



**Donders Institute**  
for Brain, Cognition and Behaviour

# Fundamentals of the analysis of neuronal oscillations

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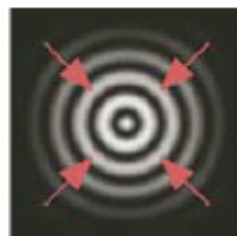
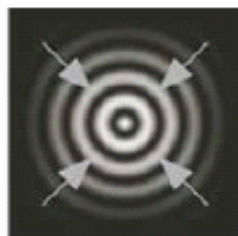
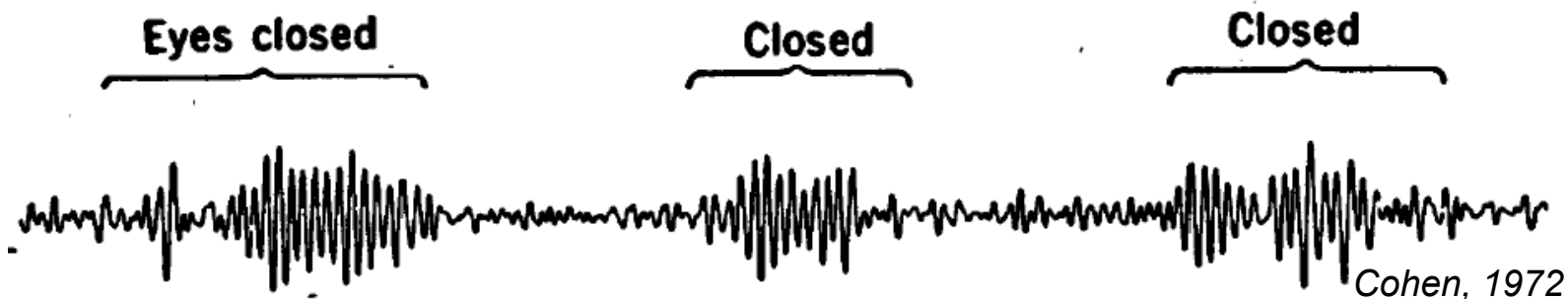
## Separating sources

- Use the temporal aspects of the data at the channel level
  - ERF latencies
  - (ERF difference waves)
  - Filtering the time-series
  - Spectral decomposition
- Use the spatial aspects of the data

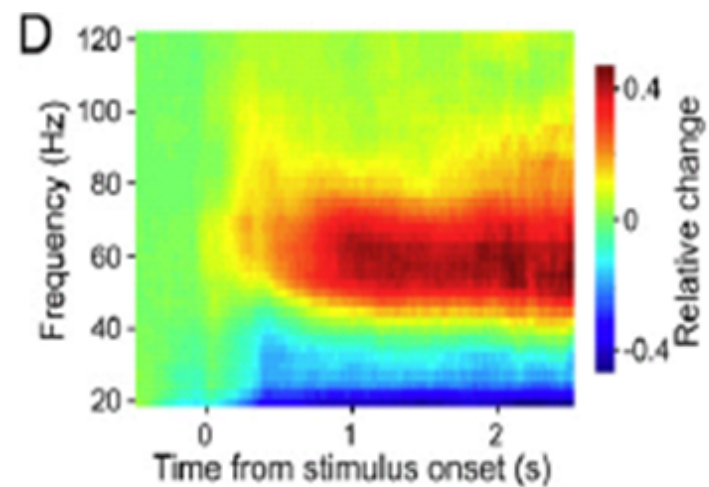




## Brain signals contain oscillatory activity at multiple frequencies



Hoogenboom et al, 2006





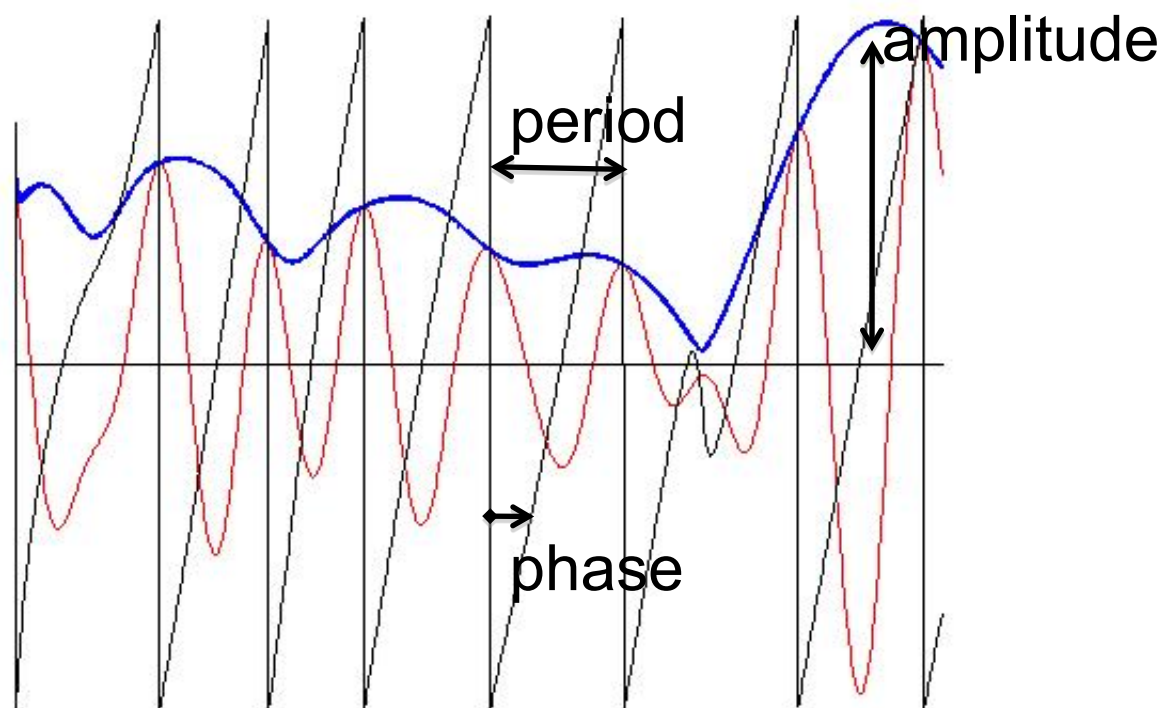
## Outline

- Spectral analysis: going from time to frequency domain
- Issues with finite and discrete sampling
- Spectral leakage and (multi-)tapering
- Time-frequency analysis





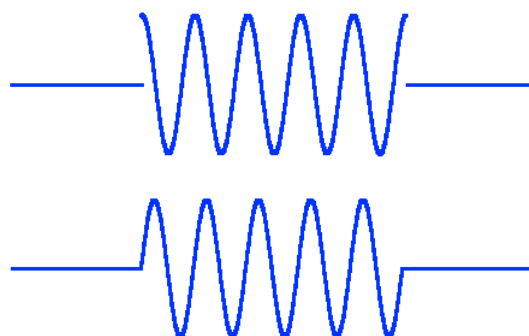
## A background note on oscillations



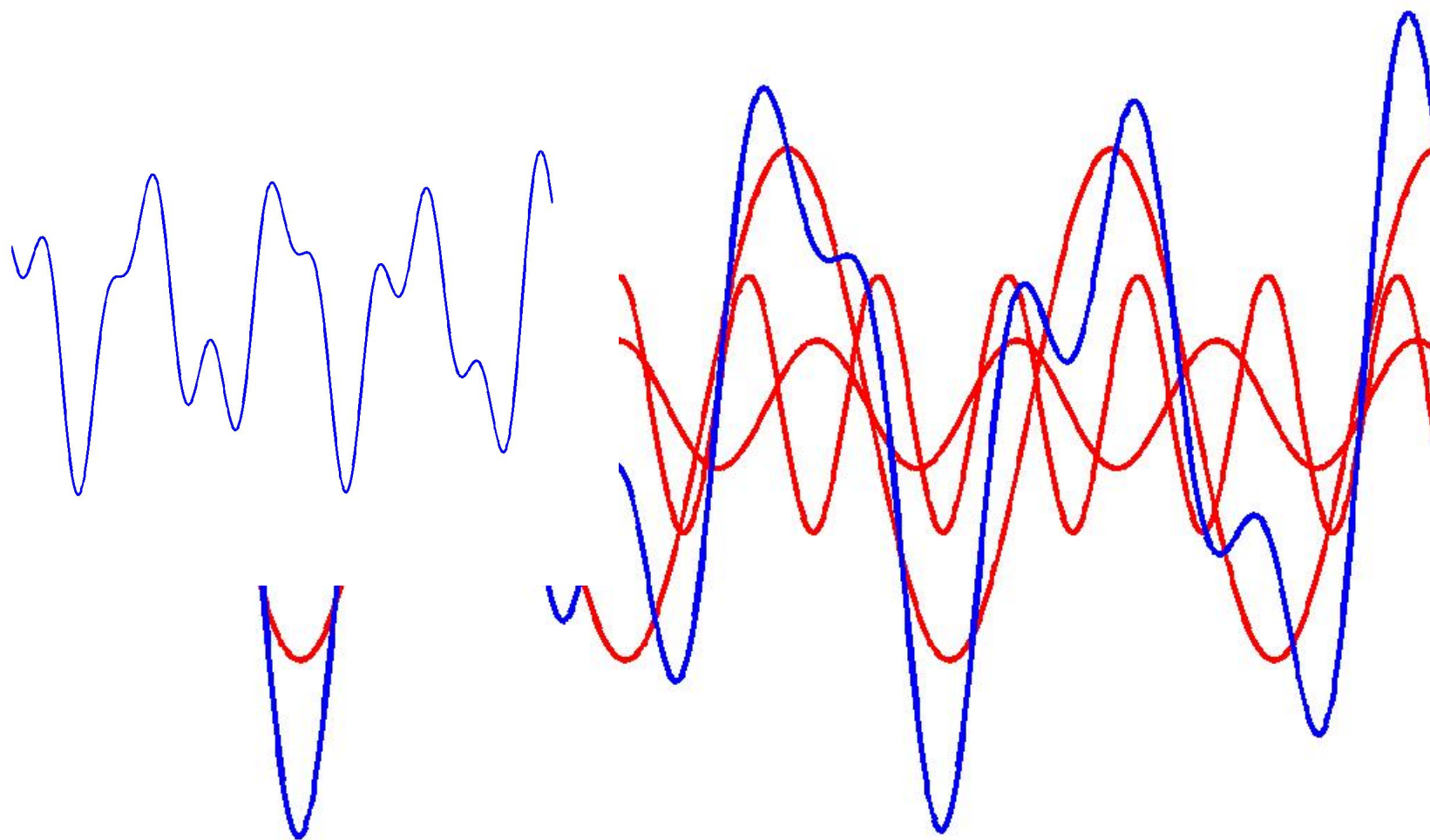


## Spectral analysis

- Deconstructing a time domain signal into its constituent oscillatory components, a.k.a. Fourier analysis
- Using simple oscillatory functions: cosines and sines

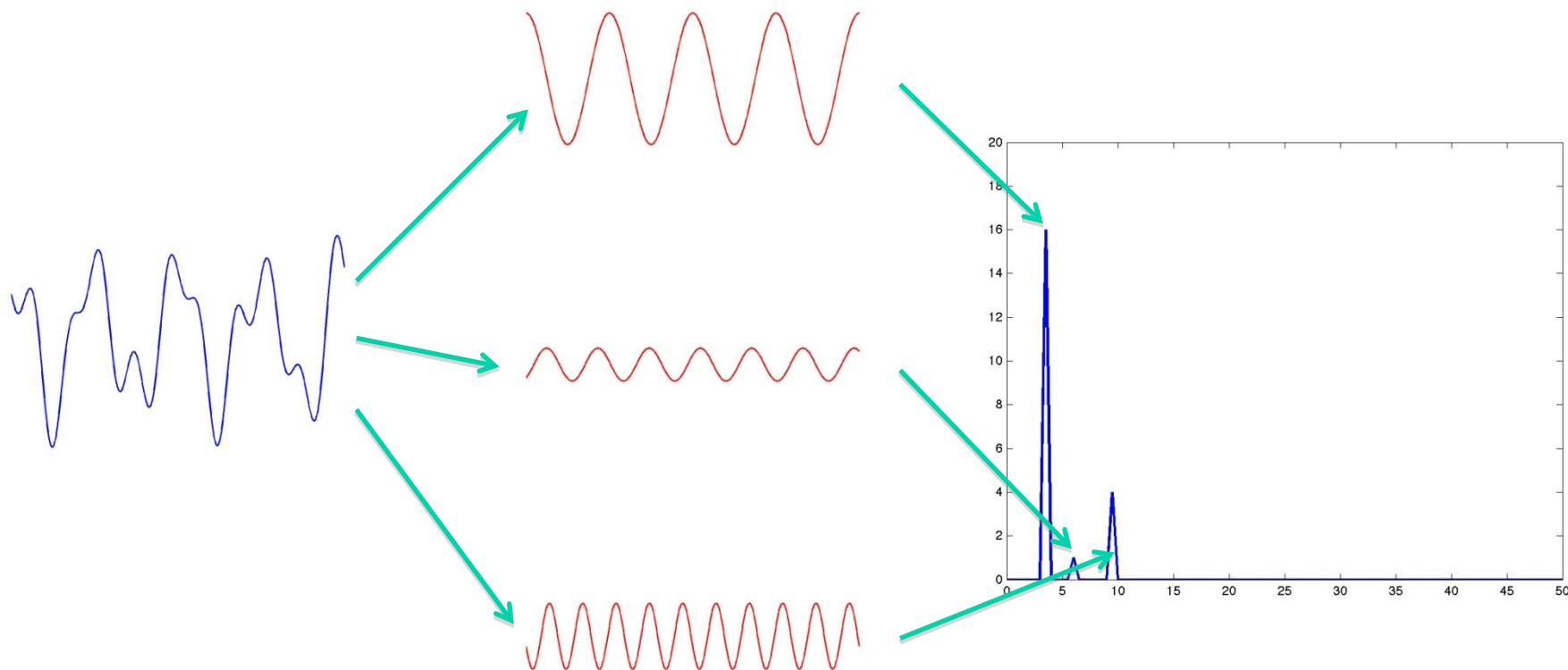


## Spectral decomposition: the principle





# Spectral decomposition: the power spectrum







## Spectral analysis

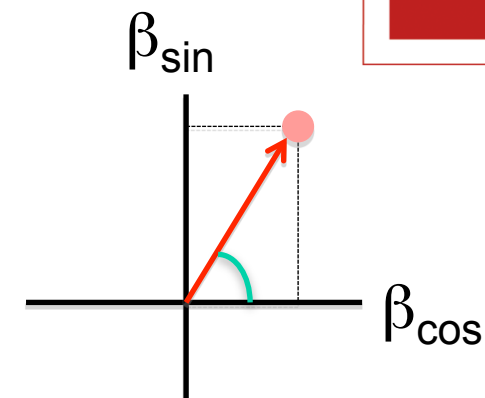
- Deconstructing a time domain signal into its constituent oscillatory components, a.k.a. Fourier analysis
- Using simple oscillatory functions: cosines and sines
- Express signal as function of frequency, rather than time
- Concept: linear regression using oscillatory basis functions



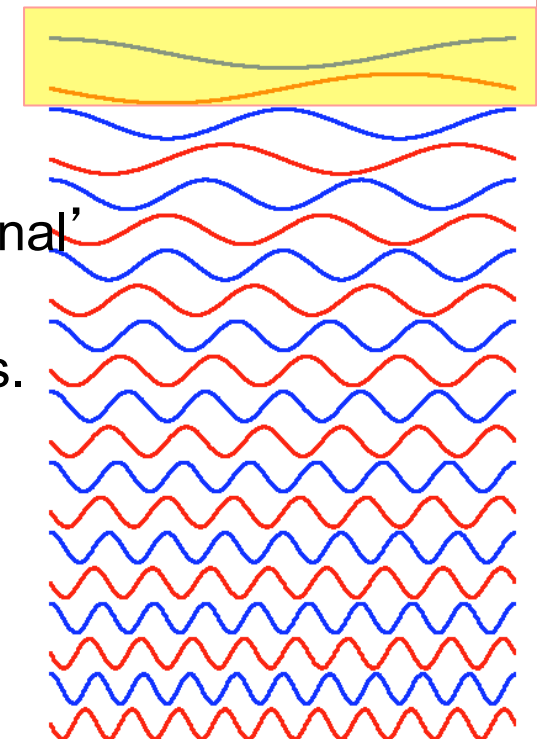


## Spectral analysis ~ regression

- $\mathbf{Y} = \beta \times \mathbf{X}$
- $\mathbf{X}$  : set of basis functions
- $\beta_i \sim$  'goodness-of-fit' of basis function  $i$  with data
- $\beta$  for cosine and sine components for a given frequency map onto amplitude and phase estimate.



