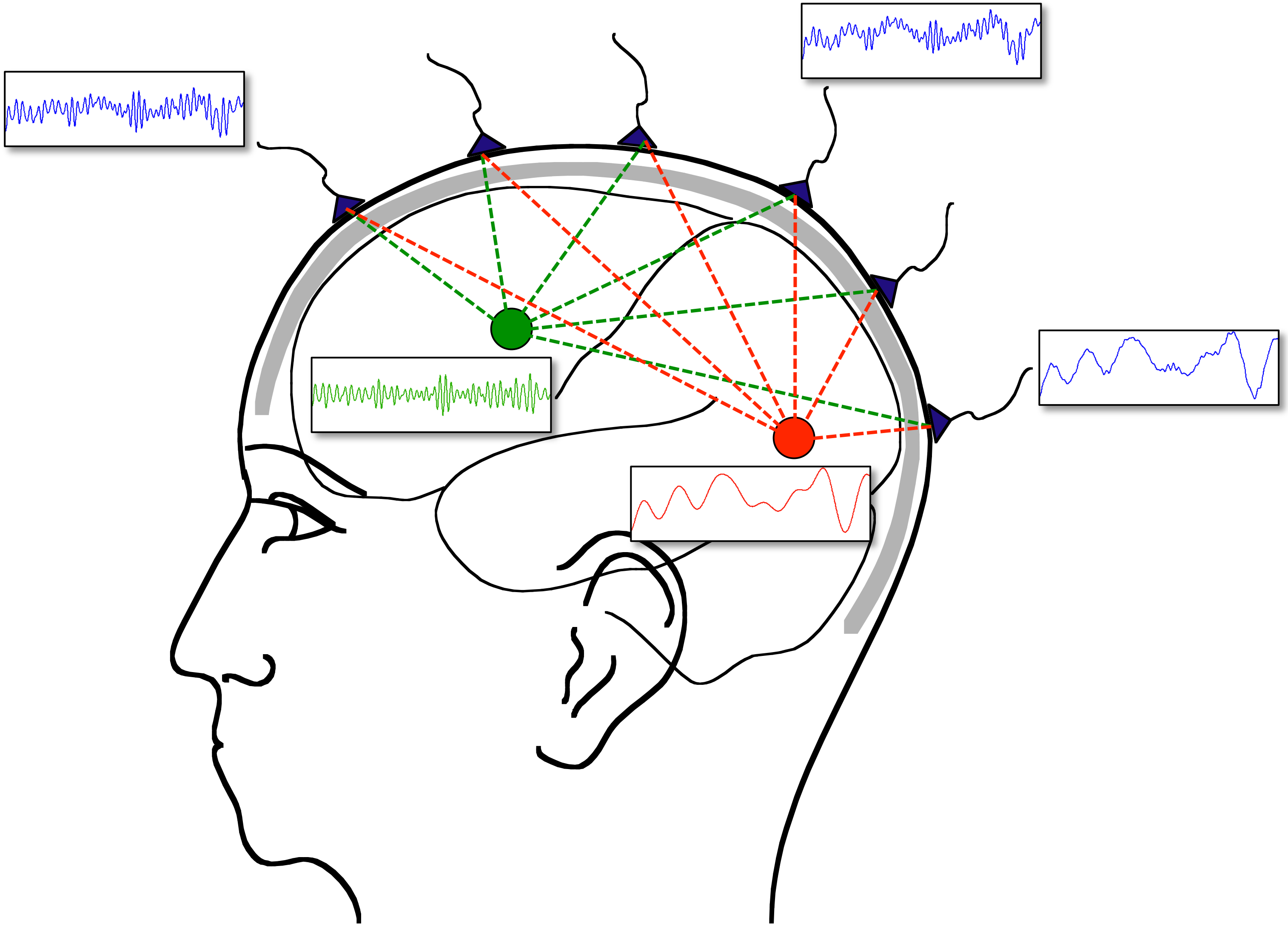
If you find a ERP/ERF component, you want to characterize it in physiological terms

- Time or frequency are the “natural” characteristics
- “Location” requires interpretation of the scalp topography

Forward and inverse modeling helps to interpret the topography

Forward and inverse modeling helps to disentangle overlapping source timeseries

Superposition of source activity



Superposition of source activity

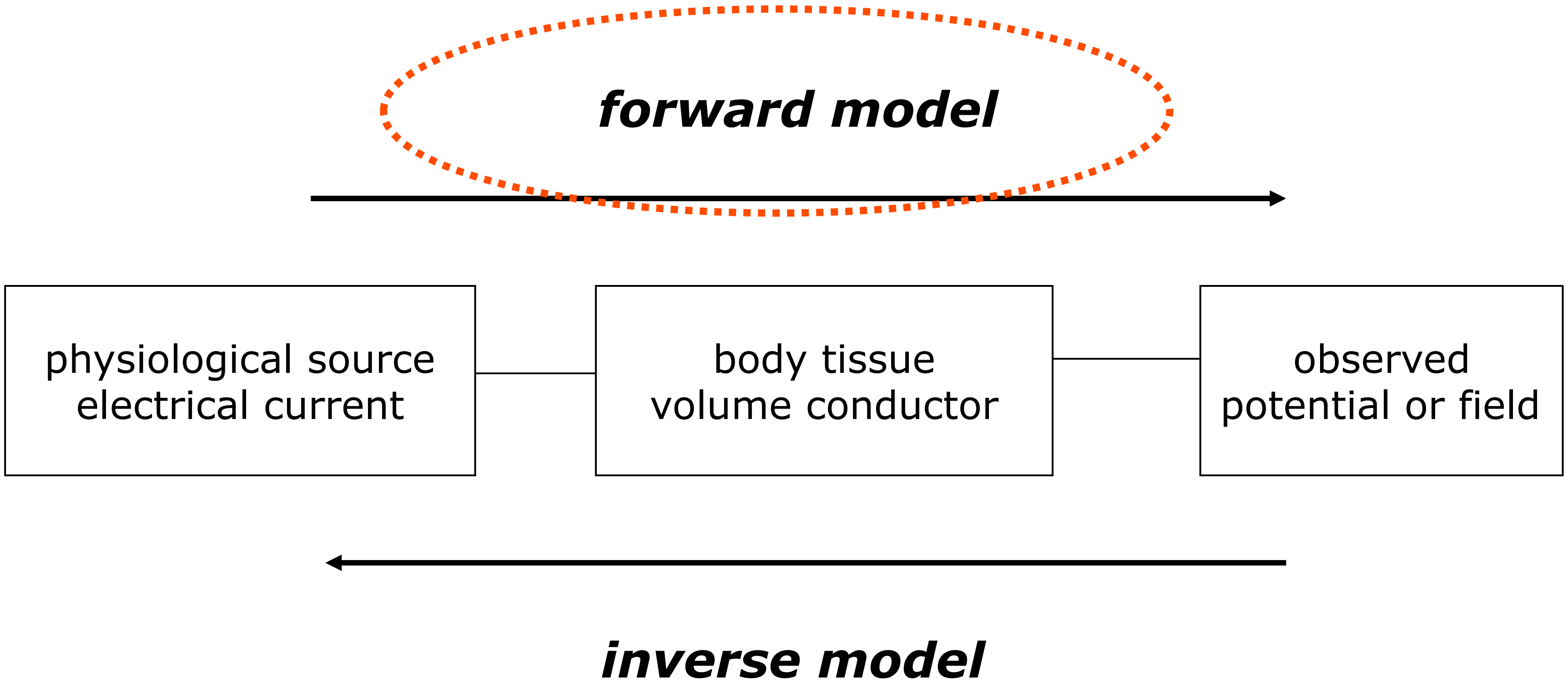
Varying “visibility” of each source to each channel

Timecourse of each source contributes to each channel

The contribution of each source depends on its “visibility”

