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for Brain, Cognition and Behaviour

Connectivity analysis in electro-physiological data: metrics and issues

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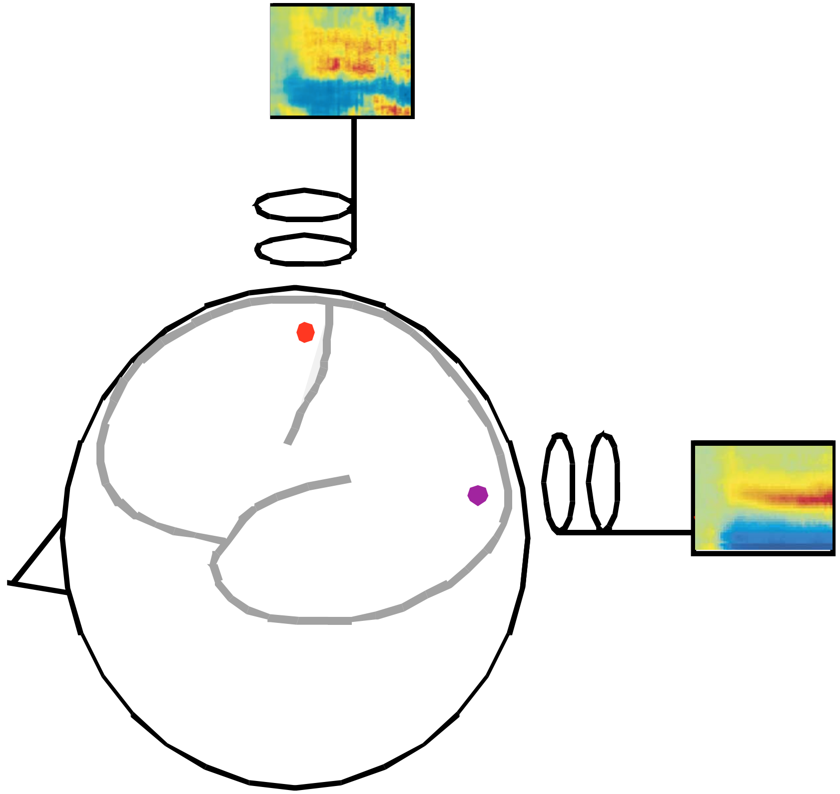
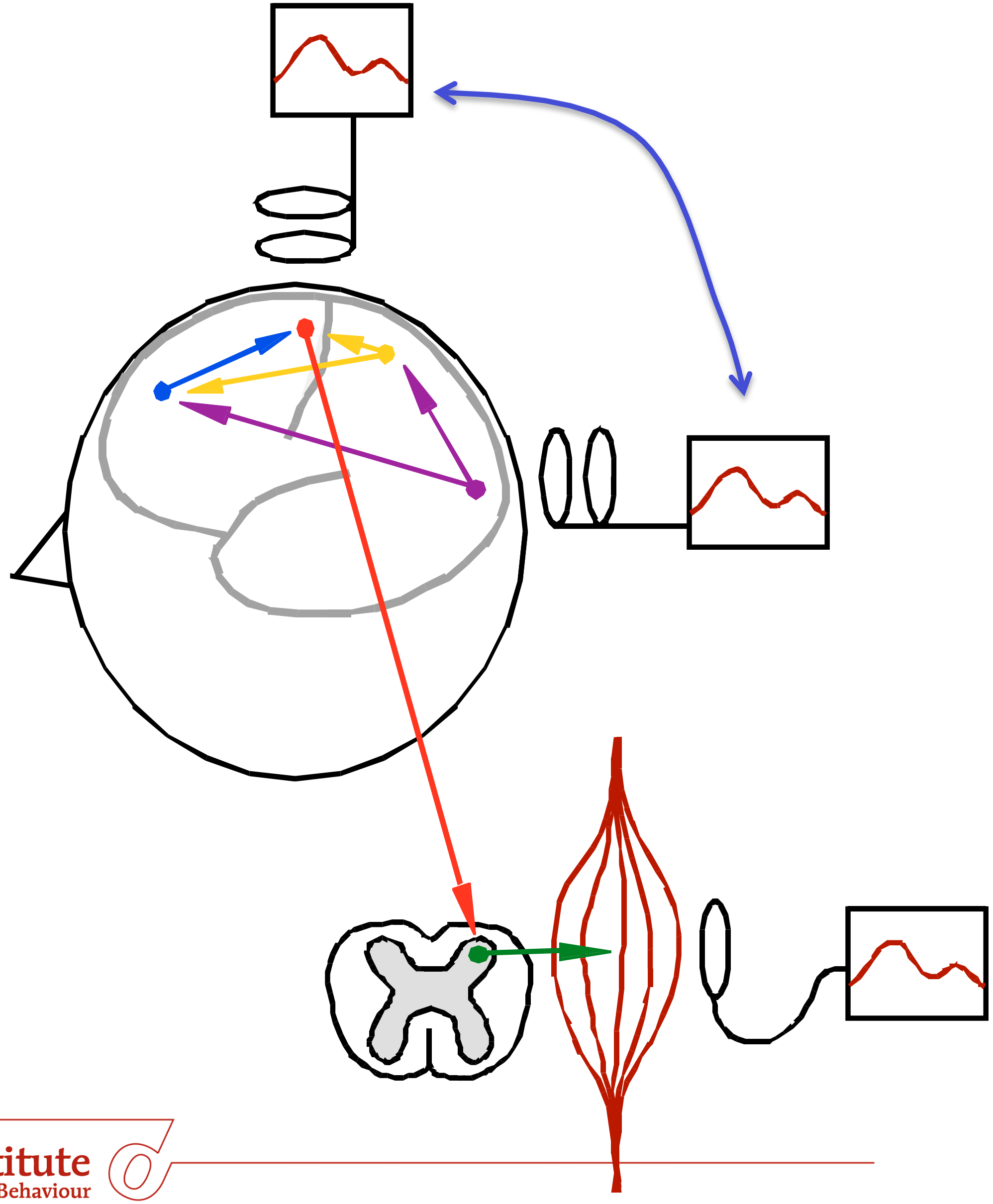
Connectivity analysis in electro-physiological data: metrics …