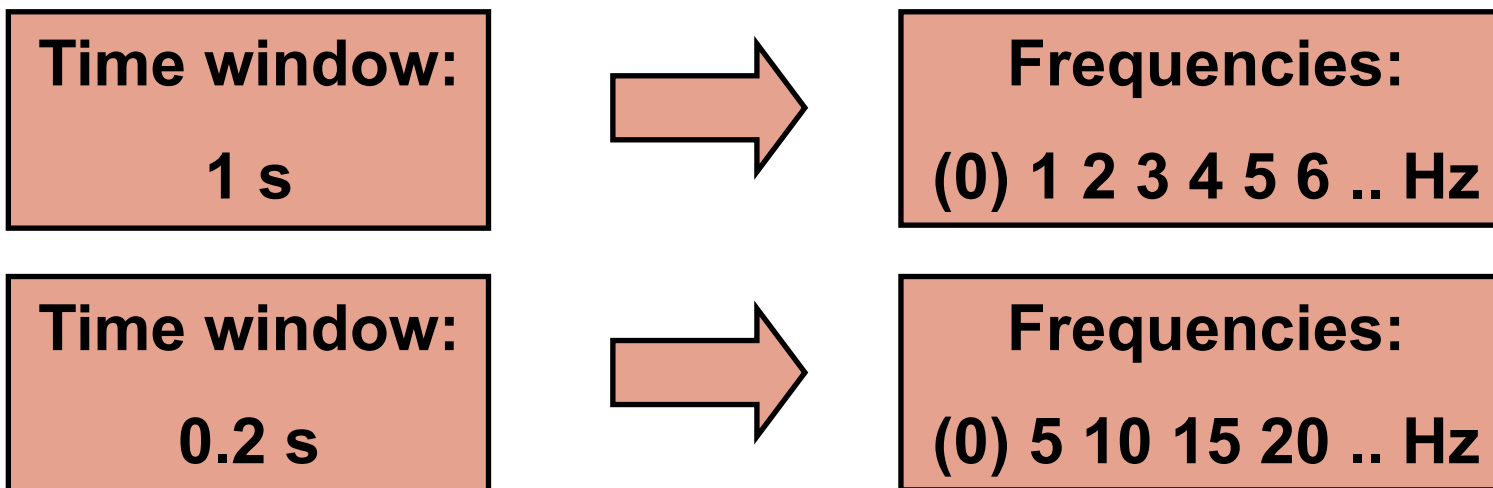
- Restriction: basis functions should be 'orthogonal'
- Consequence 1: frequencies not arbitrary -> integer amount of cycles should fit into N points.
- Consequence 2: N-point signal -> N basis functions





## Time-frequency relation

- Consequence 1: frequencies not arbitrary -> integer amount of cycles should fit into N points (of length T).
- The frequency resolution is determined by the length of the data segments (T)
- Rayleigh frequency =  $1/T = \Delta f$  = frequency resolution



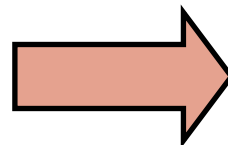


## Time-frequency relation

- Consequence 2: N-point signal  $\rightarrow$  N basis functions
- N basis functions  $\rightarrow$  N/2 frequencies
- The highest frequency that can be resolved depends on the sampling frequency F
- Nyquist frequency =  $F/2$

**Sampling freq 1 kHz**

**Time window 1 s**

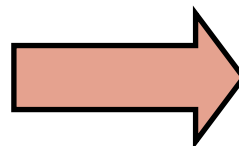


**Frequencies:**

**(0) 1 2 ... 499 500 Hz**

**Sampling freq 400 Hz**

**Time window 0.25 s**



**Frequencies:**

**(0) 4 8... 196 200 Hz**





## Spectral analysis

- Deconstructing a time domain signal into its constituent oscillatory components, a.k.a. Fourier analysis
- Using simple oscillatory functions: cosines and sines
- Express signal as function of frequency, rather than time
- Concept: linear regression using oscillatory basis functions
- Each oscillatory component has an amplitude and phase
- Discrete and finite sampling constrains the frequency axis





## Goal and challenges

- Estimate the true oscillations from the observed data
- Limited time available for Fourier transform
- You are looking at the activity through a time restricting window



- This implicitly means that the data are ‘tapered’ with a boxcar
- Data are discretely sampled

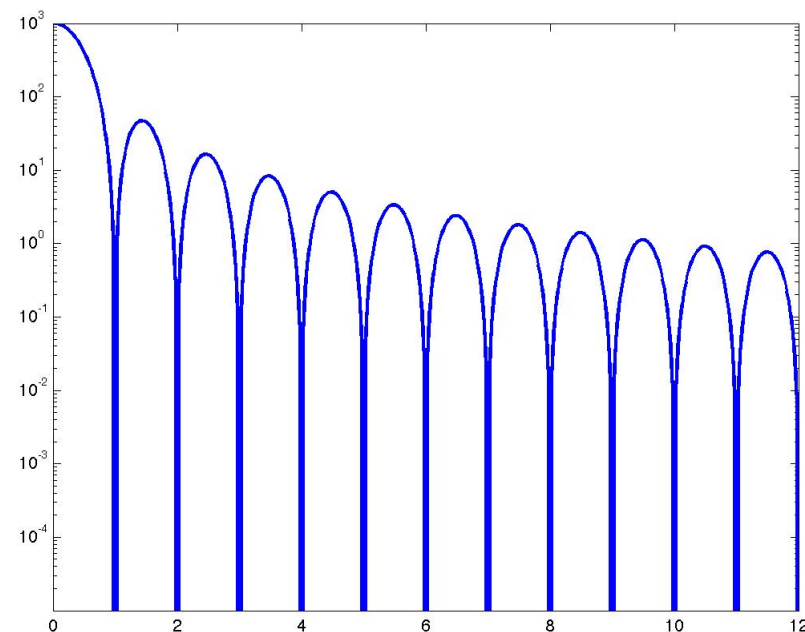
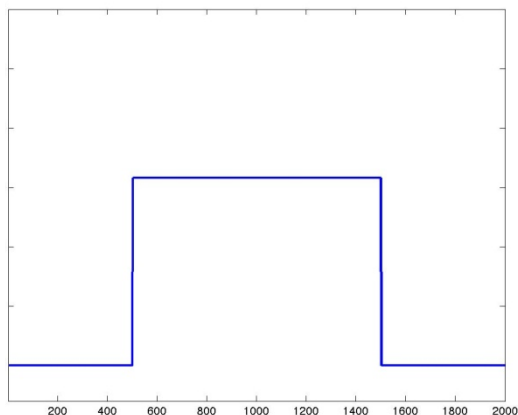






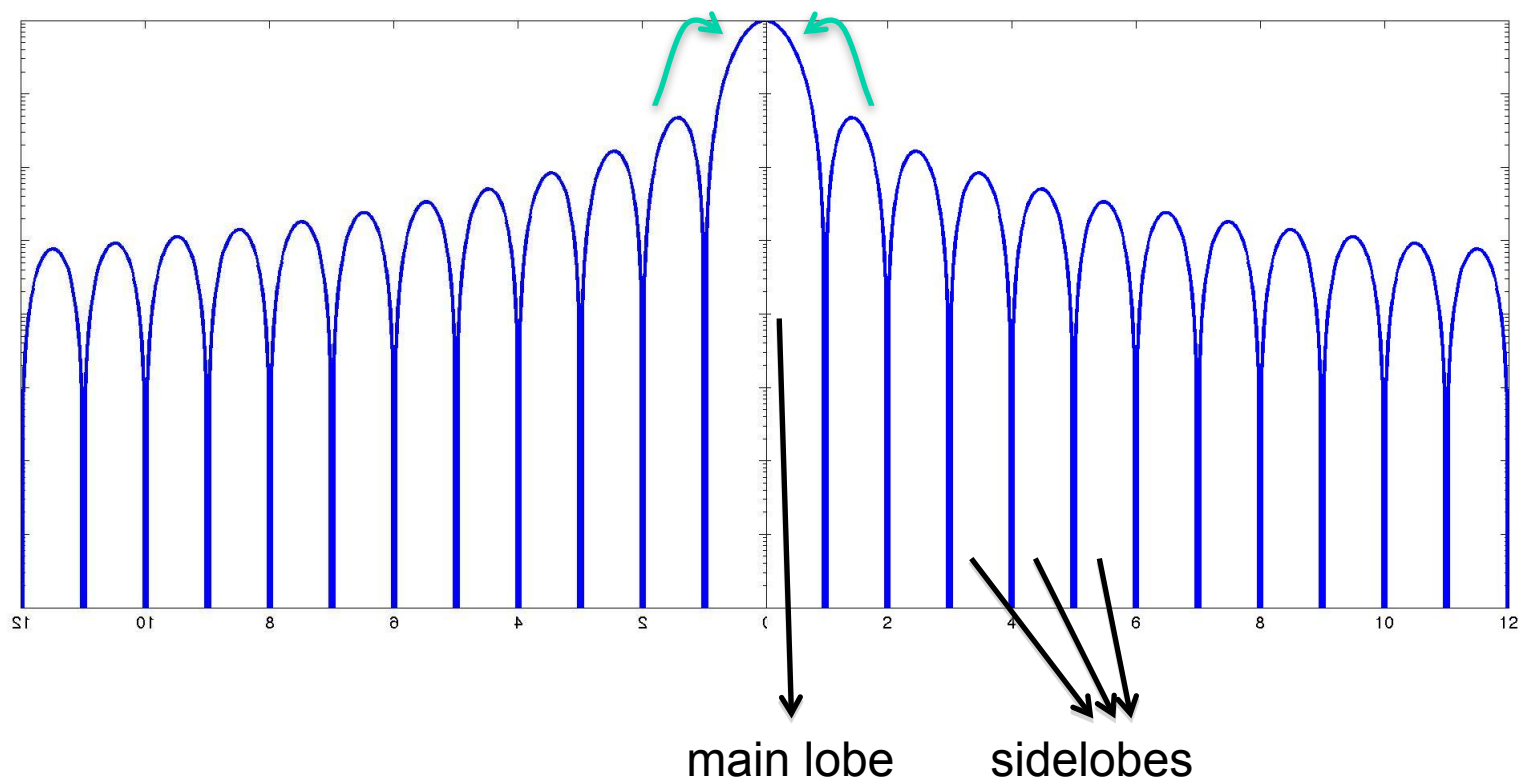
## Spectral leakage and tapering

- True oscillations in data at frequencies **not sampled** with Fourier transform **spread their energy** to the sampled frequencies
- Not tapering = applying a boxcar taper
- Each type of taper has a specific leakage profile