Activity on each channel is a superposition of all source activity

Source modelling: overview



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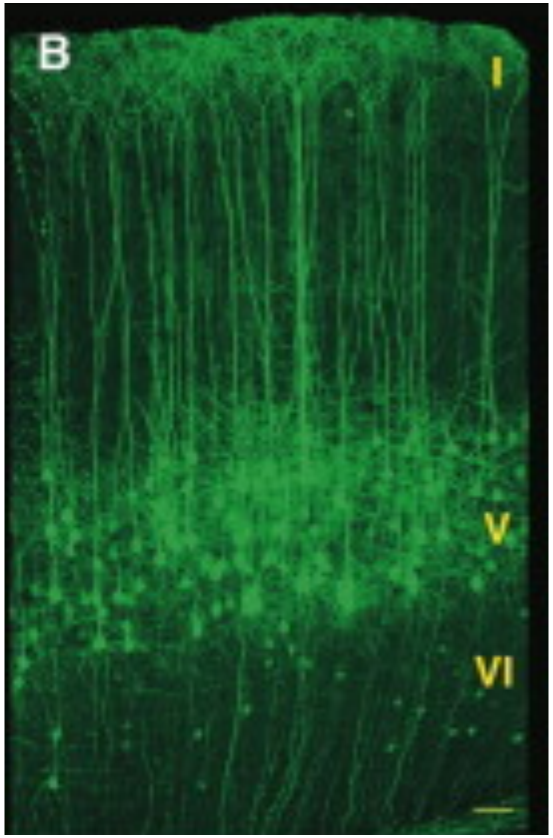
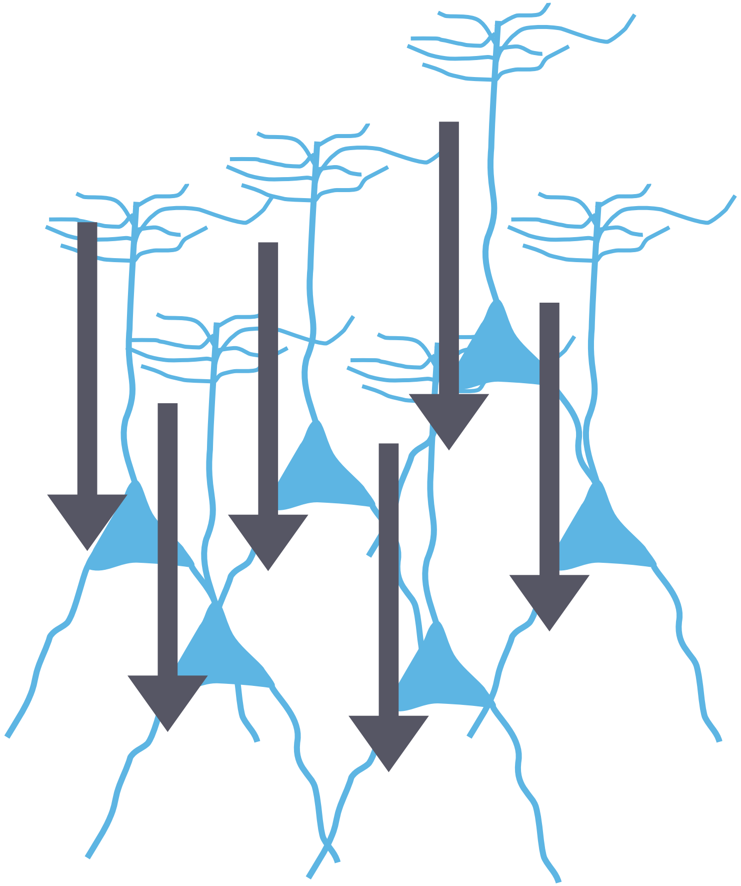
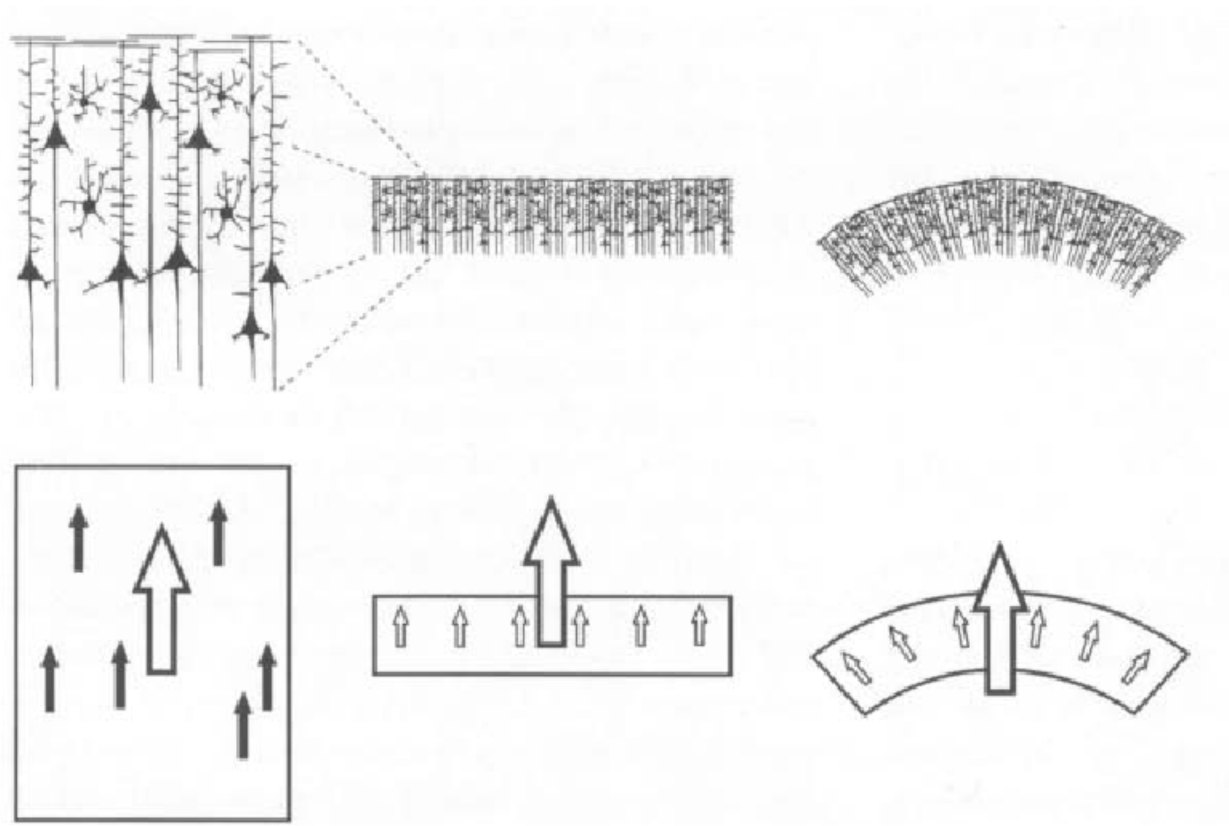
What produces the electric current





Equivalent current dipoles



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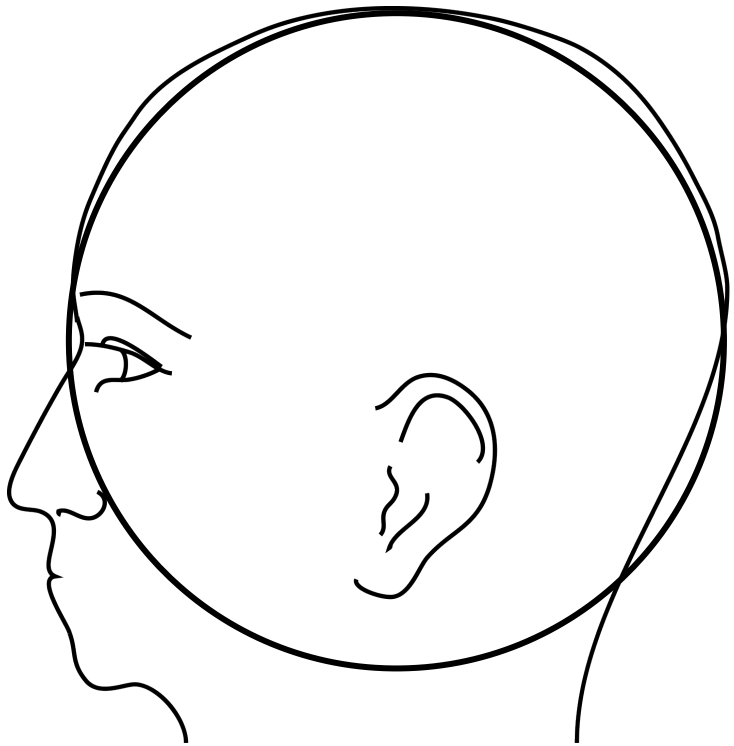
Volume conductor

described electrical properties of tissue

describes geometrical model of the head

describes **how** the currents flow, not where they originate from

same volume conductor as used in tDCS, tACS and TMS



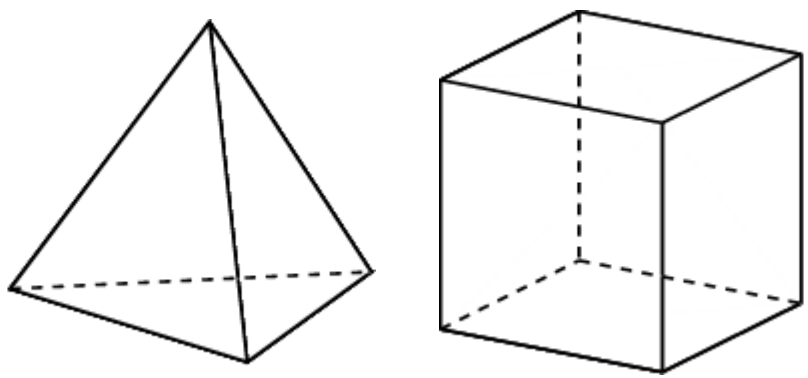
Volume conductor

Computational methods for volume conduction problem that allow for realistic geometries

Boundary Element Method (BEM)
Finite Element Method (FEM)
Finite Difference Method (FDM)

Geometrical description

triangles
tetraeders
hexaheders (cubes)