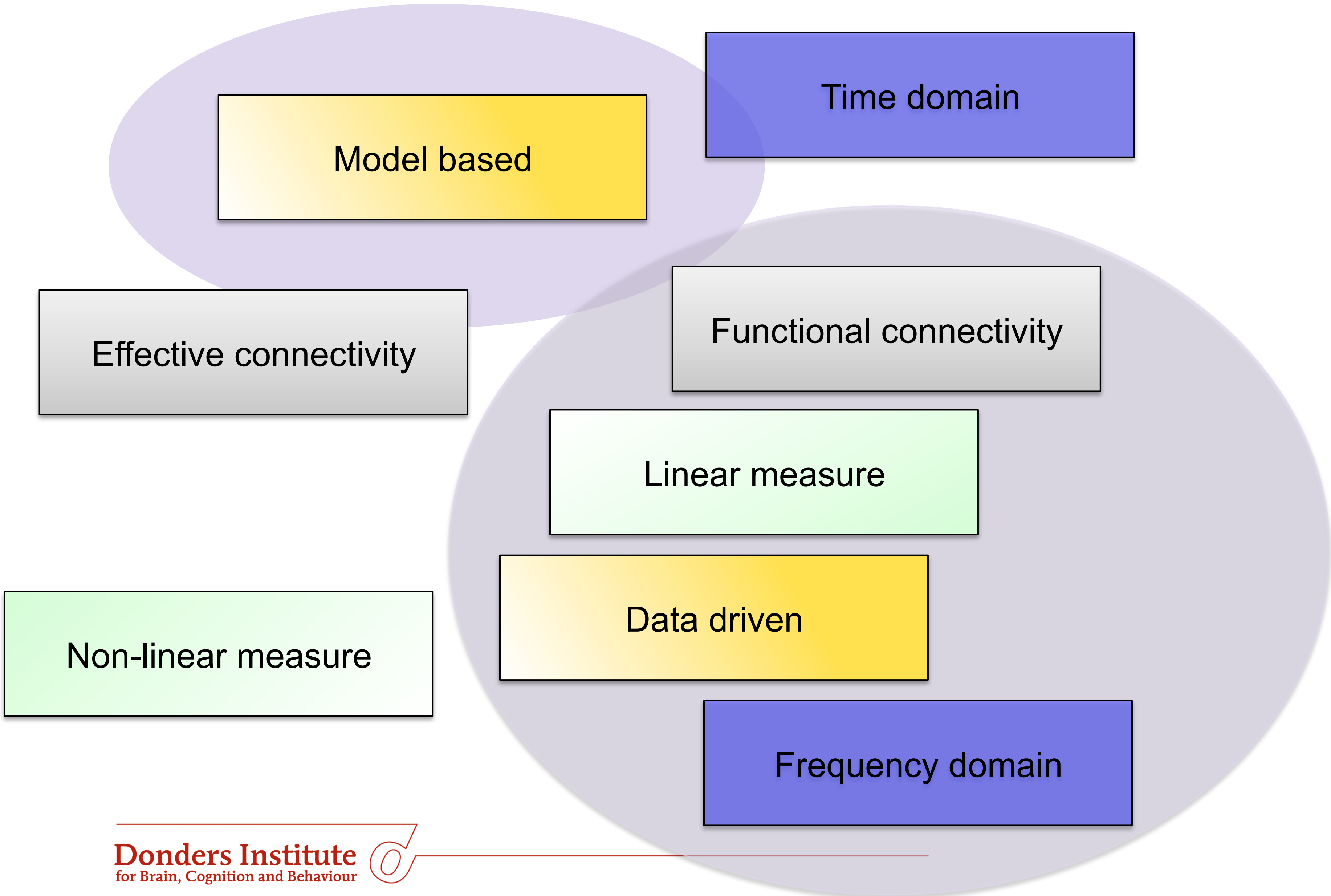
Univariate analysis



Connectivity analysis: Beyond univariate analysis



Measures of connectivity



Measures of frequency domain connectivity

Coherence coefficient

Phase lag index

Phase synchronization

Partial directed coherence

Directed transfer function

Phase locking value