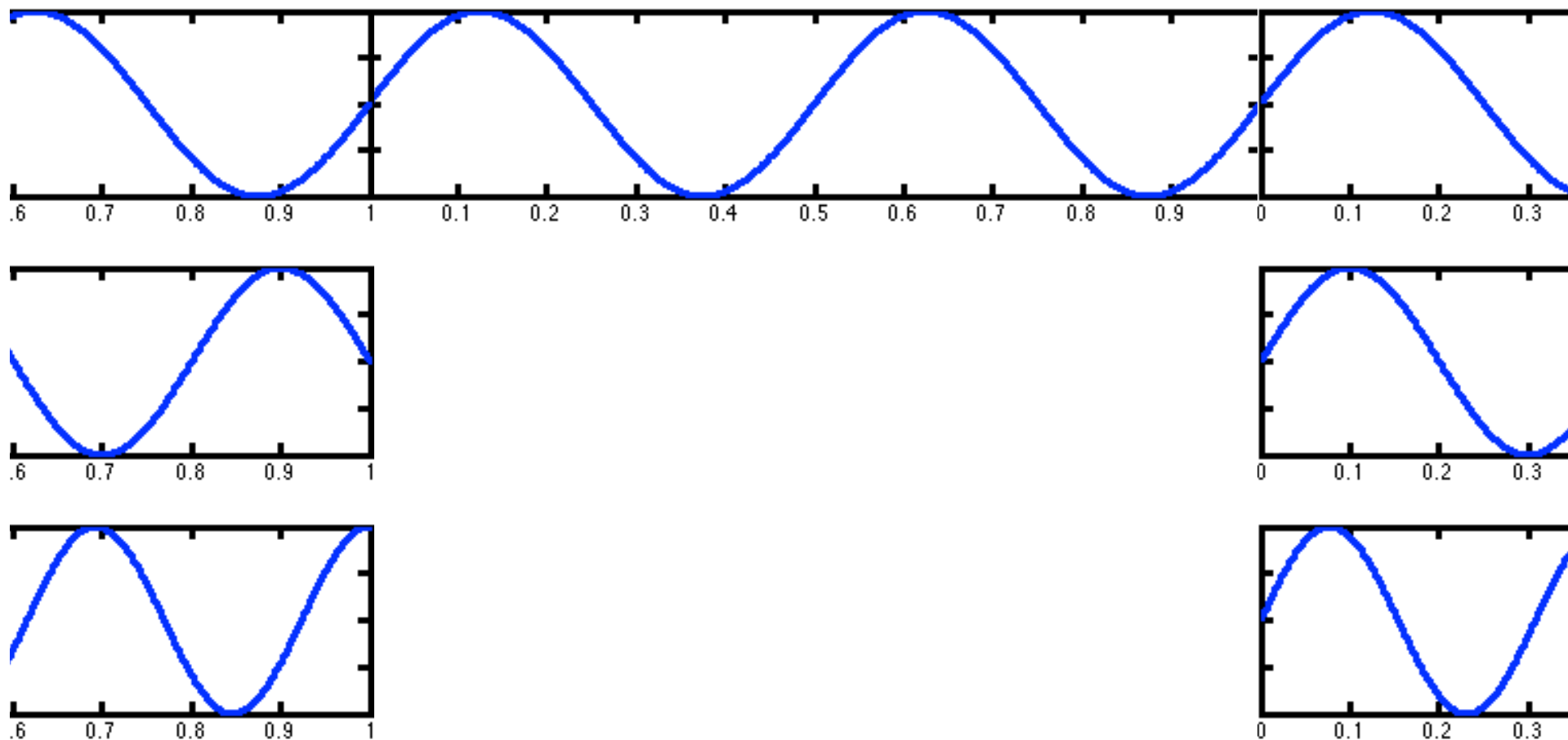
# Spectral leakage





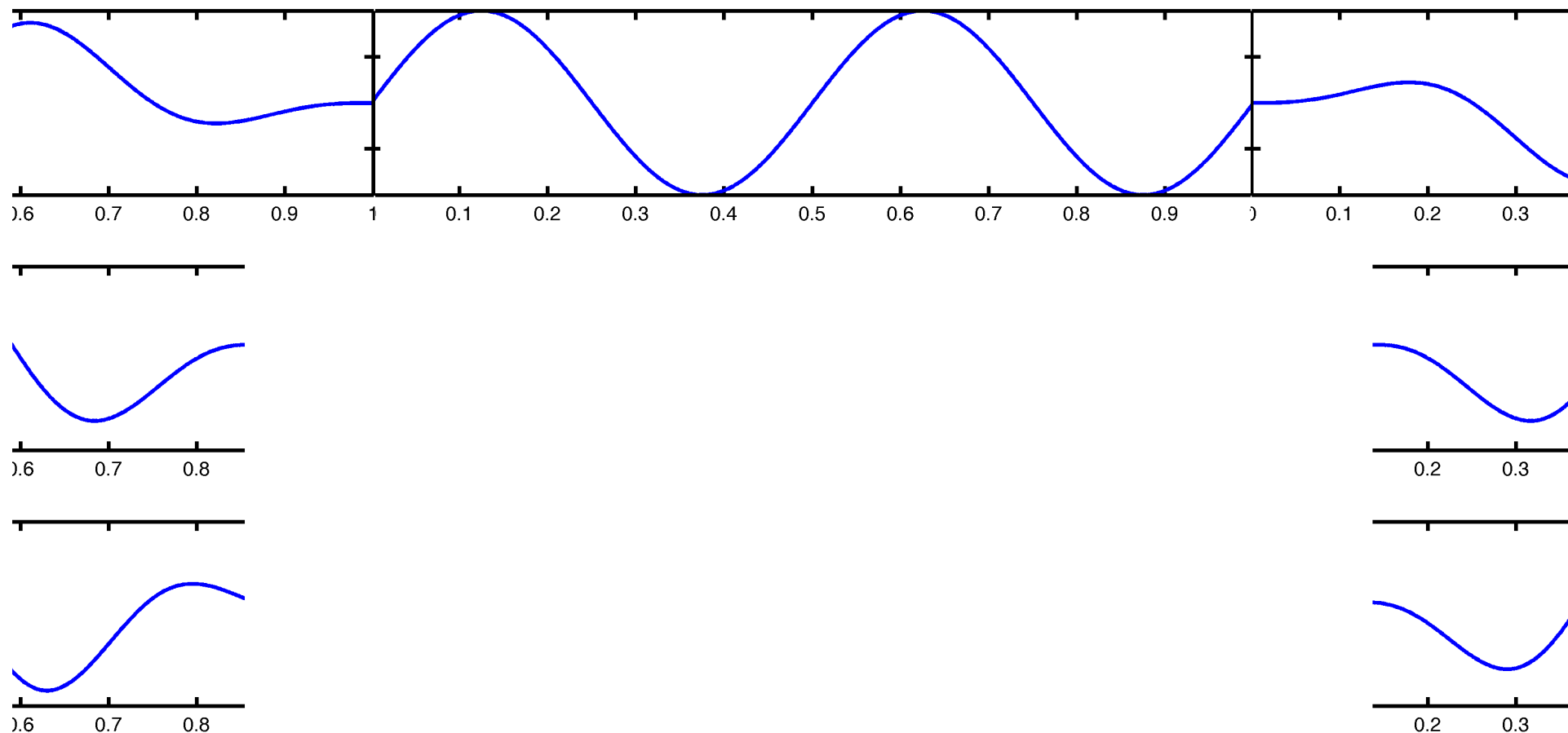


## Tapering in spectral analysis



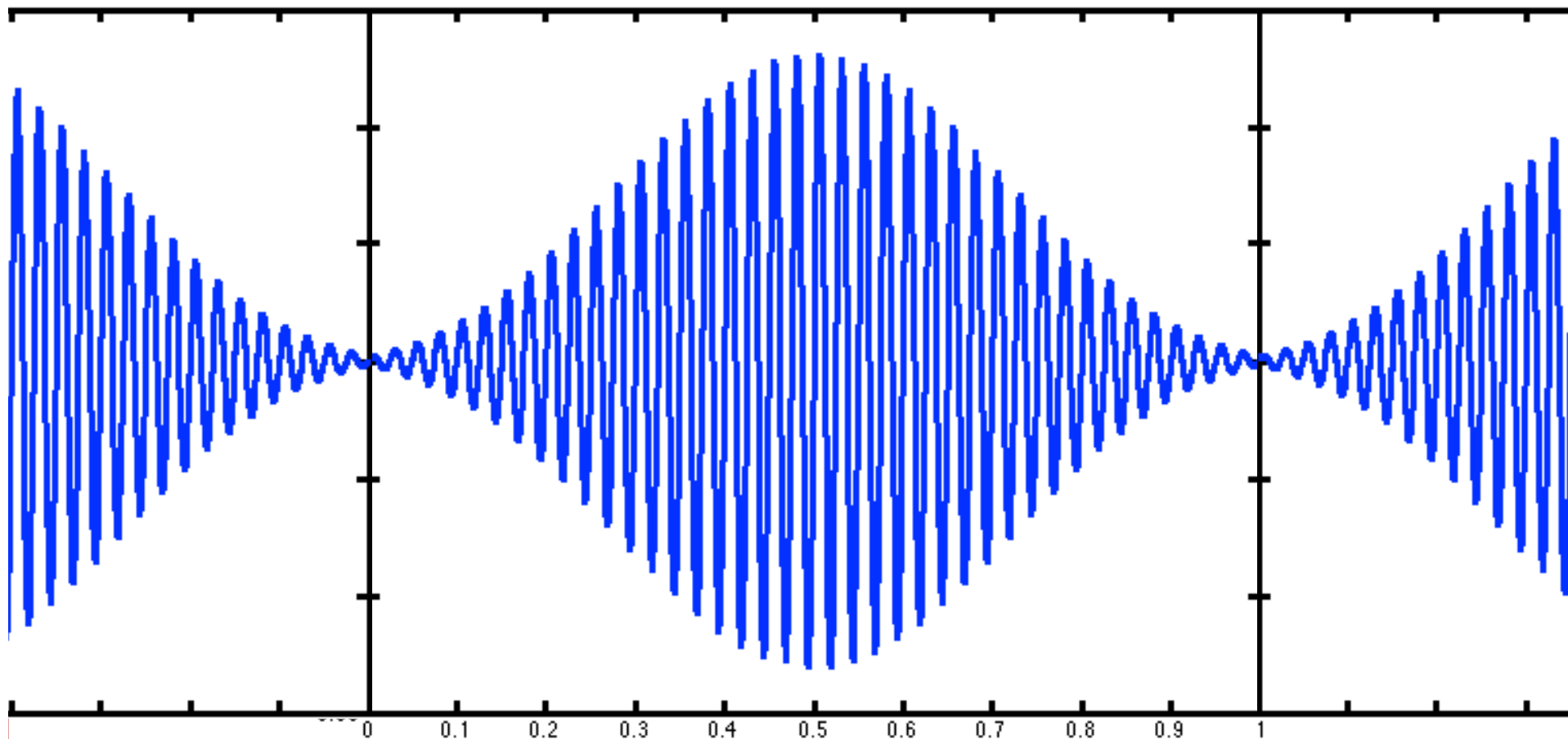


# Tapering in spectral analysis





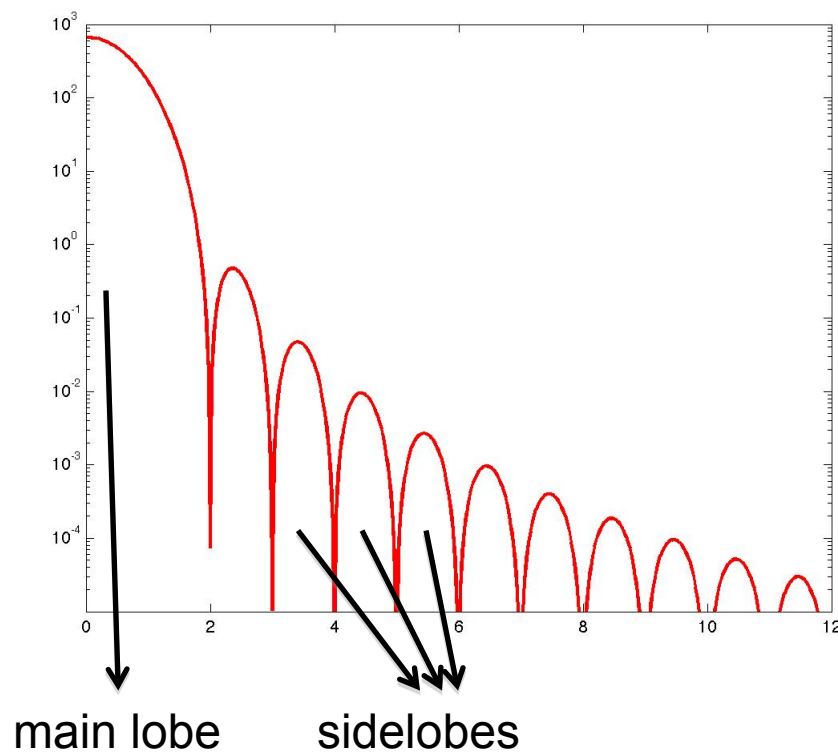
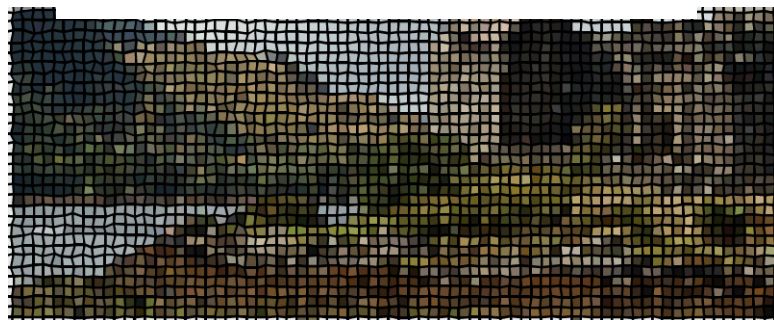
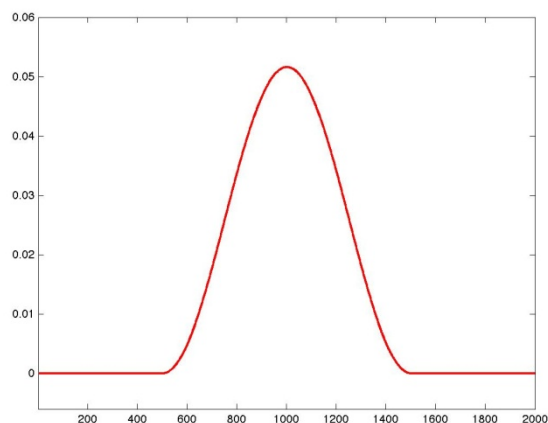
## Tapering in spectral analysis





## Spectral leakage and tapering

- True oscillations in data at frequencies not sampled with Fourier transform spread their energy to the sampled frequencies
- Not tapering = applying a boxcar taper
- Each type of taper has a specific leakage profile



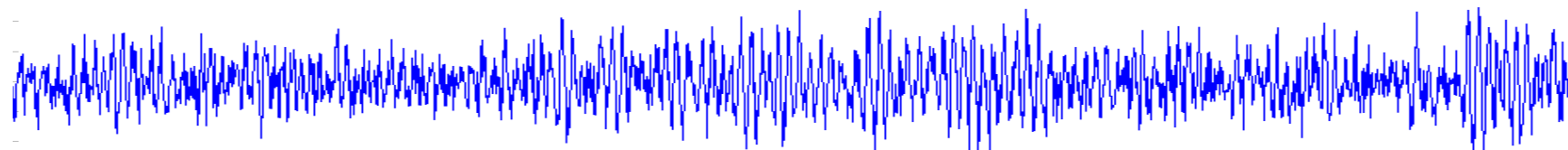


## Multitapers

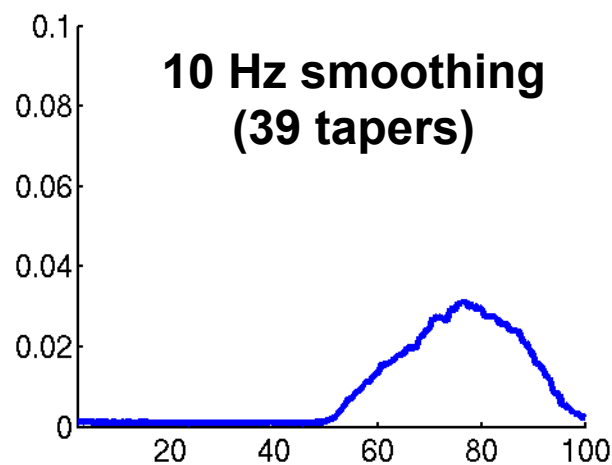
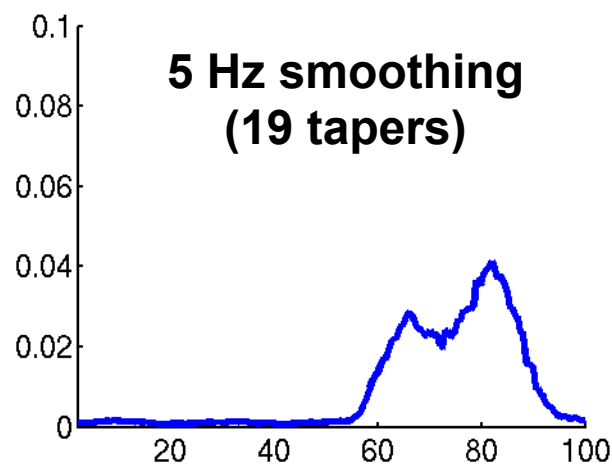
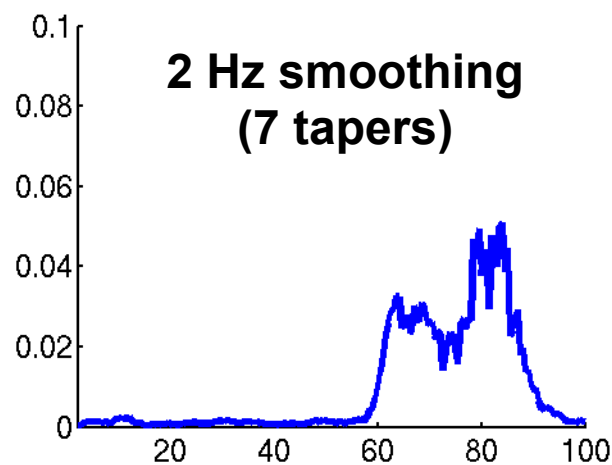
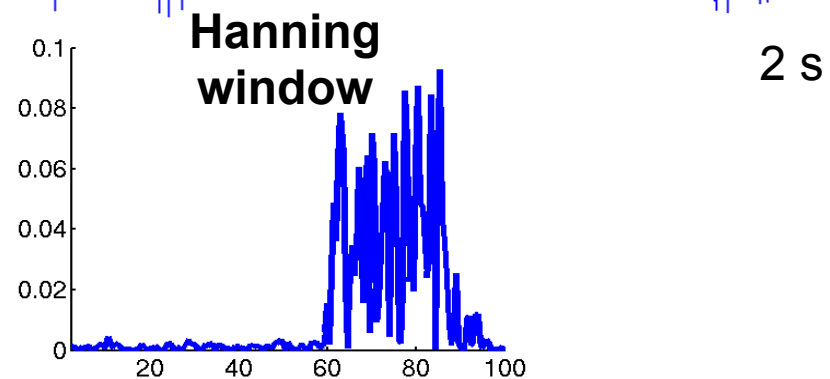
- Make use of more than one taper and combine their properties
- Used for smoothing in the frequency domain
- Instead of “smoothing” one can also say “controlled leakage”