Volume conductor: Boundary Element Method

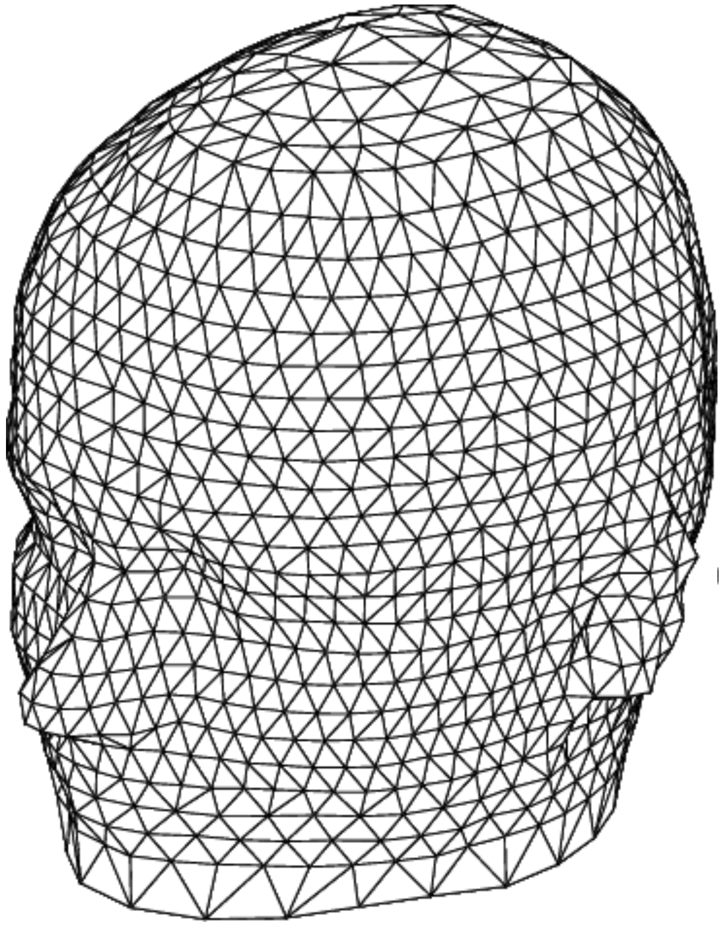
Each compartment is

- homogenous
- isotropic

Important tissues

- skin
- skull
- brain
- (CSF)

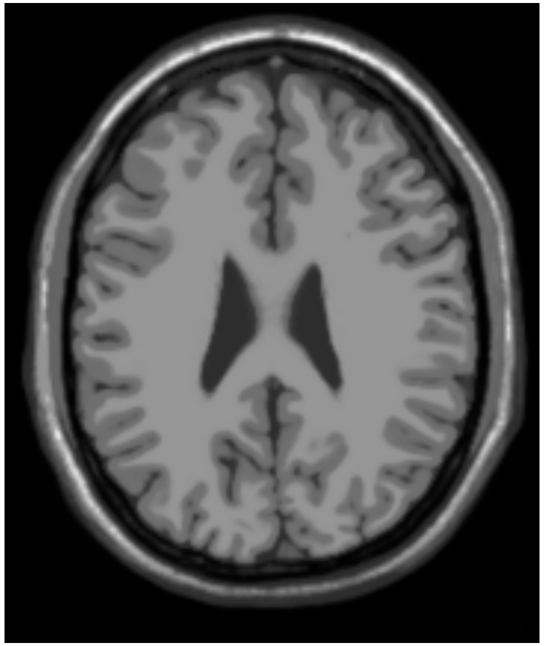
Triangulated surfaces describe boundaries

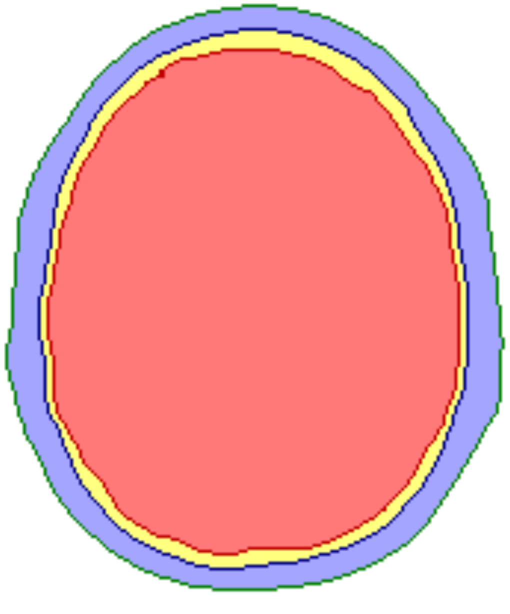


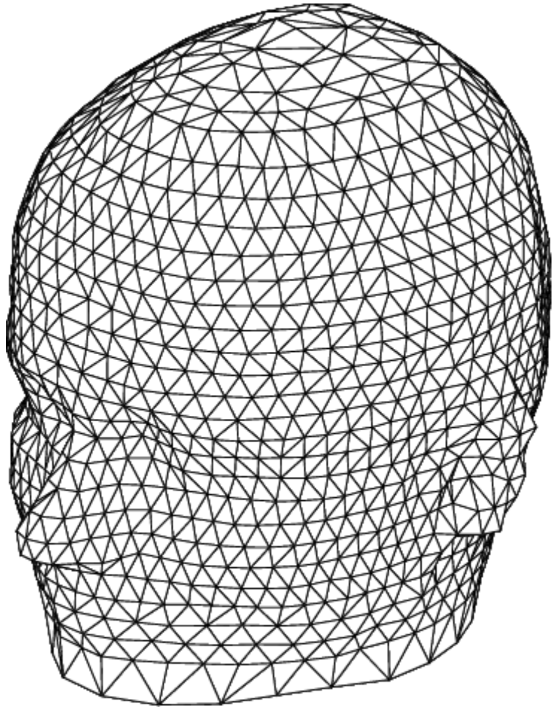
Volume conductor: Boundary Element Method

Construction of geometry

- segmentation in different tissue types
- extract surface description
- downsample to reasonable number of triangles







Volume conductor: Boundary Element Method

- Construction of geometry
  - segmentation in different tissue types
  - extract surface description
  - downsample to reasonable number of triangles
- Computation of model
  - independent of source model
  - only one lengthy computation
  - fast during application to real data
- Can (almost) be arbitrary complex
  - ventricles
  - holes in skull

Volume conductor: Finite Element Method

Tesselation of 3D volume in tetraeders

Large number of elements

Simplify the tesselation in regions were less accuracy is required

Each tetraeder can have its own conductivity

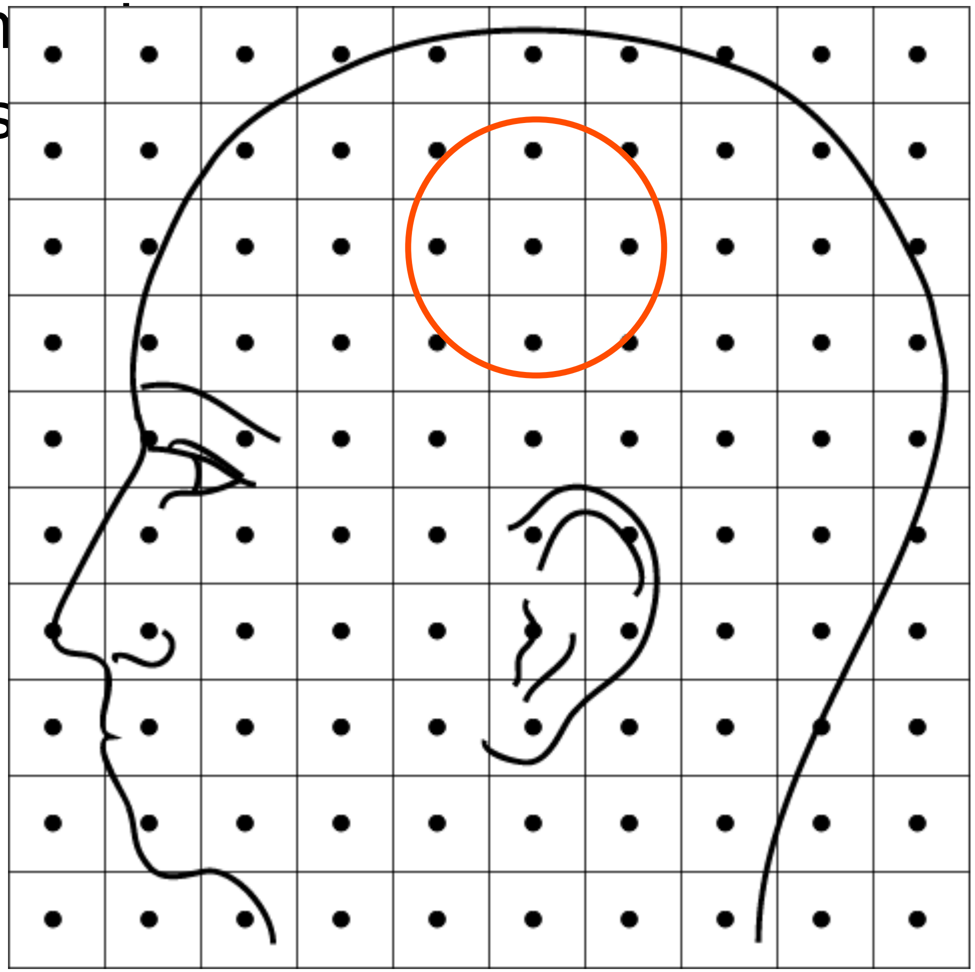
FEM is the most accurate numerical method

Computationally more expensive

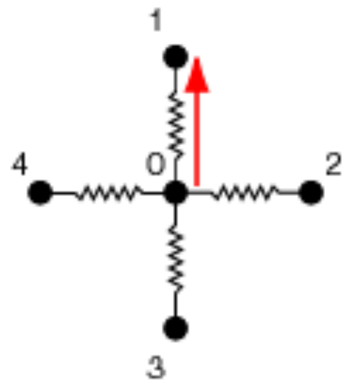
Volume conductor: Finite Difference Method

Easy to con

Not very us



Volume conductor: Finite Difference Method



$$\left.\begin{array}{l} I_1 + I_2 + I_3 + I_4 = 0 \\ V = I{*}R \end{array}\right\} \Longrightarrow$$

$$\Delta V_1/R_1 + \Delta V_2\,/R_2 + \Delta V_3\,/R_3 + \Delta V_4\,/R_4 = 0 \quad \Longrightarrow$$

$$(V_1\text{-}V_0)/R_1 + (V_2\text{-}V_0)/R_2 + (V_3\text{-}V_0)/R_3 + (V_4\text{-}V_0)/R_4 = 0$$