Imaginary part of coherency

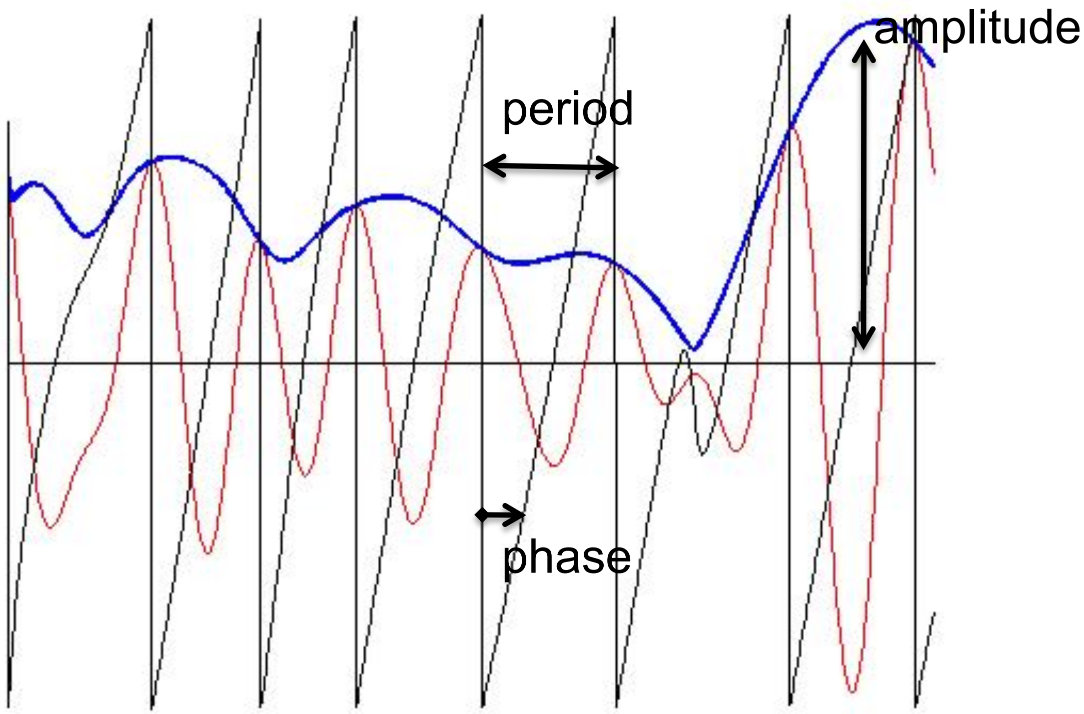
Pairwise phase consistency

Phase slope index

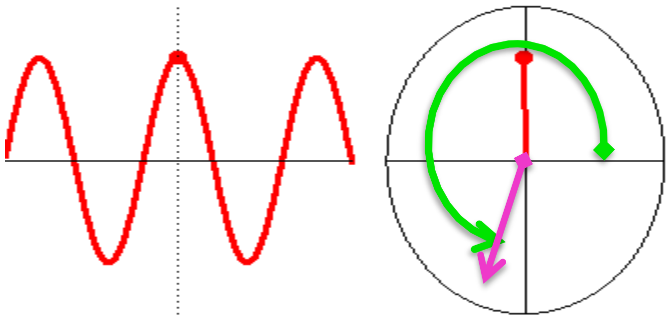
Synchronization likelihood

Frequency domain granger causality

What constitutes an oscillation? (recap)



What constitutes an oscillation? (the movie)