# Multitapered spectral analysis

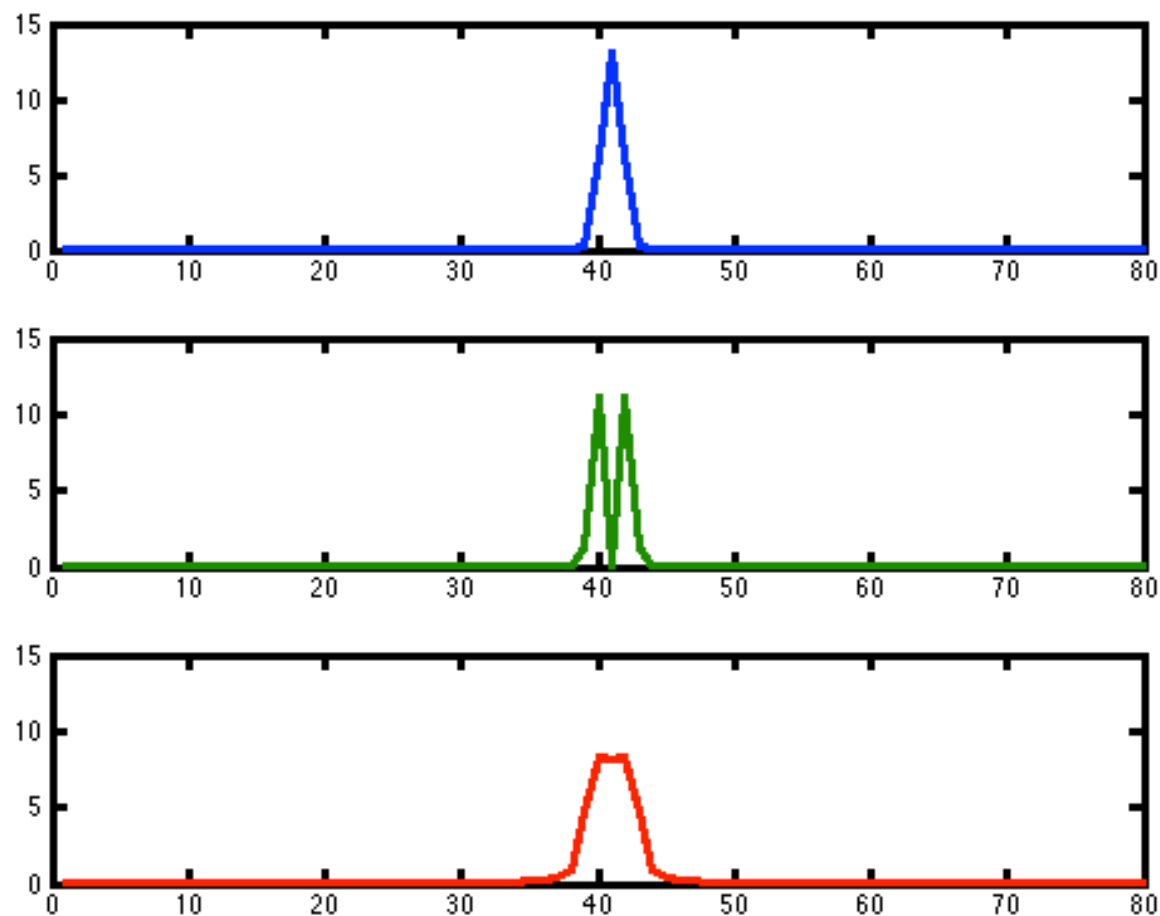


**broadband activity  
between 60-90 Hz**



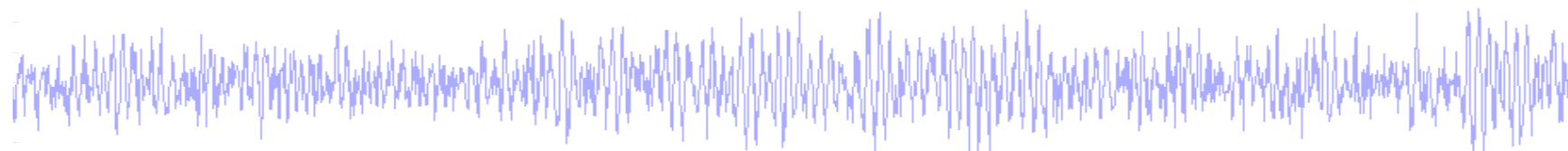


## Multitapered spectral analysis

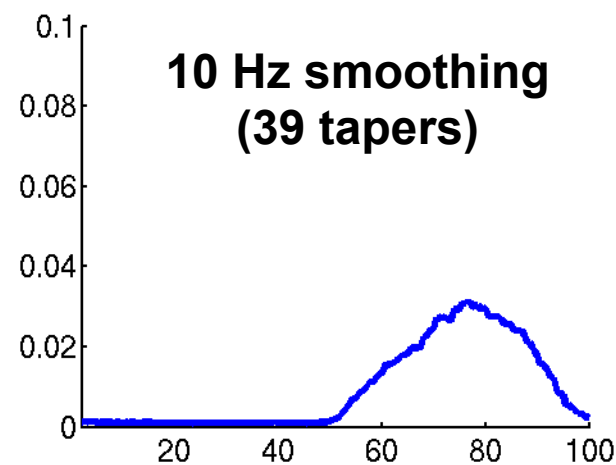
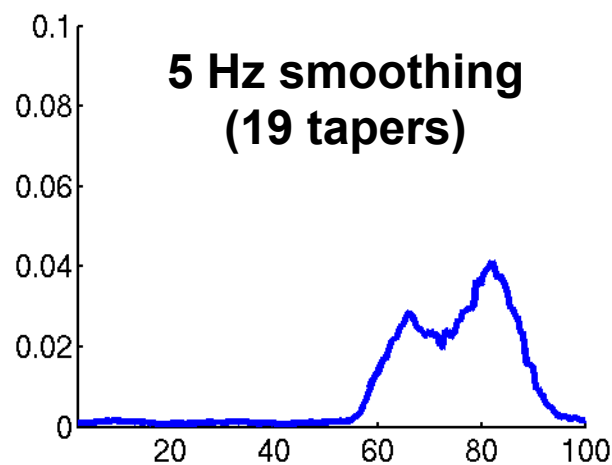
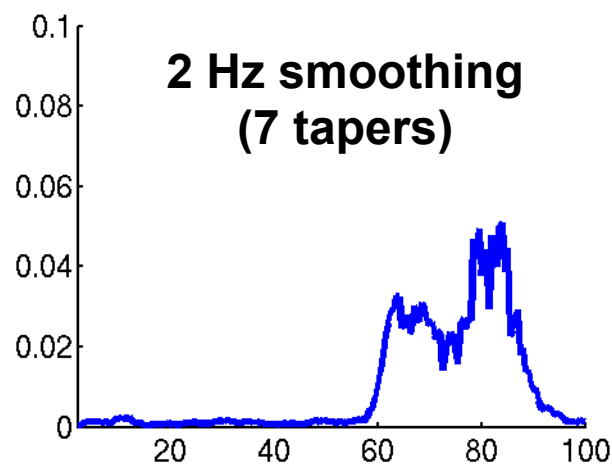
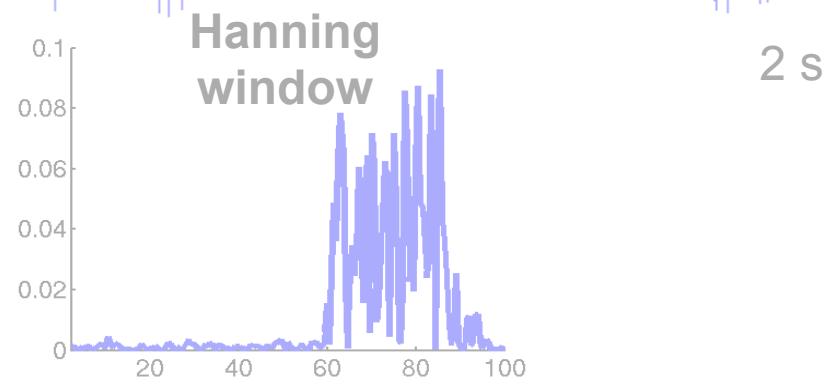




# Multitapered spectral analysis



broadband activity  
between 60-90 Hz







## Multitapers

- Multitapers are useful for reliable estimation of high frequency components
- Low frequency components are better estimated using a single (Hanning) taper

```
%estimate low frequencies
```

```
cfg = [];  
cfg.method = 'mtmfft';  
cfg.foilim = [1 30];  
cfg.taper = 'hanning';  
.  
.  
.  
freq=ft_freqanalysis(cfg, data);
```

```
%estimate high frequencies
```

```
cfg = [];  
cfg.method = 'mtmfft';  
cfg.foilim = [30 120];  
cfg.taper = 'dpss';  
cfg.tapsmofrq = 8;  
.  
.  
freq=ft_freqanalysis(cfg, data);
```





## Sub summary

- Spectral analysis
  - Decompose signal into its constituent oscillatory components
  - Focused on ‘stationary’ power
- Tapers
  - Boxcar, Hanning, Gaussian
- Multitapers
  - Control spectral leakage/smoothing





## Time-frequency analysis

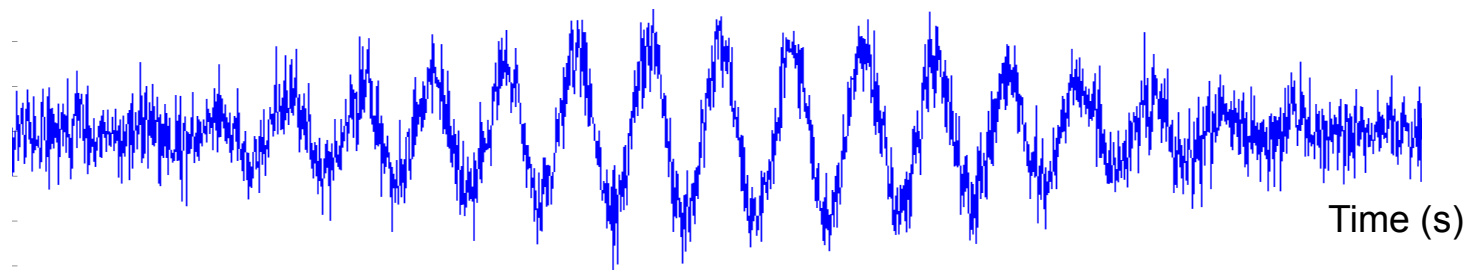
- Typically, brain signals are not ‘stationary’
- Divide the measured signal in shorter time segments and apply Fourier analysis to each signal segment
- Everything we saw so far with respect to frequency resolution applies here as well

```
cfg = [];  
cfg.method = 'mtmconvol';  
.  
.  
.  
freq = ft_freqanalysis(cfg, data);
```