Volume conductor: Finite Difference Method

Unknown potential Vi at each node

Linear equation for each node

- approx. 100x100x100 = 1.000.000 linear equations
- just as many unknown potentials

Add a source/sink

- sum of currents is zero for all nodes, except
- sum of current is I+ for a certain node
- sum of current is I- for another node

Solve for unknown potential

Methods implemented in FieldTrip

```
mri              = ft_read_mri(filename);

cfg              = [];
cfg.output       = {'brain','skull','scalp'};
segmentedmri     = ft_volumesegment(cfg, mri);

cfg              = [];
cfg.tissue       = {'brain','skull','scalp'};
cfg.numvertices = [3000 2000 1000];
bnd              = ft_prepare_mesh(cfg,segmentedmri);
```

```
cfg              = [];
cfg.method       = ’concentricspheres';
...
headm
```

```
cfg              = [];
cfg.method       = ’bemcp';
...
head
```

```
cfg              = [];
cfg.method       = ’simbio';
...
headmodel        = ft_prepare_headmodel(cfg, segmentedmri);
```

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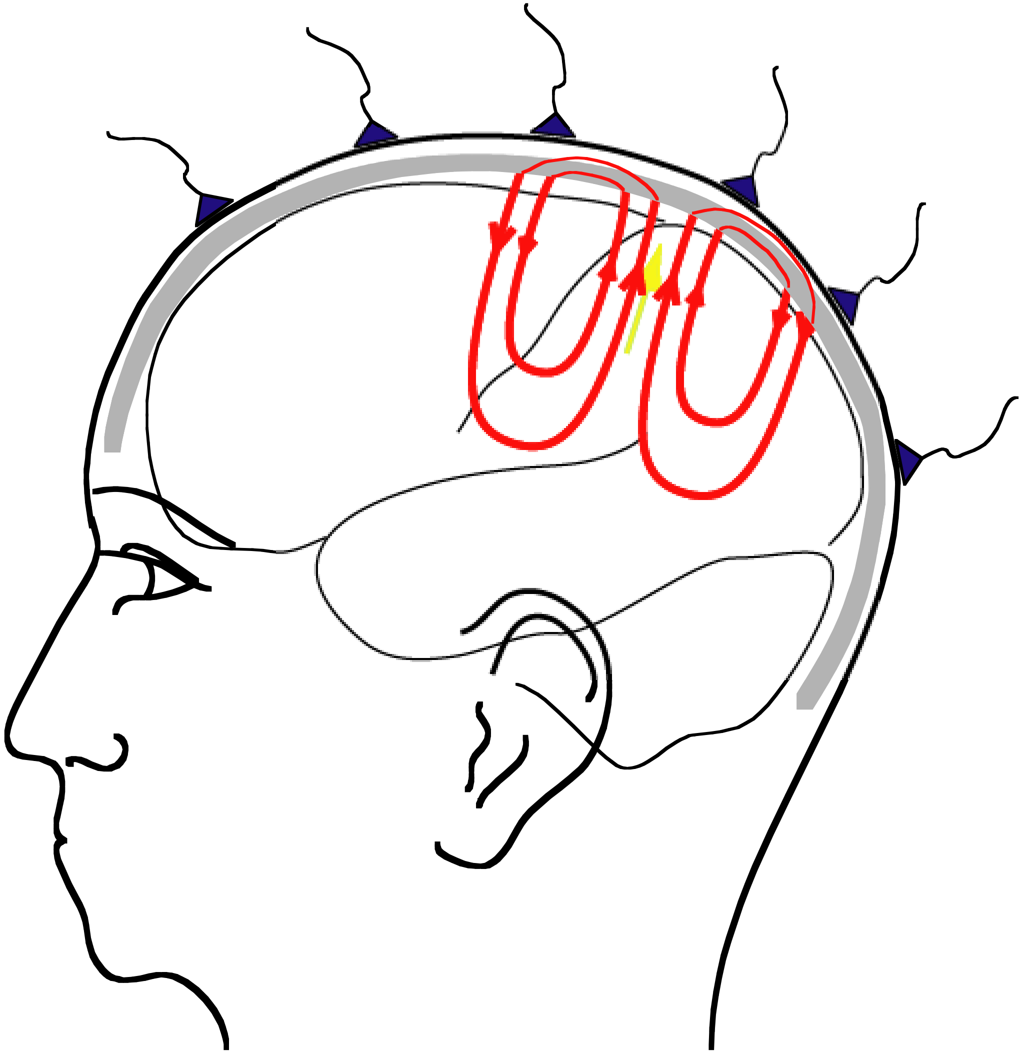
EEG volume conduction



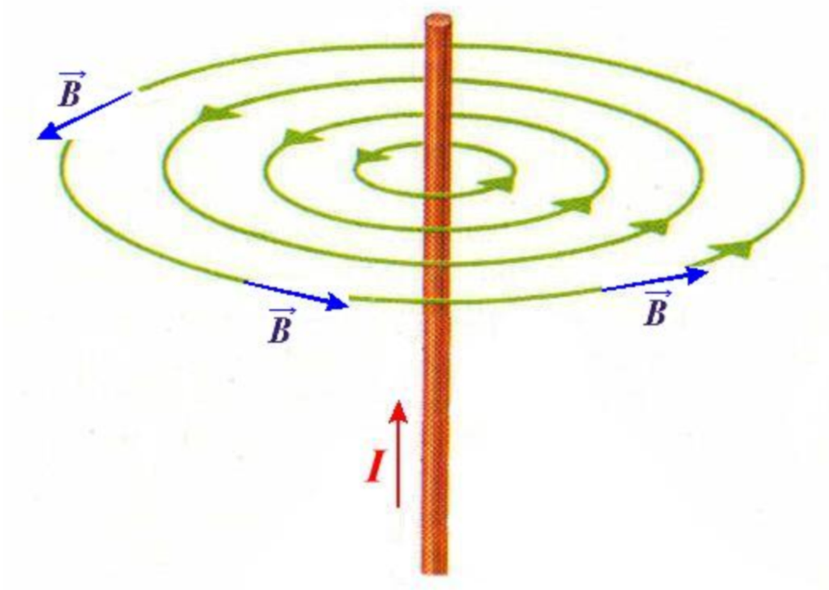
EEG volume conduction

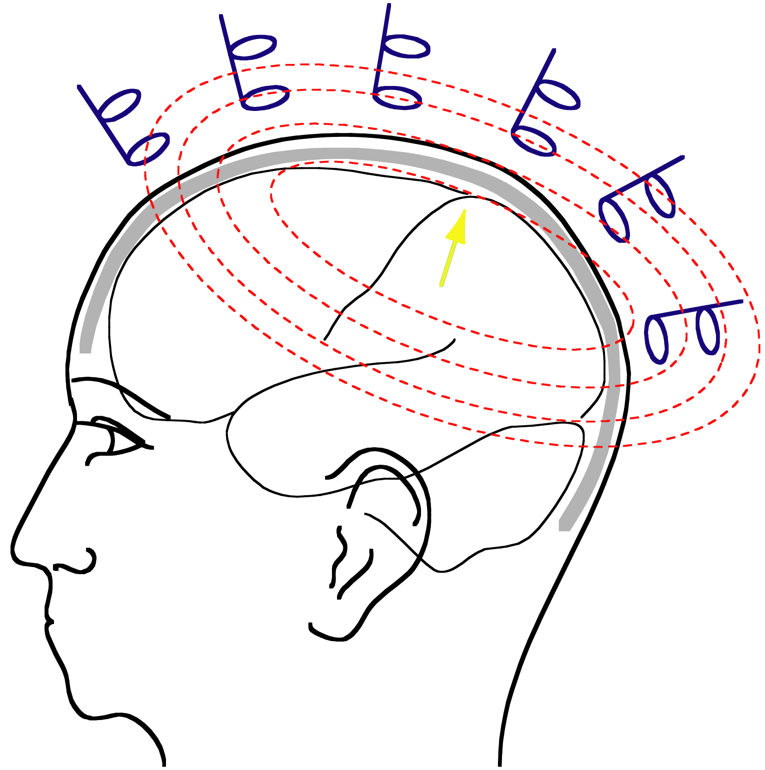
Potential difference between electrodes corresponds to current flowing through skin

Only tiny fraction of current passes through skull

Therefore the model should describe the skull and skin as accurately as possible

Electric current → magnetic field





MEG volume conduction

MEG measures the magnetic field due to the primary neuronal current, but also due to the volume currents

Only tiny fraction of current passes through the poorly conductive skull

Therefore skull and skin are usually neglected in MEG model

Similarities between EEG and MEG

Identical source model

Similar volume conductor model

Identical inverse methods apply!

For EEG you have to consider the referencing scheme, which has to be consistent between data and model.