$$x = Ae^{i\varphi}$$

What about 2 oscillations? Let's look at the phase difference

phase signal 1

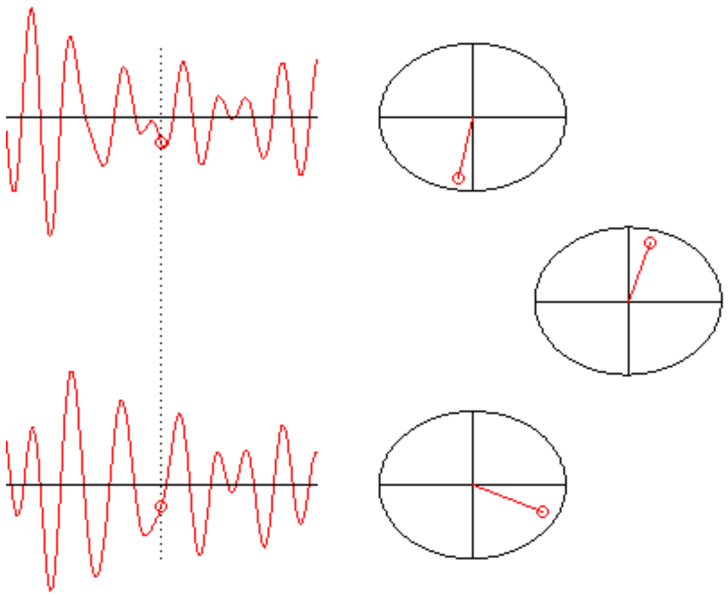
phase difference

phase signal 2

Phase difference is scattered:
Low synchrony

Let's look at the phase difference

$$x_1 = A_1 e^{i\varphi_1}$$



phase signal 1

phase difference

phase signal 2

$$x_2 = A_2 e^{i\varphi_2}$$

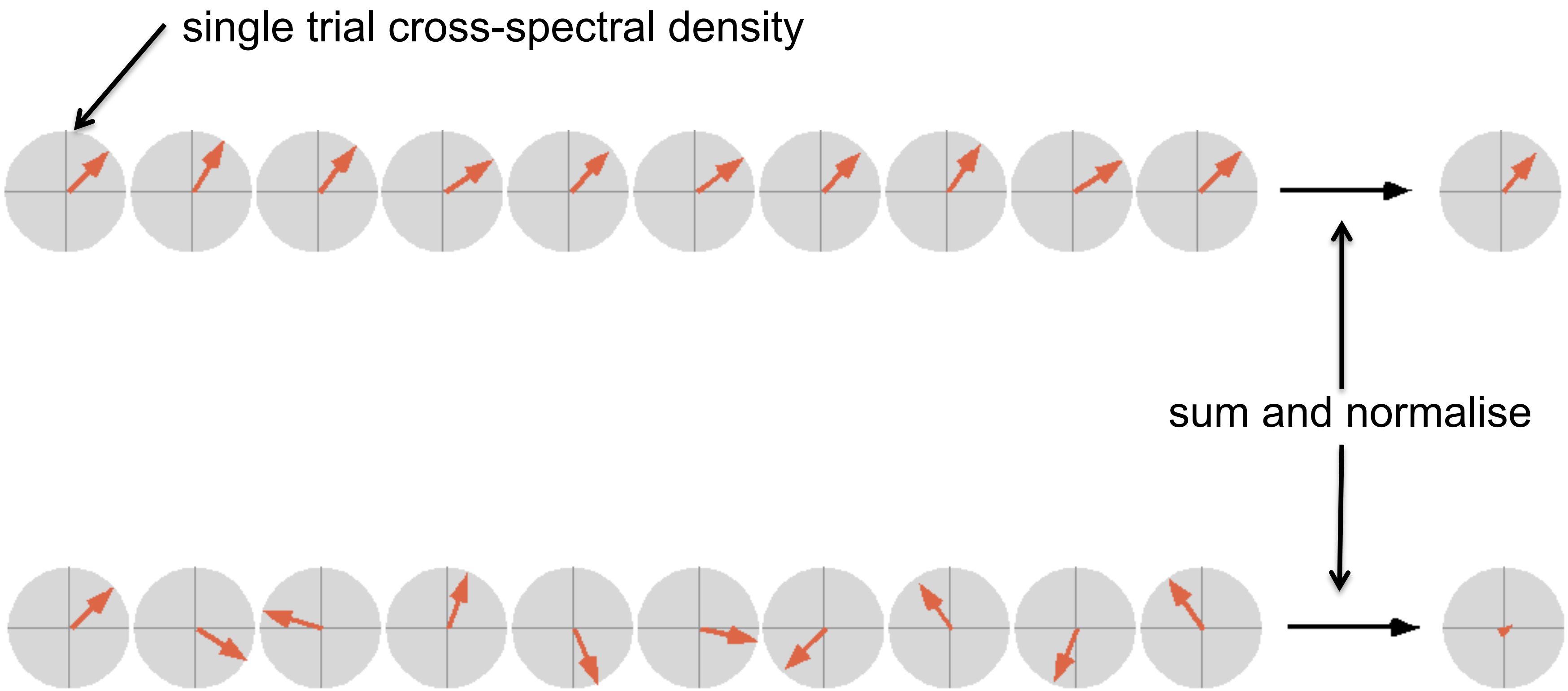
Phase difference is clustered:
High synchrony

$$x_1 x_2^* = \langle A_1 A_2 e^{i(\varphi_1 - \varphi_2)} \rangle$$

Measures of connectivity: coherence (the math view)