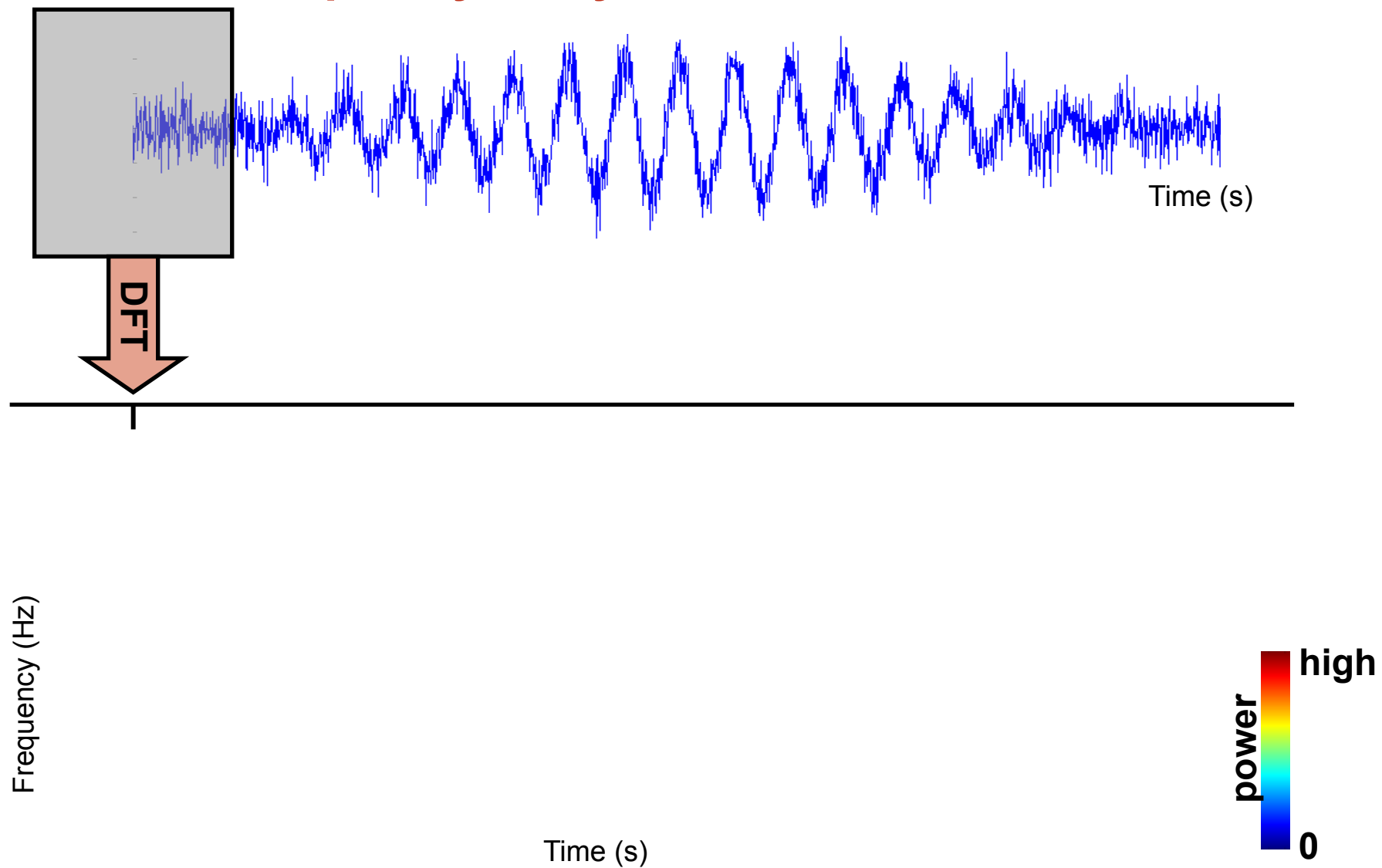


## Time frequency analysis



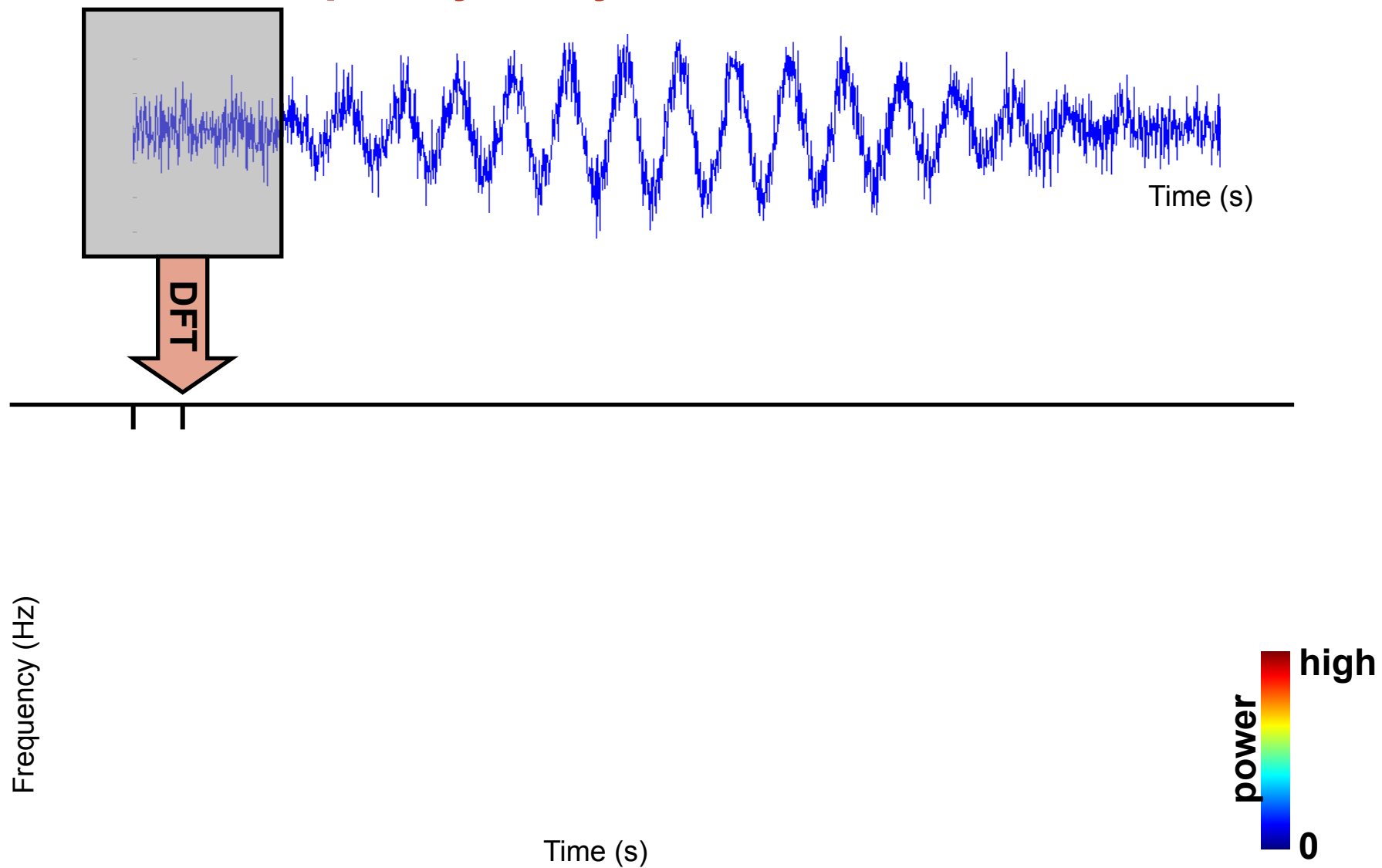


# Time frequency analysis



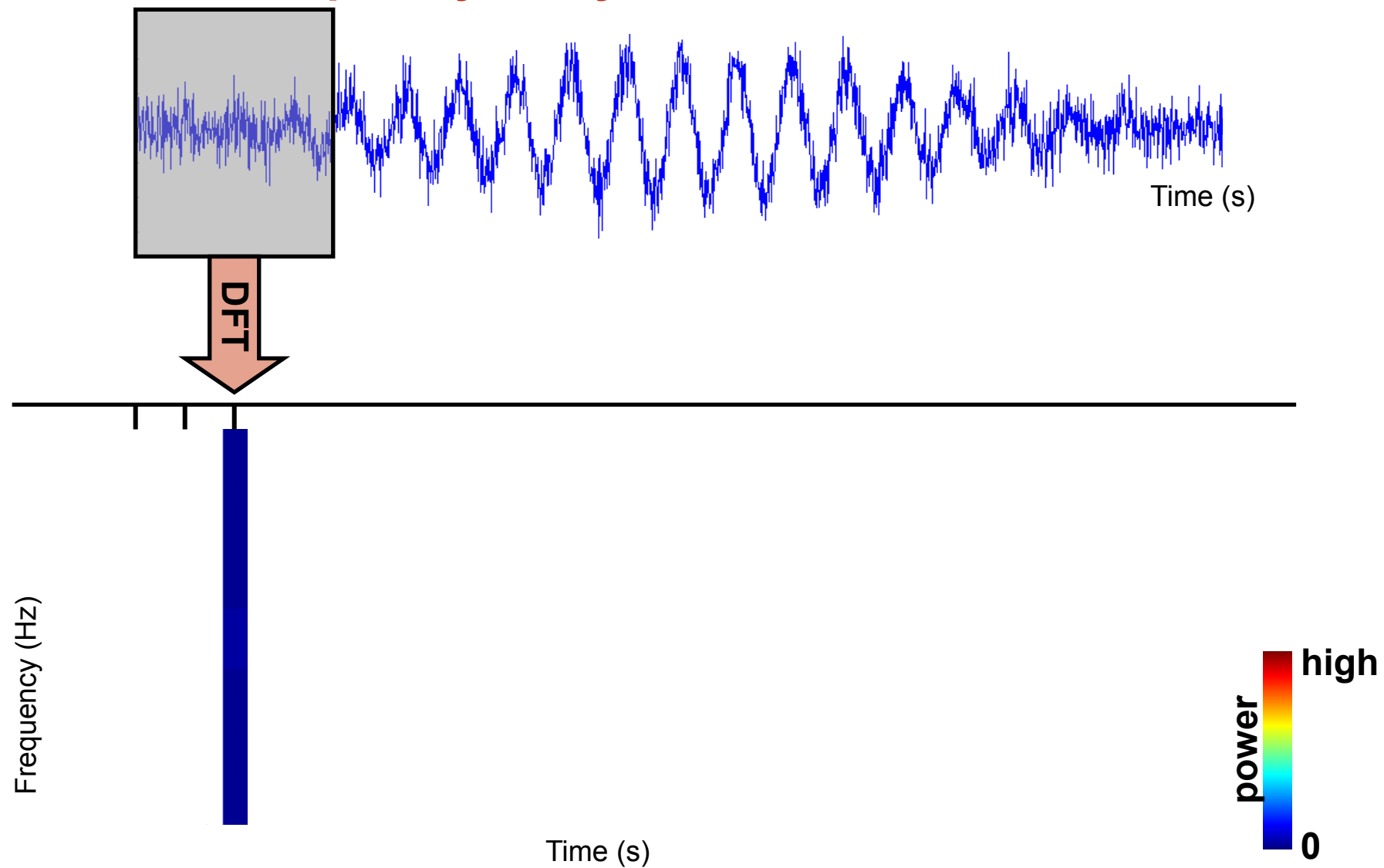


# Time frequency analysis



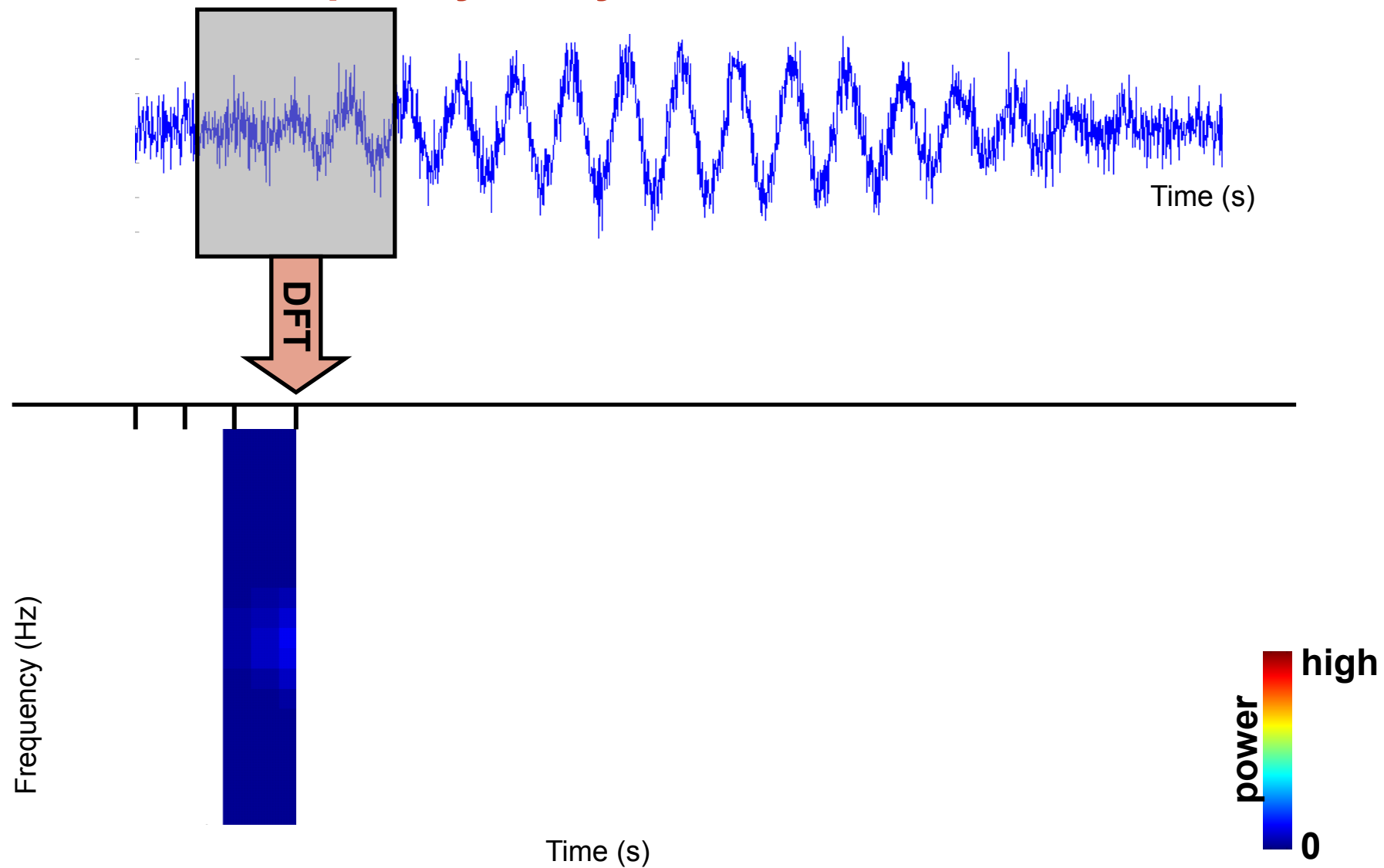


# Time frequency analysis





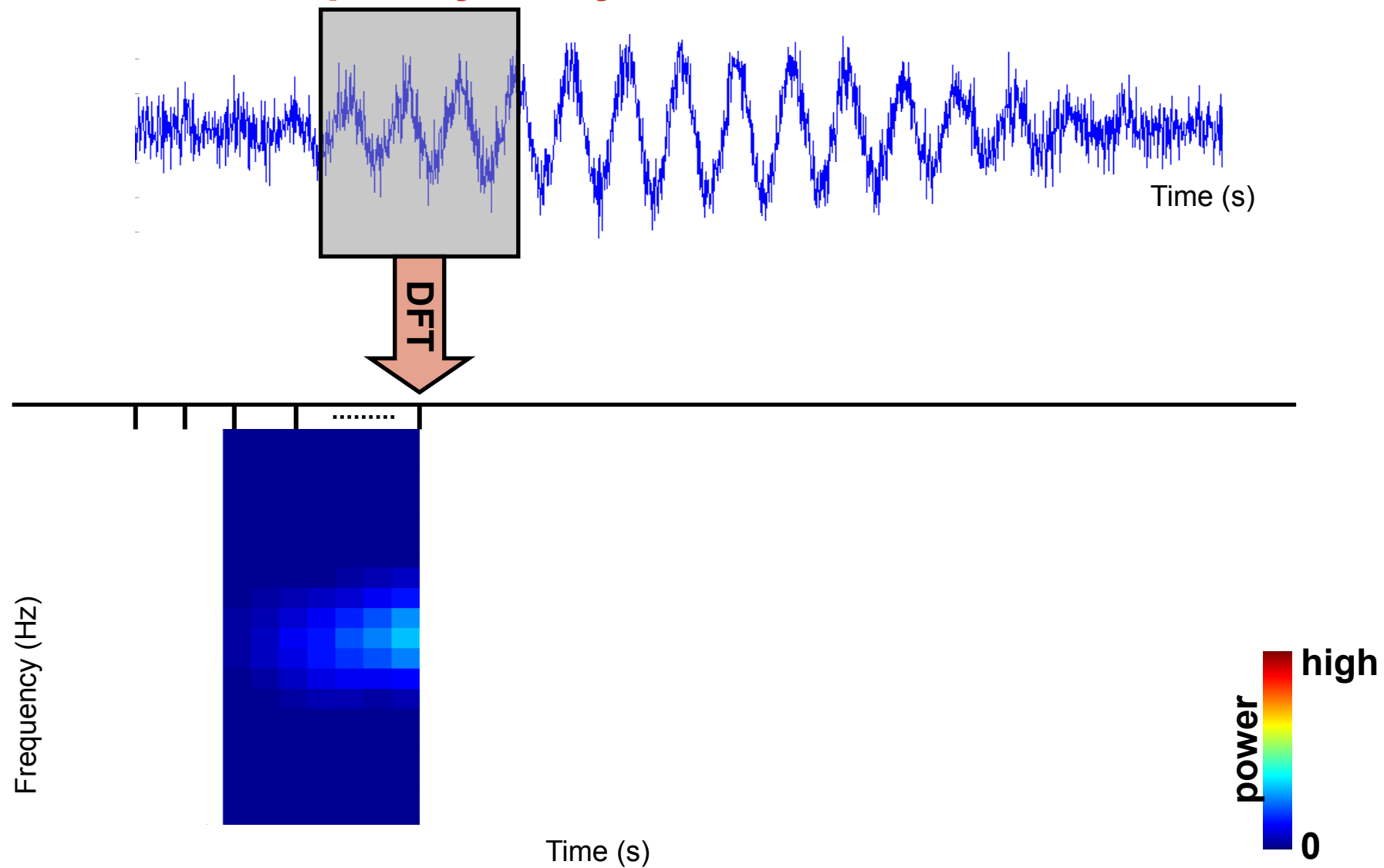
# Time frequency analysis





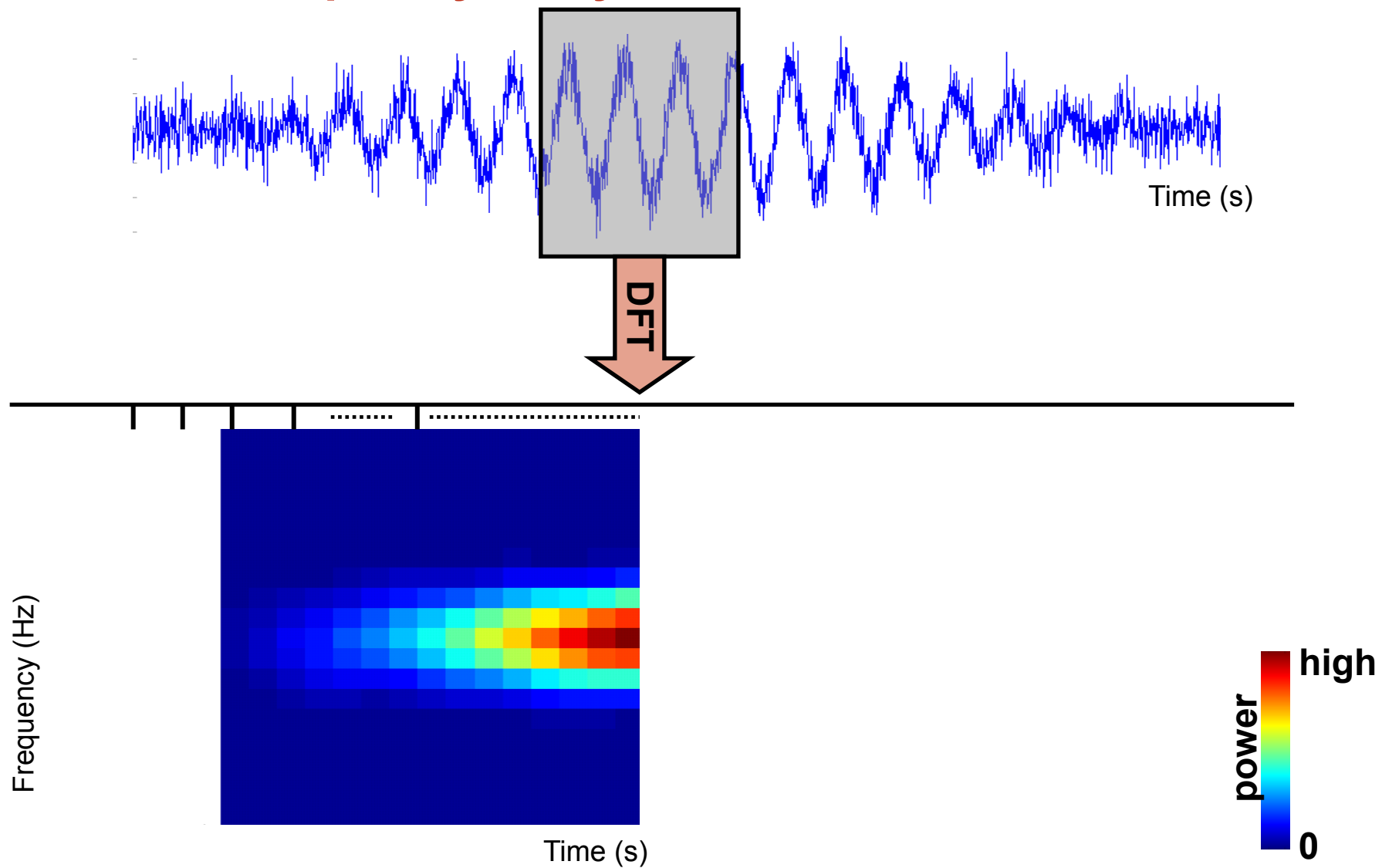