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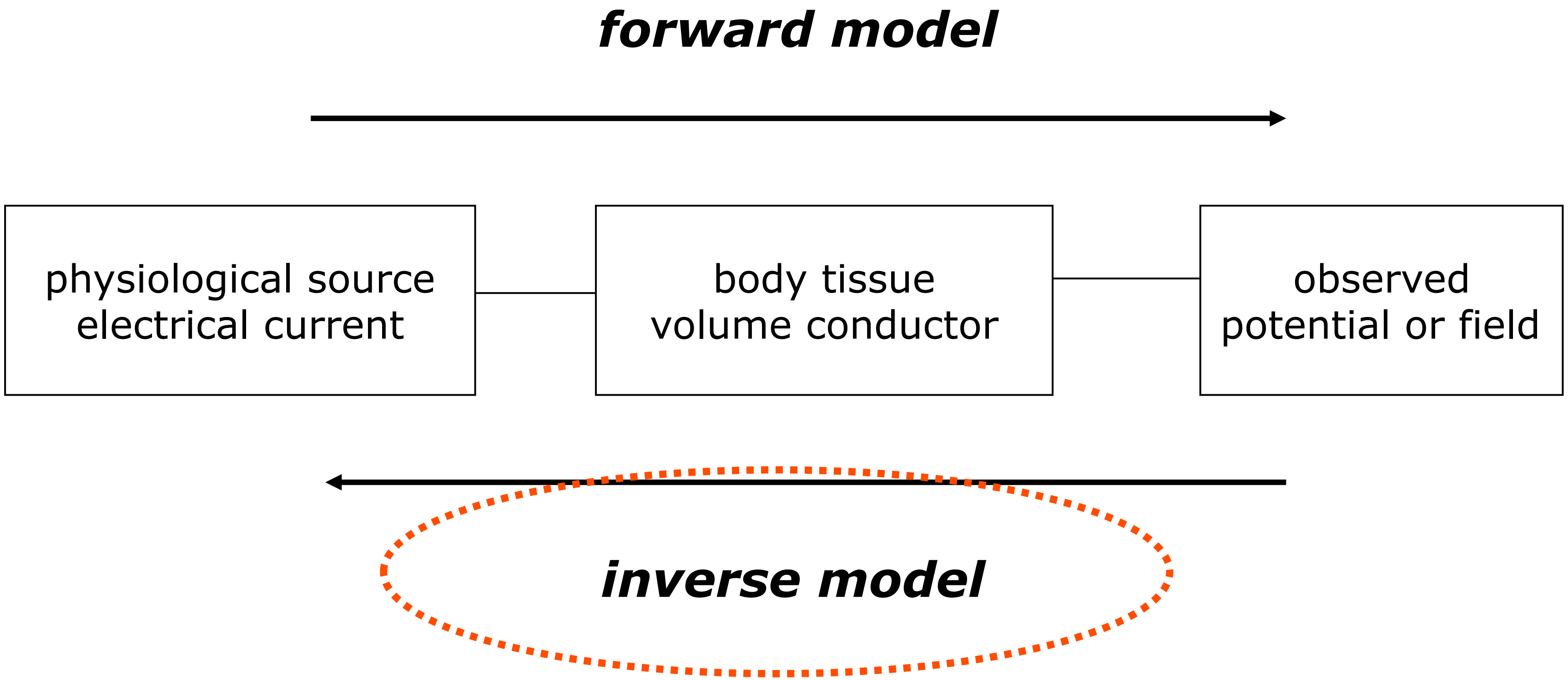
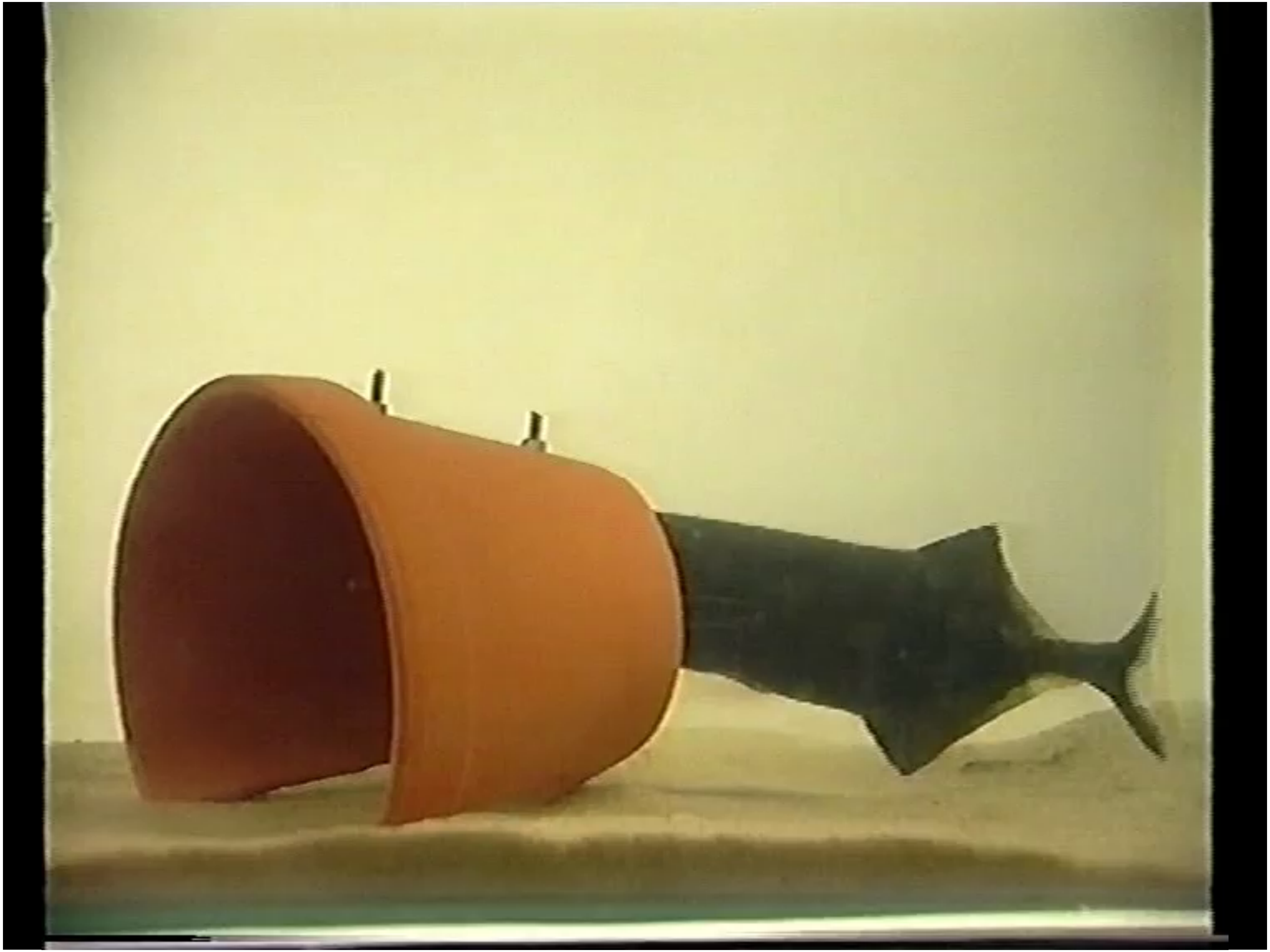
Source analysis: overview



Inverse localization: demo



Inverse methods

Single and multiple dipole models

Minimize error between model and measured potential/field

Distributed source models

Perfect fit of model to the measured potential/field

Additional constraint on source smoothness, power or amplitude

Spatial filtering

Scan the whole brain with a single dipole and compute the filter output at every location

Beamforming (e.g. LCMV, SAM, DICS)

Multiple Signal Classification (MUSIC)

Methods implemented in FieldTrip

```
cfg  = [];
      .
source = ft_dipolefitting(cfg, data);
```

```
cfg  = [];
cfg.method = ‘mne’;

so
```

```
cfg  = [];
Cfg.method = ‘dics’;

sou
```

```
cfg  = [];
cfg.method = ‘lcmv’;
      .
      .
      .
source = ft_sourceanalysis(cfg, data);
```

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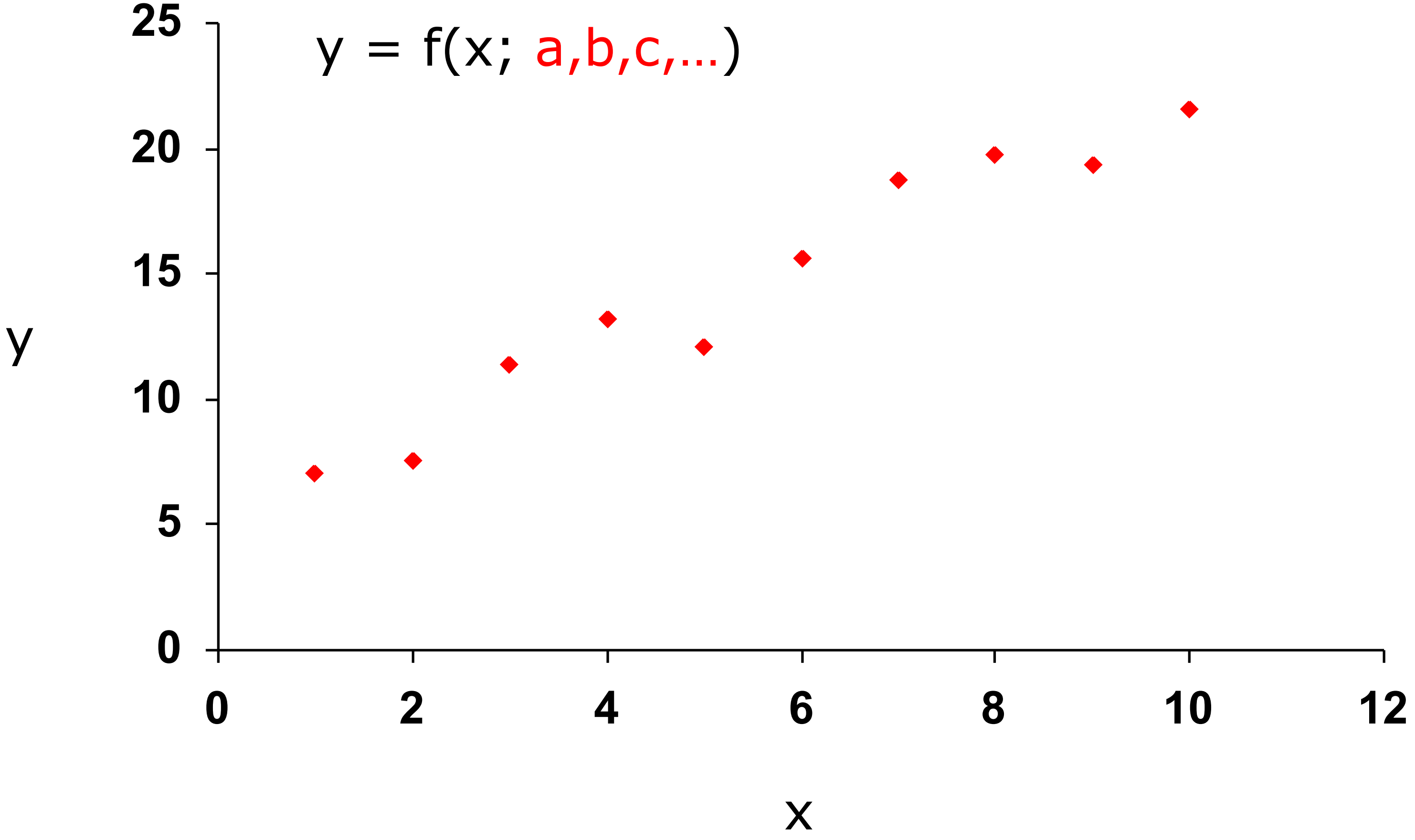
Single or multiple dipole models