Coherence is computed from the *cross-spectral density*, which is obtained by *conjugate multiplication* of the frequency domain representation of the signals

$$x_1 x_2^{*} = A_1 e^{i\varphi_1} \times A_2 e^{-i\varphi_2} = A_1 A_2 e^{i(\varphi_1 - \varphi_2)}$$



Measures of connectivity: coherence & co

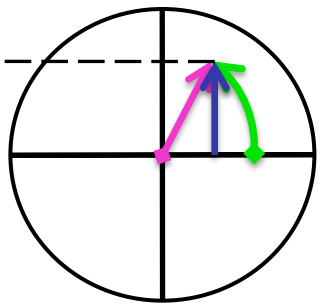
$$\text{Coherence} = \left| \frac{1/N\ \Sigma A_1 A_2 e^{i(\varphi_1 - \varphi_2)}}{\sqrt{(1/N\ \Sigma A_1^2)(1/N\ \Sigma A_2^2)}} \right|$$

$$\text{PLV} = \left| \frac{1/N\ \Sigma 1 \times 1 \times e^{i(\varphi_1 - \varphi_2)}}{\sqrt{(1/N\ \Sigma 1^2)(1/N\ \Sigma 1^2)}} \right| = \left| \frac{\Sigma e^{i(\varphi_1 - \varphi_2)}}{N} \right|$$