# Time frequency analysis



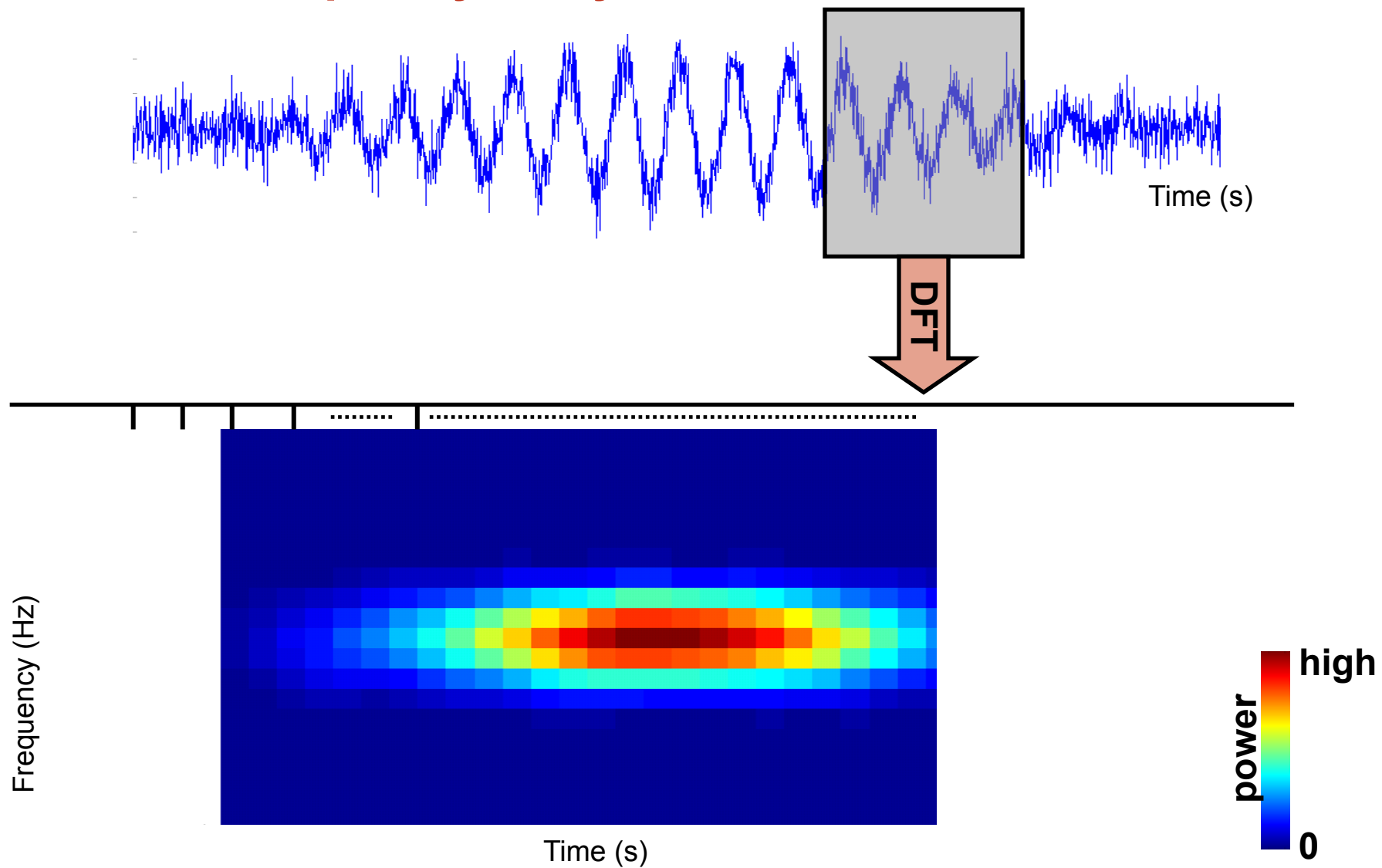


# Time frequency analysis



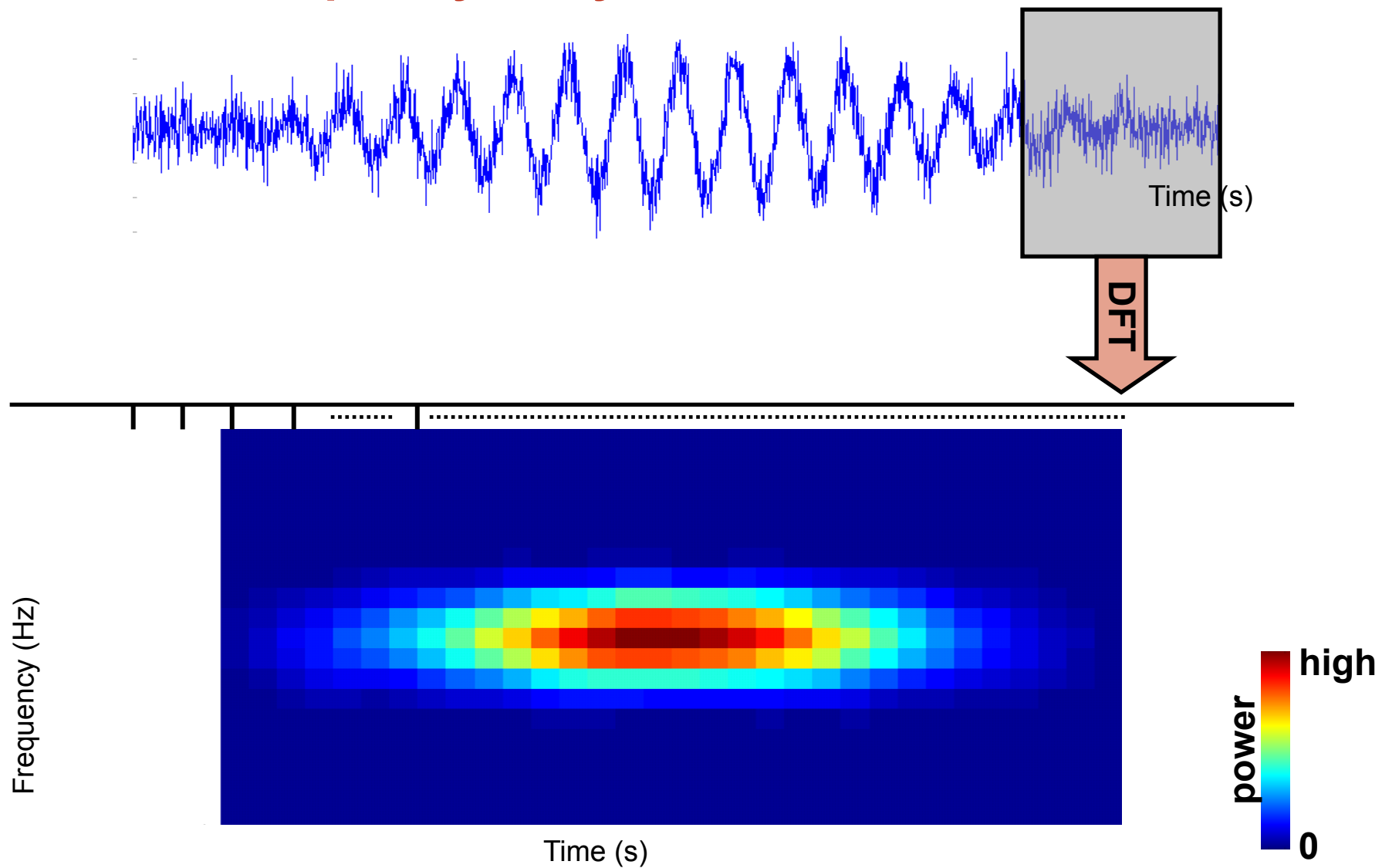


# Time frequency analysis



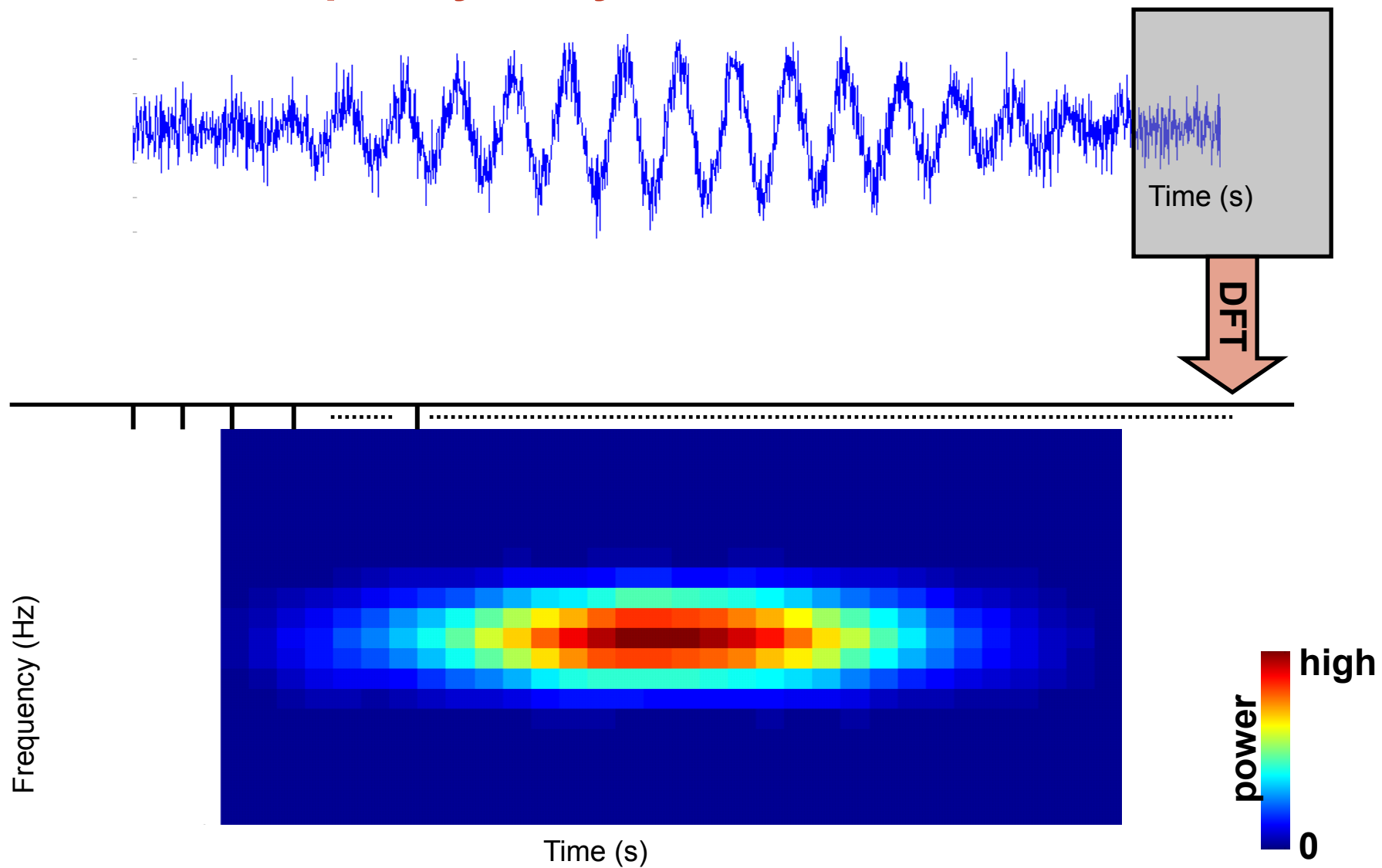


# Time frequency analysis



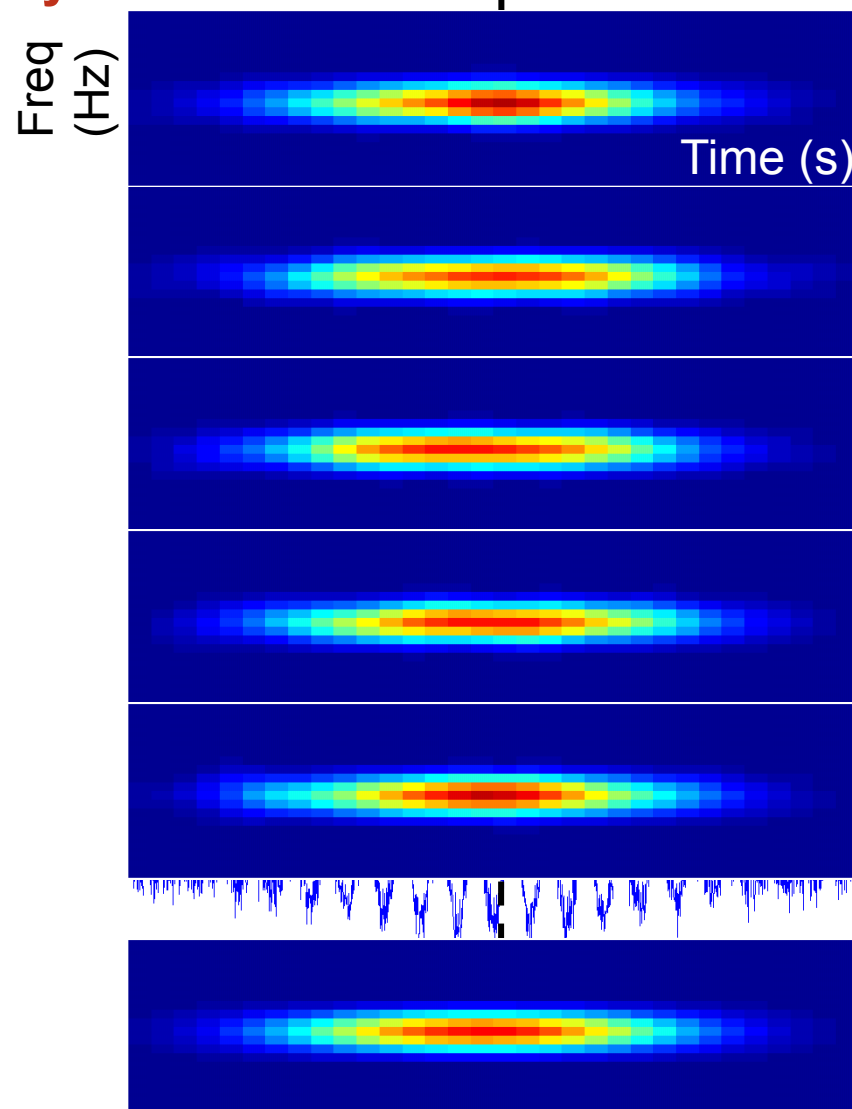
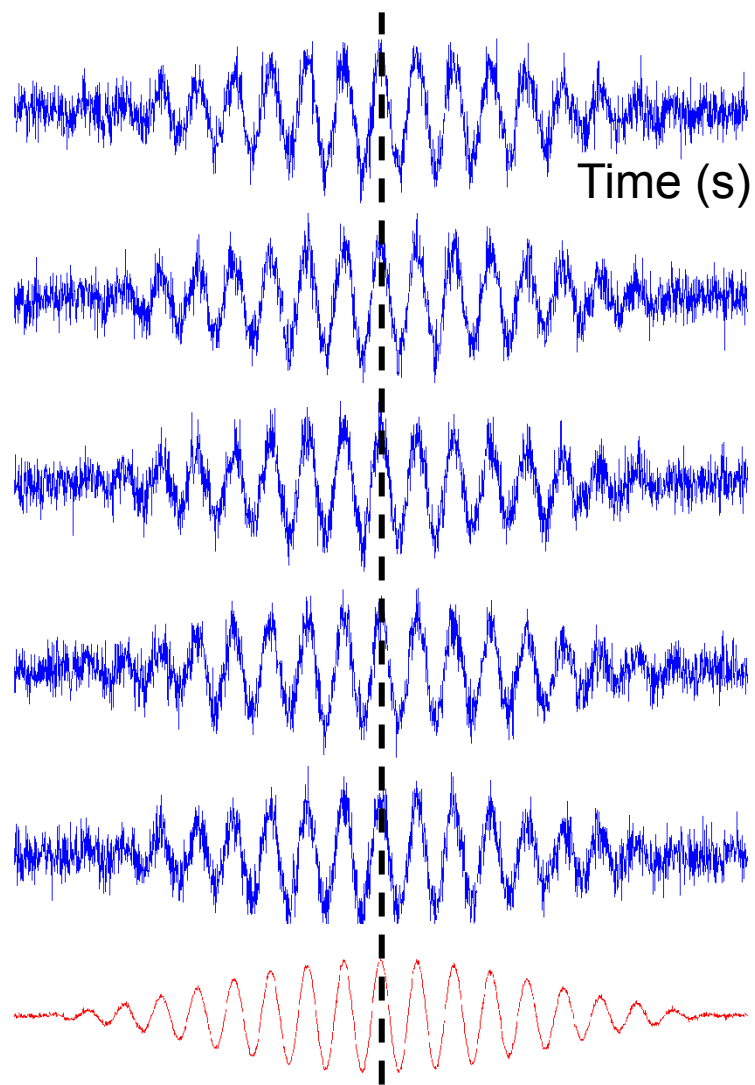


# Time frequency analysis



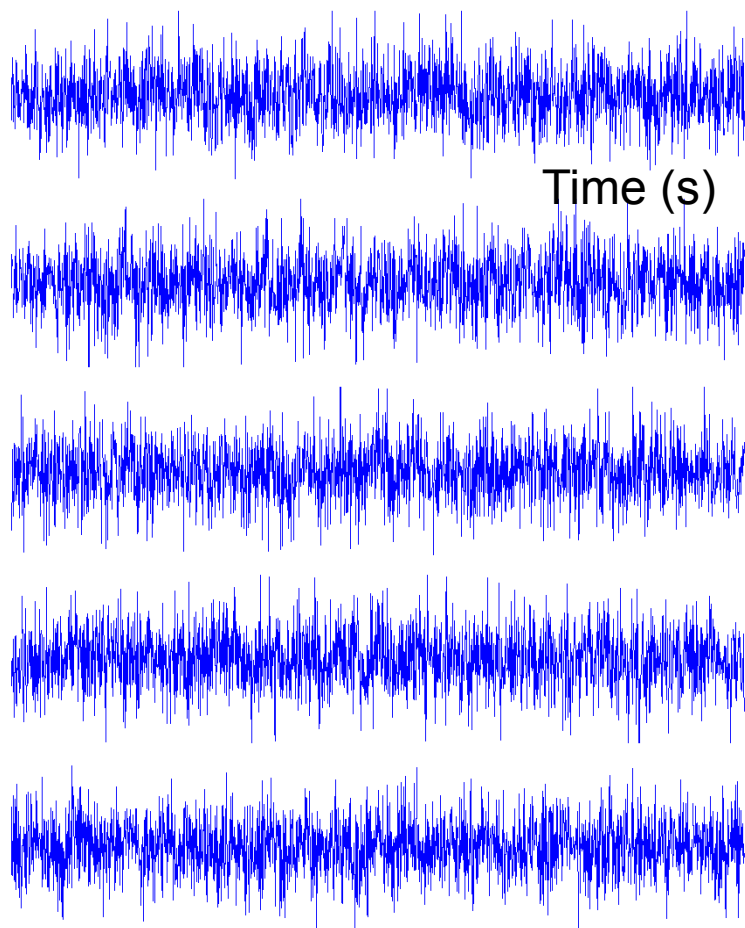


## Evoked versus induced activity

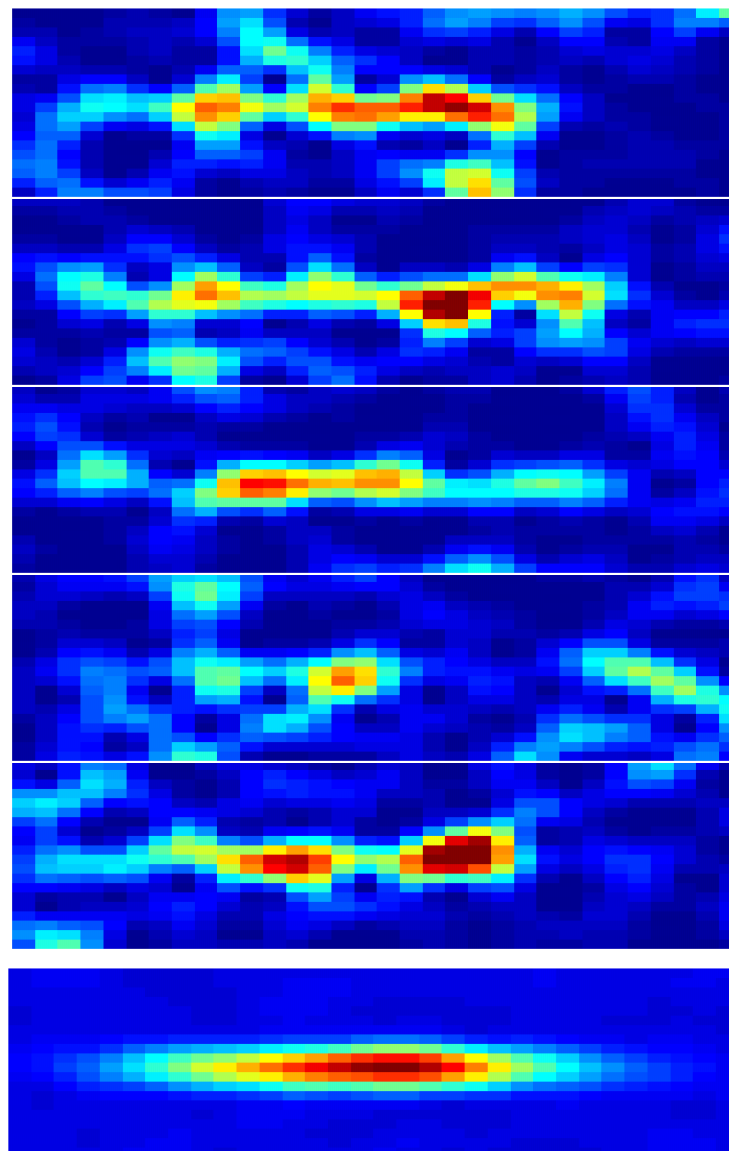