- Manipulate source parameters to minimize error between measured and model data
  - Location of each source
  - Orientation of each source
  - Strength of each source
- Orientation and strength together correspond to the “dipole moment” and can be estimated linearly
- Position is estimated non-linearly
- Source parameter estimation

Parameter estimation



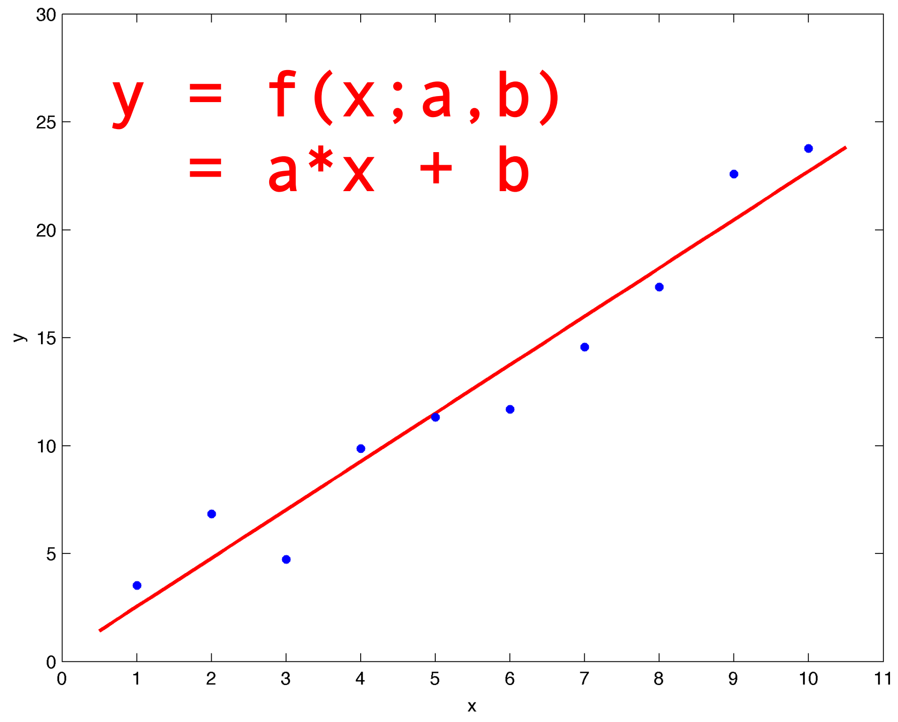
Parameter estimation: dipole parameters

source model with few parameters

- position
- orientation
- strength

compute the model data

minimize difference between actual and model data



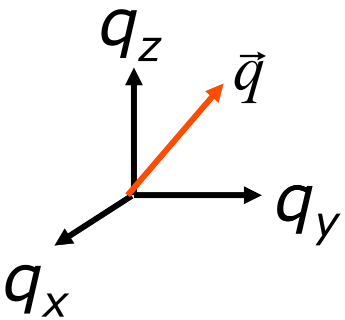
Linear parameters: superposition of sources

three sources with parameters ζ_1, ζ_2 and ζ_3

$$\left.\begin{array}{l}\Psi(\zeta_1) \\ \\ \Psi(\zeta_2) \\ \\ \Psi(\zeta_3)\end{array}\right\} \quad \Psi_{combined} = \Psi(\zeta_1) + \Psi(\zeta_2) + \Psi(\zeta_3)$$

Linear parameters: estimation

$$\vec{\Psi} = q_x \vec{\Psi}_x + q_y \vec{\Psi}_y + q_z \vec{\Psi}_z = \begin{bmatrix} \Psi_{x,1} & \Psi_{y,1} & \Psi_{z,1} \\ \Psi_{x,2} & \Psi_{y,2} & \Psi_{z,2} \\ \vdots & \vdots & \vdots \\ \Psi_{x,N} & \Psi_{y,N} & \Psi_{z,N} \end{bmatrix} \cdot \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} = \mathrm{L} \cdot \vec{q}$$



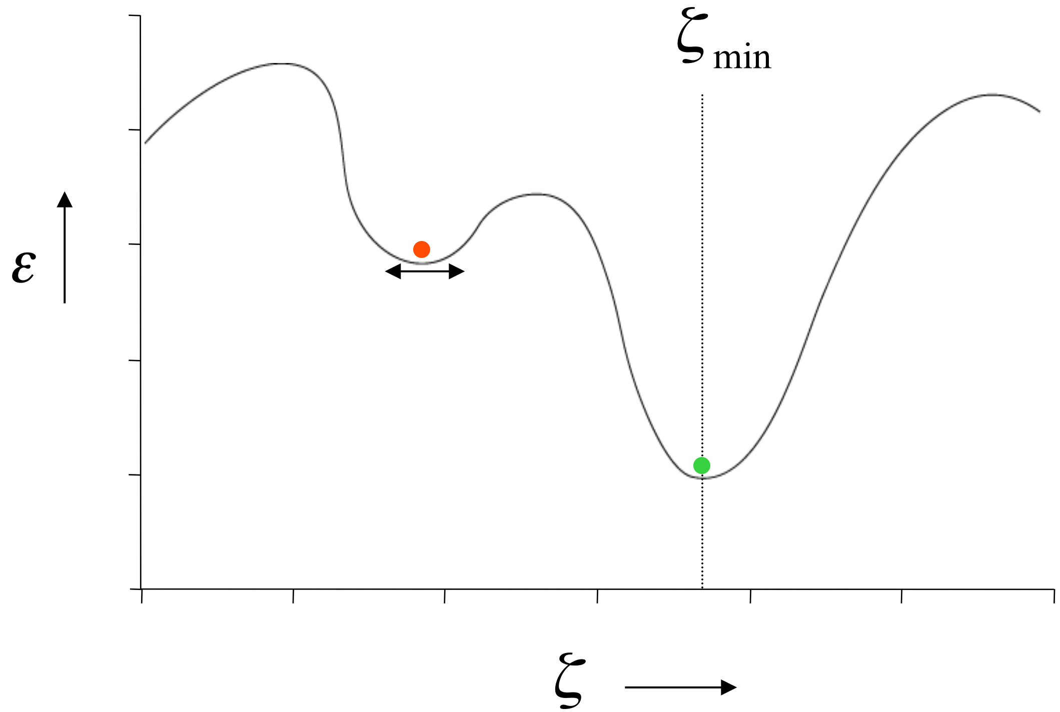
$$\vec{\Psi} = \mathrm{L} \cdot \vec{q}$$
$$= \mathrm{L}(\zeta) \cdot \vec{q}$$

$$\vec{q} = \mathrm{L}^{-1} \cdot \vec{\Psi}$$

Non-linear parameters

$$error(\zeta) = \sum_{i=1}^{N} \left(Y_i(\zeta) - V_i\right)^2 \quad \Rightarrow \quad \min_{\zeta}\left(error(\zeta)\right)$$