Measures of connectivity: coherence & co

$$\text{Coherency} = \frac{1/N \sum A_1 A_2 e^{i(\varphi_1 - \varphi_2)}}{\sqrt{(1/N \sum A_1^2)(1/N \sum A_2^2)}} = C e^{i\Delta\varphi}$$

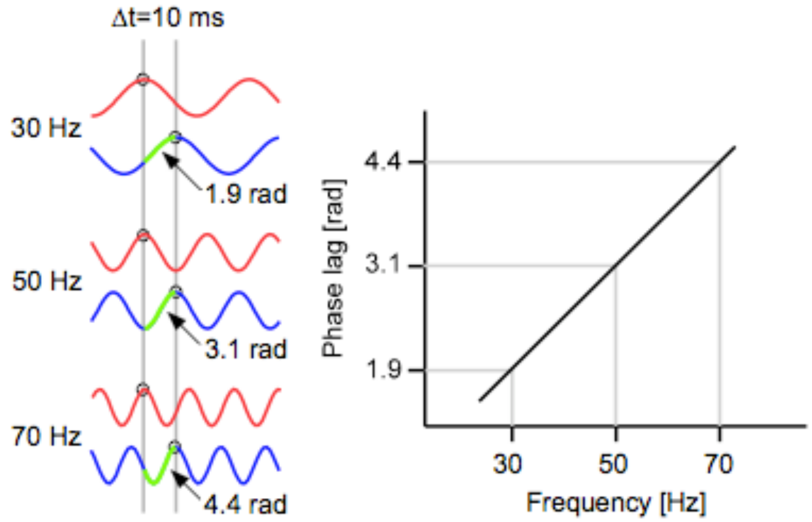
Imaginary part of coherency



Measures of connectivity: coherence & co

$$\text{Coherency} = \frac{1/N\ \Sigma A_1 A_2 e^{i(\varphi_1-\varphi_2)}}{\sqrt{(1/N\ \Sigma A_1^2)(1/N\ \Sigma A_2^2)}} = Ce^{i\Delta\varphi}$$