## Noisy signal -> many trials needed

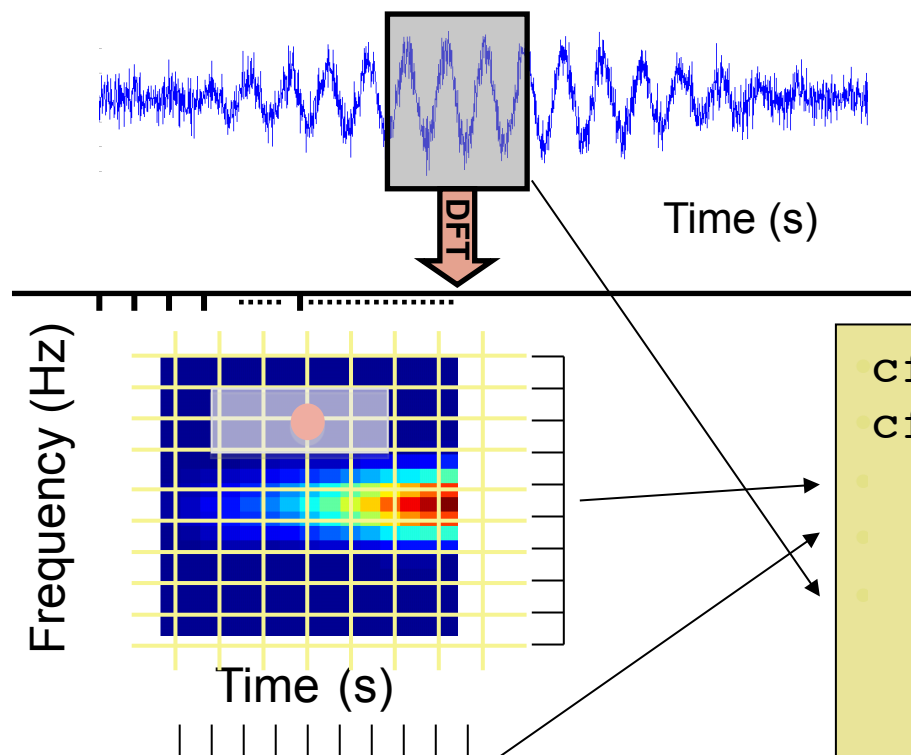


Freq  
(Hz)





## The time-frequency plane



```
cfg = [];  
cfg.method = 'mtmconvol';  
.  
.  
.  
freq = freqanalysis(cfg, data);
```

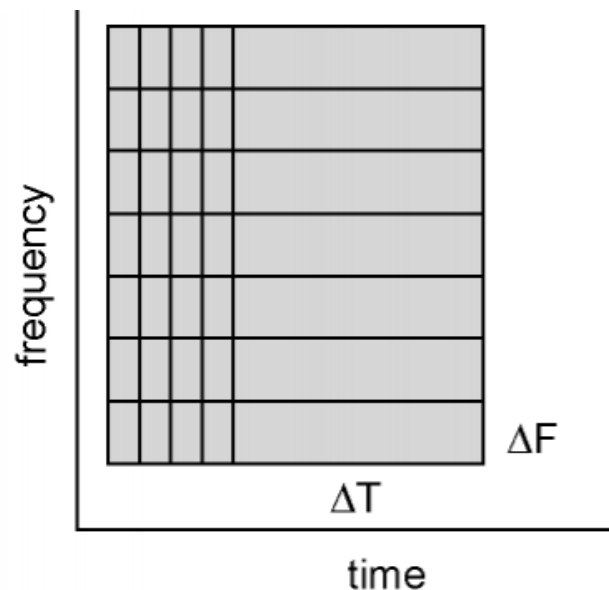






## The time-frequency plane

- Division is 'up to you'
- Depends on the phenomenon you want to investigate
  - Which frequency band?
  - Which time scale?