$$\zeta = a, b, c, \ldots$$



Non-linear parameters: grid search

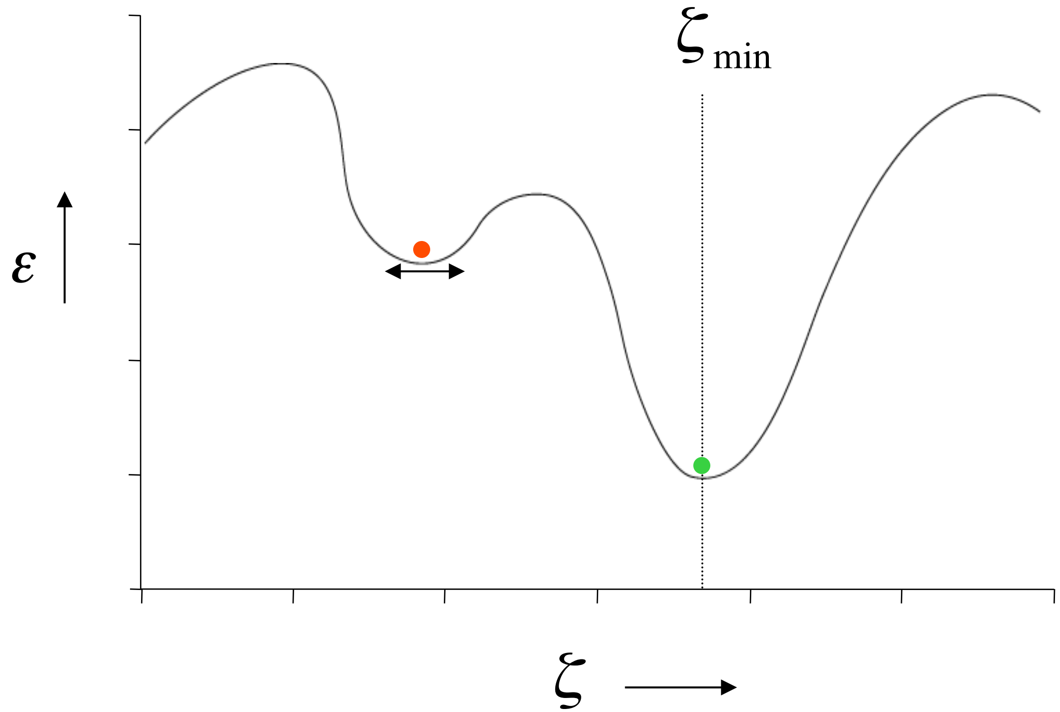
- One dimension, e.g. location along medial-lateral
  - 100 possible locations
- Two dimensions, e.g. med-lat + inf-sup
  - 100x100=10.000
- Three dimensions
  - 100x100x100 = 1.000.000 = 10^6

- Two dipoles, each with three dimensions
  - 100x100x100x100x100x100 = 10^{12}

Non-linear parameters: gradient descent optimization

$$\varepsilon rror(\zeta) = \sum_{i=1}^{N} \left(Y_i(\zeta) - V_i\right)^2 \quad \Rightarrow \quad \min_{\zeta}\left(\varepsilon rror(\zeta)\right)$$

$$\zeta = a, b, c, \ldots$$



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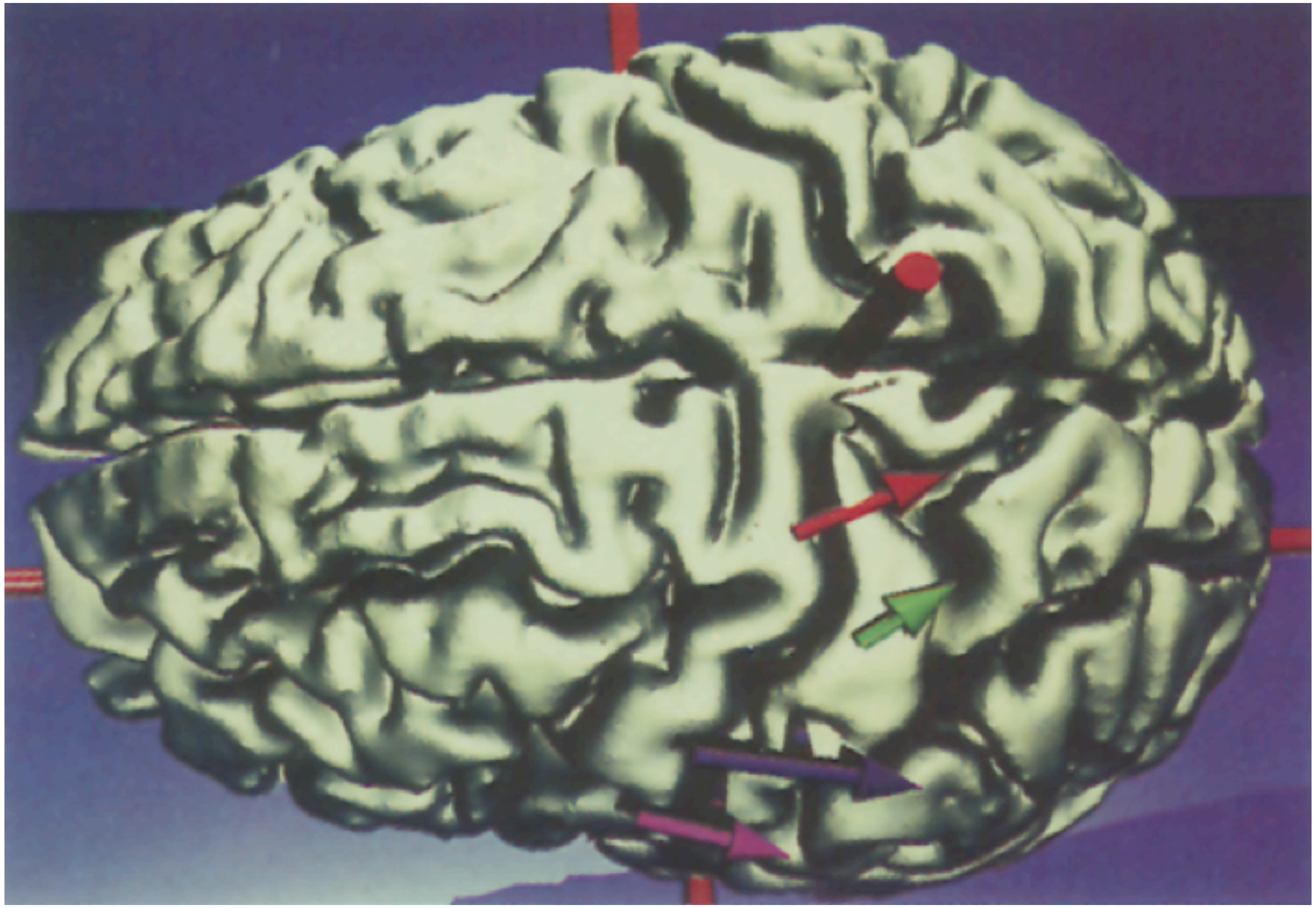
  - **Distributed source models**
  - Spatial filtering

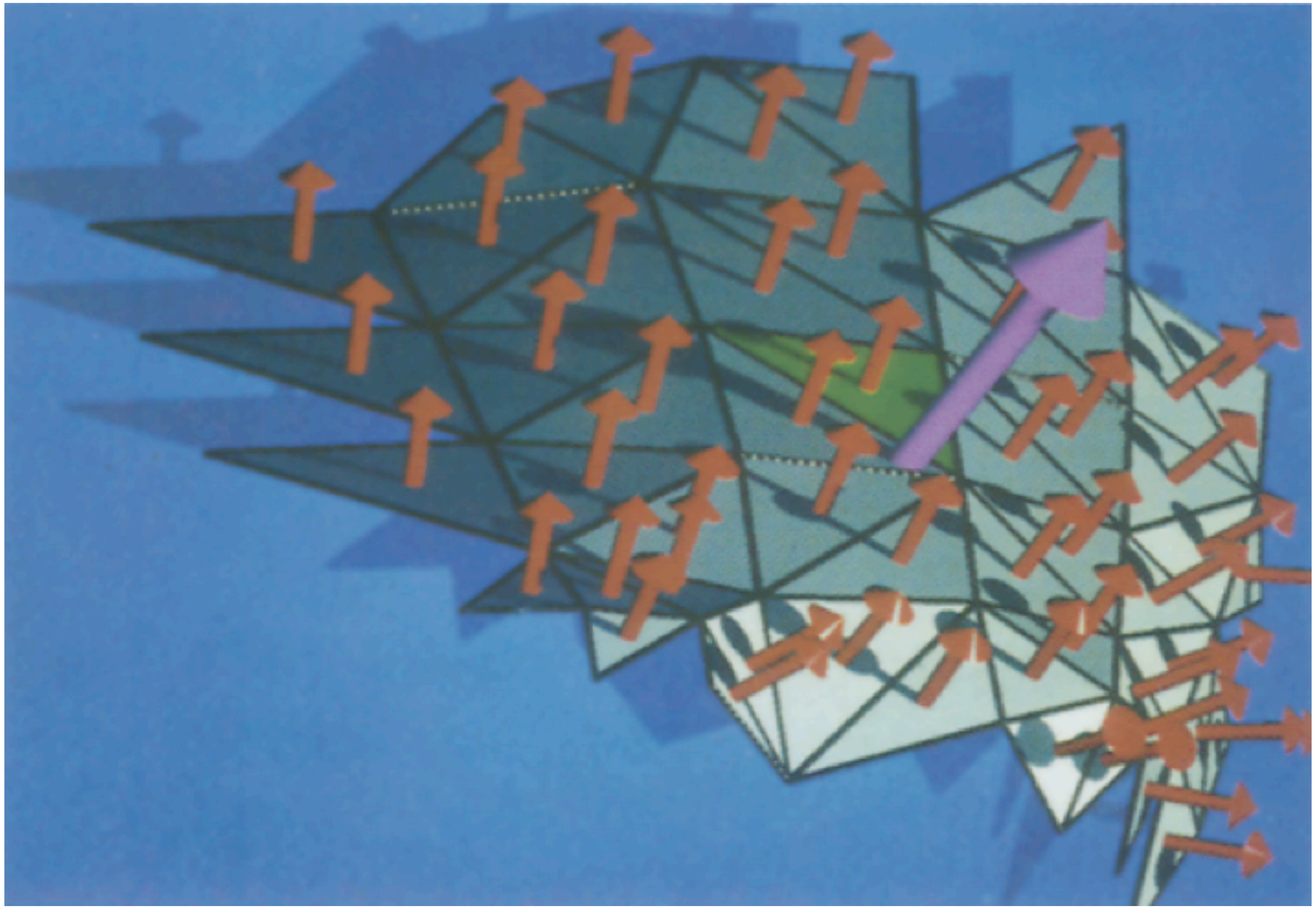
Distributed source model

- Position of the source is not estimated as such
  - Pre-defined grid (3D volume or on cortical sheet)
- Strength is estimated
  - In principle easy to solve, however…
  - More “unknowns” (parameters) than “knowns” (measurements)
  - Infinite number of solutions can explain the data perfectly
  - Additional constraints required
  - Linear estimation problem

Distributed source model



Distributed source model



Distributed source model: linear estimation

$$\vec{\Psi} = q_1\vec{\Psi}_1 + q_2\vec{\Psi}_2 + \ldots \quad = \begin{bmatrix} \Psi_{1,1} & \Psi_{2,1} & \cdots \\ \Psi_{1,2} & \Psi_{2,2} & \cdots \\ \vdots & \vdots & \ddots \\ \Psi_{1,N} & \Psi_{2,N} & \cdots \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ \vdots \end{bmatrix} \quad = \mathrm{L} \cdot \vec{q}$$

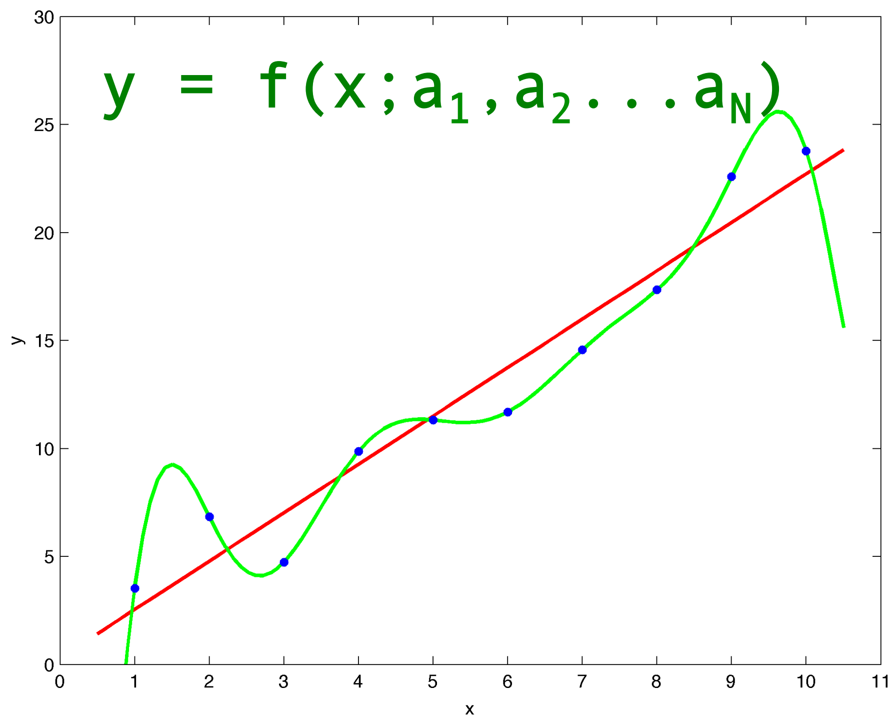
$$\vec{q} = \mathrm{L}^{-1} \cdot \vec{\Psi}$$

Distributed source model: linear estimation

distributed source model with **many dipoles** throughout the whole brain

estimate the strength of all dipoles

data and noise can be perfectly explained



Distributed source model: regularization

$$V = L \cdot q + Noise$$

$$\min_q \{\| V - L \cdot q \|^2\} = 0 \;\; !!$$

Regularized linear estimation:

$$\rightarrow \min_q \{\| V - L \cdot q \|^2 + \lambda \cdot \| D \cdot q \|^2\}$$

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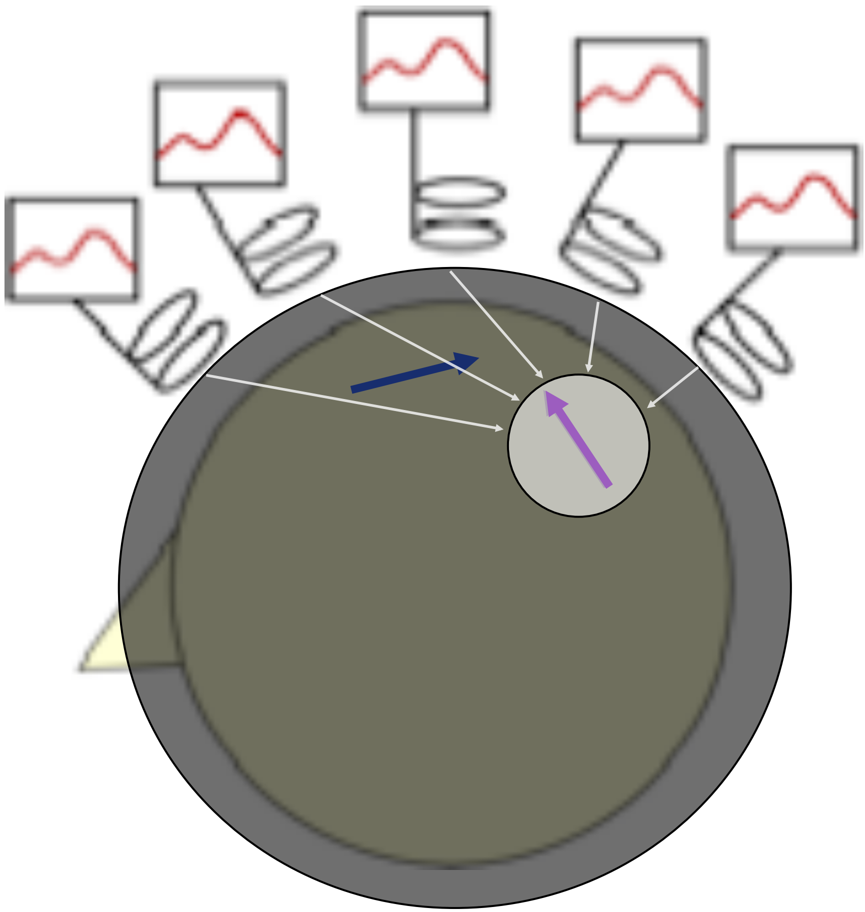
Spatial filtering, beamforming

- Position of the source is not estimated as such
- Manipulate filter properties, not source properties
  - No explicit assumptions about source constraints (implicit: single dipole)
  - Assumptions about data (multiple sources should be sufficiently uncorrelated)

Spatial filtering, beamforming



$w^T * h = 1$

$w^T * h = 1$

$w^T * h = 0$

Estimating source timecourse activity

$$M = G_1X_1 + G_2X_2 + \ldots + G_nX_n + \text{noise}$$

$$M = G\ X\ + \text{noise}$$

WARNING: the letters are used differently in various slides

here	**G** = gain matrix, **X** = source activity, **M** = measurement
elsewhere	**H** = gain matrix, **S** = source activity, **X** = measurement
or	**L** = gain matrix, **Q** = source activity, **V** = measurement

Estimating source timecourse activity using dipole fitting

$$M = G_1X_1 + G_2X_2 + \ldots + G_nX_n + \text{noise}$$

n is typically small

$$(M - G_1X_1 \qquad\qquad\qquad) = \text{residual}$$

measured data model data

$X' = W\,M$, where $W = G^T\,(G\,G^T)^{-1}$

Estimating source timecourse activity using distributed source models

$M = G_1X_1 + G_2X_2 + ... + G_nX_n + \text{noise}$

n is typically large (> # channels)

$M = (G_1X_1 + G_2X_2 + ... + G_nX_n) + \text{noise}$

$M = G\,X + \text{noise}$

$X' = W\,M$, where W ensures $\min_X \{\| M - G \cdot X \|^2 + \lambda \cdot \| X \|^2\}$

Estimating source timecourse activity using spatial filtering

$$M = G_1X_1 + G_2X_2 + \dots + G_nX_n + \text{noise}$$

any number of n

$$M = (G_1X_1 + G_2X_2 + \dots) + G_nX_n + (\text{noise})$$

$$X'_n = W_n M, \quad \text{where} \quad W^T = [G_n^T C_M^{-1} G_n]^{-1} G_n^T C_M^{-1}$$

Estimating source timecourse activity

$$M = G_1X_1 + G_2X_2 + \ldots + G_nX_n + \text{noise}$$

few sources

distributed sources

one at a time

$$X'(t) = W\,M(t)$$

dipole fitting

minimum norm estimate

beamforming

Estimating source spectral activity

$$M = G_1X_1 + G_2X_2 + \ldots + G_nX_n + \text{noise}$$

few sources

distributed sources

one at a time

$$X'(f) = W\,M(f)$$

dipole fitting

minimum norm estimate

beamforming

Summary 1

Forward modelling

- Required for the interpretation of scalp topographies
- Interpretation of scalp topography *is* "source estimation"
- Mathematical techniques are available that aid in interpreting scalp topographies -> inverse modelling

Summary 2

Inverse modeling

- Model assumption for volume conductor
- Model assumption for source, i.e. dipole
- Additional assumptions on source
  - Single point-like source
  - Multiple point-like sources
  - Distributed source
- Different mathematical approaches
  - Dipole fitting (linear and nonlinear part)
  - Linear estimation (regularized)
  - Spatial filtering

Summary 3: disentangling the superposition