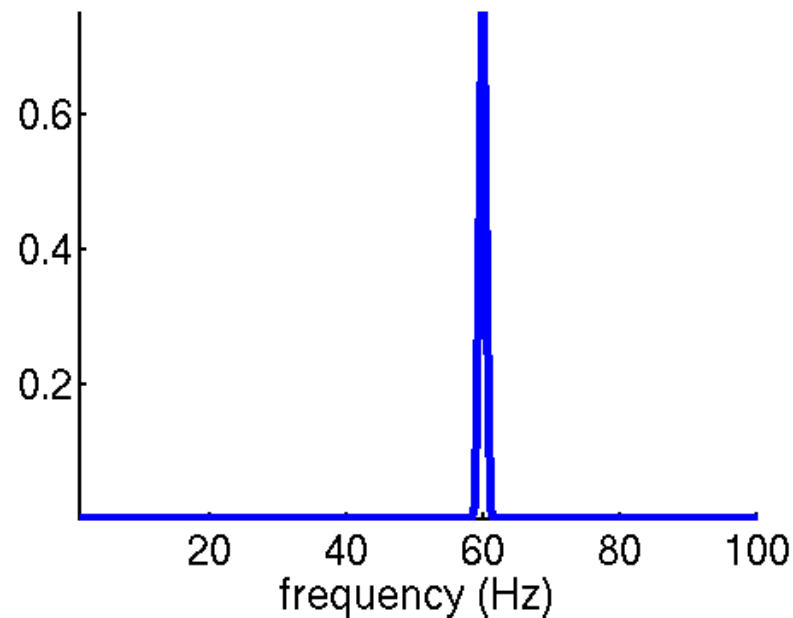
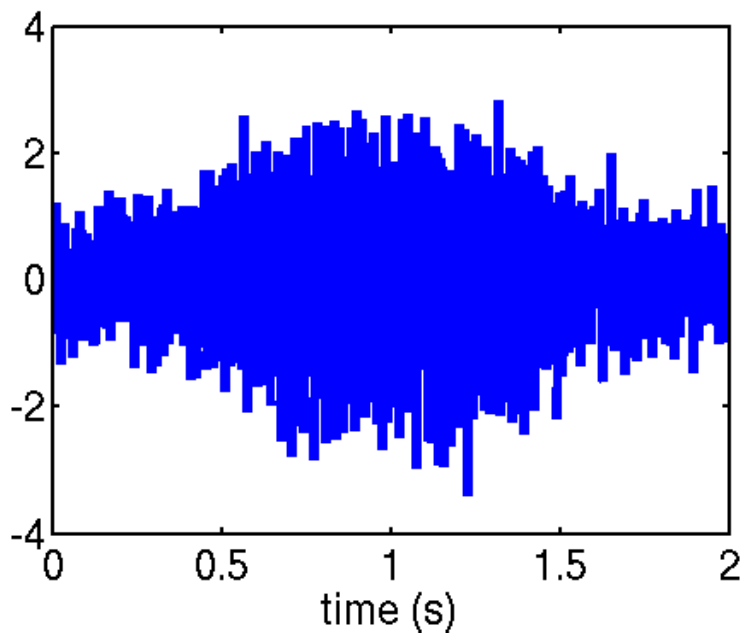


```
cfg = [];  
cfg.method      = 'mtmconvol';  
cfg.foi         = [2 4 ... 40];  
cfg.toi         = [0:0.050:1.0];  
cfg.t_ftimwin   = [0.5 0.5 ... 0.5];  
cfg.tapsmofrq   = [4 4 ... 4];  
.  
.  
freq = freqanalysis(cfg, data);
```



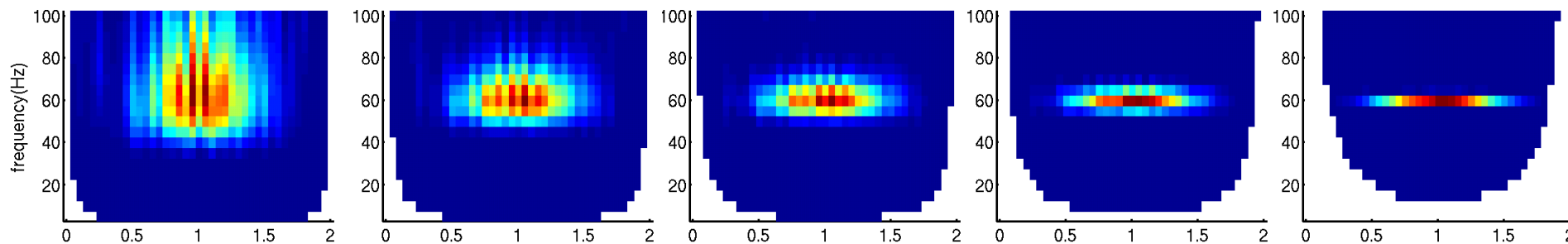


## Time versus frequency resolution



short timewindow

long timewindow





## Sub summary

- Time frequency analysis
  - Fourier analysis on shorter sliding time window
- Evoked & Induced activity
- Time frequency resolution trade off



## Wavelet analysis

- Popular method to calculate time-frequency representations
- Is based on convolution of signal with a family of ‘wavelets’ which capture different frequency components in the signal
- Convolution  $\sim$  local correlation





## Wavelet analysis

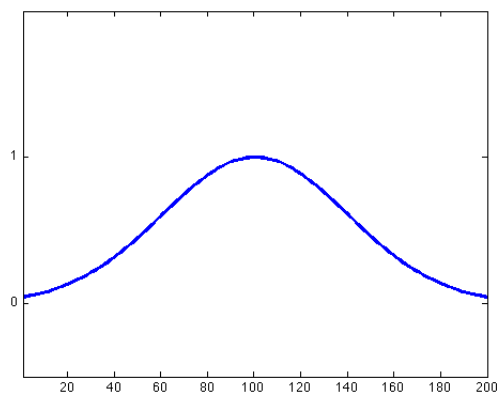
```
cfg = [];  
cfg.method = 'wavelet';  
.  
.  
.  
freq=ft_freqanalysis(cfg, data);
```





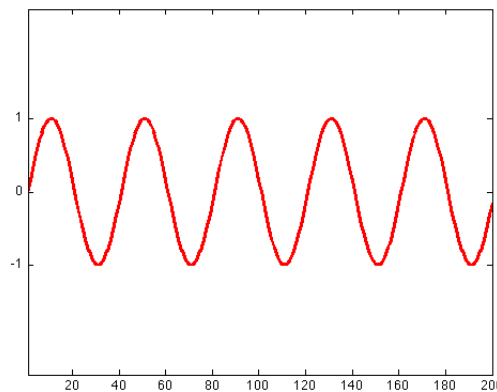
# Wavelets

Taper

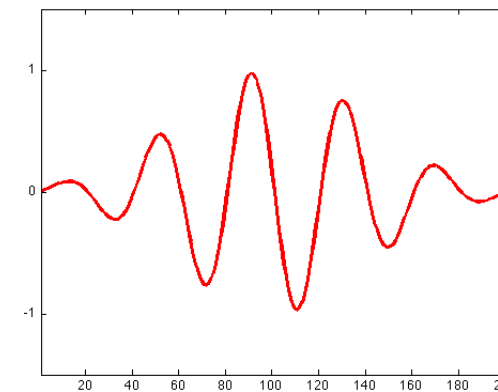


**X**

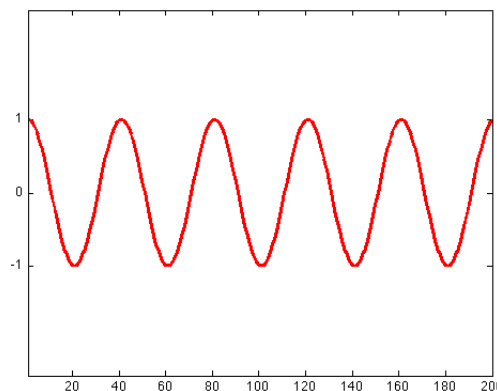
Sine wave



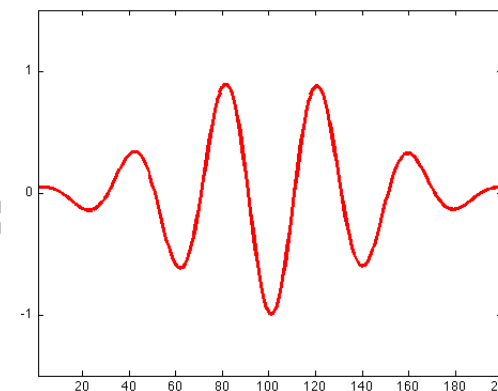
**=**

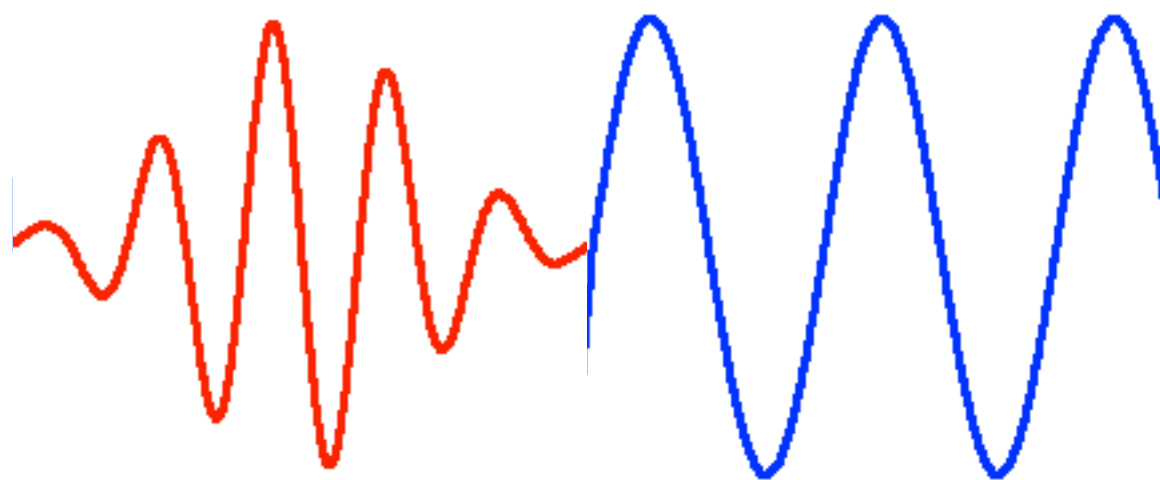


Cosine wave



**=**

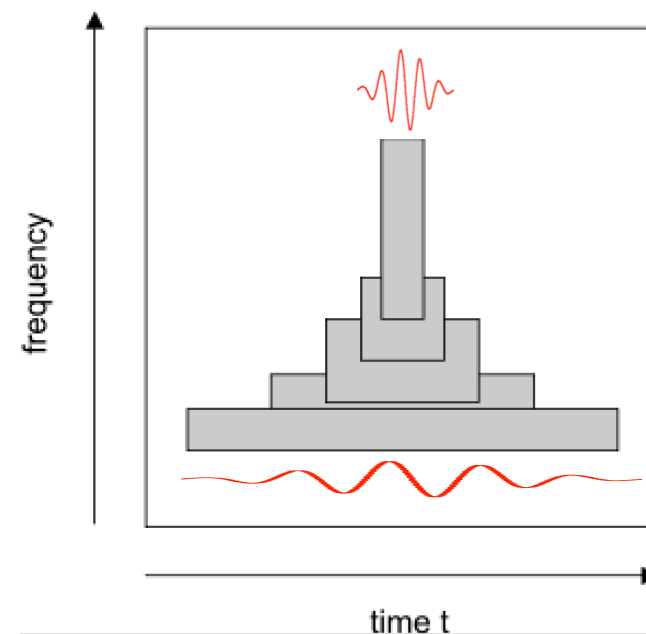






## Wavelet analysis

- Wavelet width determines time-frequency resolution
- Width function of frequency (often 5 cycles)
- ‘Long’ wavelet at low frequencies leads to relatively narrow frequency resolution but poor temporal resolution
- ‘Short’ wavelet at high frequencies leads to broad frequency resolution but more accurate temporal resolution







## Wavelet analysis

- Similar to Fourier analysis, but
  - Computationally slow
  - Tiles the time frequency plane in a particular way with few degrees of freedom

```
%time frequency analysis with  
%multitapers
```

```
cfg = [];  
cfg.method      = 'mtmconvol';  
cfg.toi         = [0:0.05:1];  
cfg.foi         = [4 8 ... 80];  
cfg.t_ftimwin   = [0.5 0.5 ... 0.5];  
cfg.tapsmofrq   = [2 2 ... 10];  
.  
.  
freq=ft_freqanalysis(cfg, data);
```

```
%time frequency analysis with  
%wavelets
```

```
cfg = [];  
cfg.method      = 'wavelet';  
cfg.toi         = [0:0.05:1];  
cfg.foi         = [4 8 ... 80];  
cfg.gwidth      = 5;  
.  
.  
freq=ft_freqanalysis(cfg, data);
```



# Summary

## Spectral analysis

Relation between time and frequency domains

Tapers

## Time frequency analysis

Time vs frequency resolution

## Wavelets

*Hands-on: Time-frequency analysis of power*

*Hanning window*

*fixed and variable length.*

*Wavelets*

*Multi-tapers*