Slope of relative phase spectrum indicates time delay

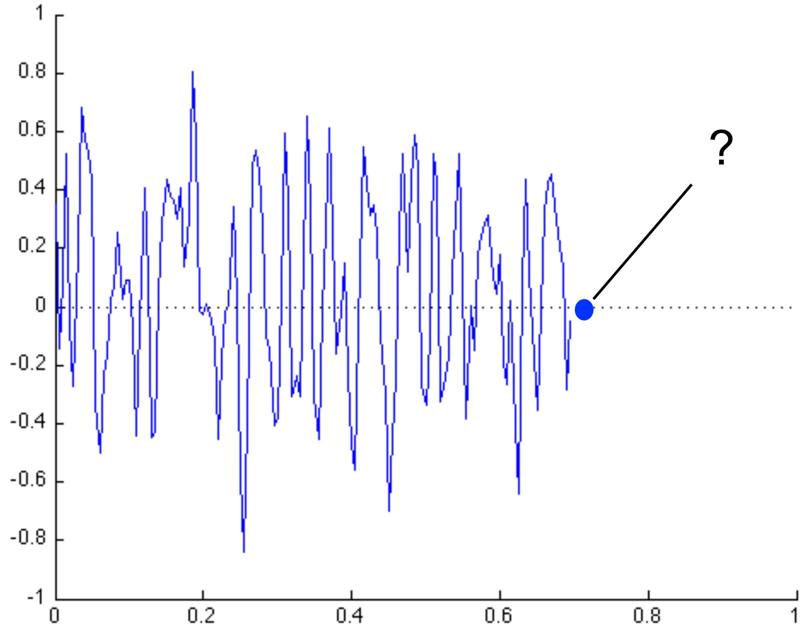


Coherence and linear prediction

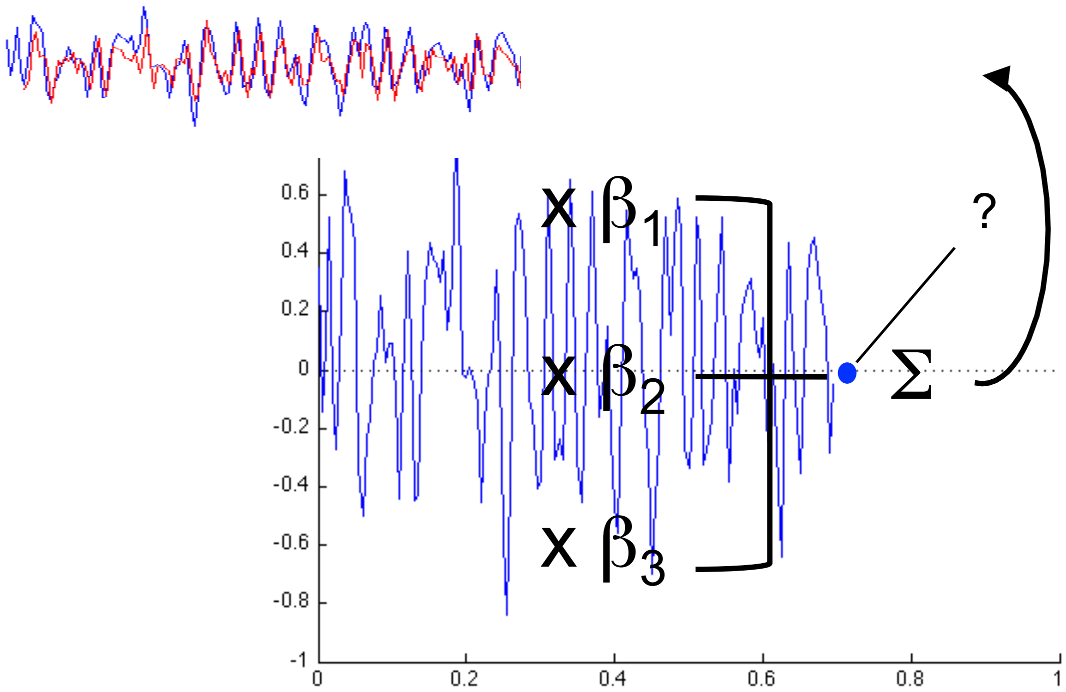
- Coherence coefficient ~ cross-correlation coefficient
- $|\text{Coherence}|^2$ ~ % variance explained
- Coherence coefficient similar to frequency domain regression
- Conceptual difference with regression: independent and dependent variable are interchangeable
- Slope of relative phase spectrum indicates temporal precedence (~ directed influence)
- Slope often hard to estimate or close to zero

Linear prediction and directed interaction: the concept of Granger causality

Linear prediction and directed interaction: the concept of Granger causality



Linear prediction: autoregressive models



$$X(t) = \Sigma\ \beta_\tau X(t-\tau) + \eta$$

Two signals: bivariate autoregressive models

$$X(t) = \Sigma\ \beta_{\tau 1} X(t-\tau) + \eta_1$$

$$Y(t) = \Sigma\ \beta_{\tau 2} Y(t-\tau) + \eta_2$$

$$X(t) = \Sigma\ \beta_{\tau 11} X(t-\tau) + \Sigma\ \beta_{\tau 21} Y(t-\tau) + \varepsilon_1$$

$$Y(t) = \Sigma\ \beta_{\tau 12} X(t-\tau) + \Sigma\ \beta_{\tau 22} Y(t-\tau) + \varepsilon_2$$

Granger causality: compare the residuals

$$X(t) = \sum \beta_{\tau 1} X(t-\tau) + \eta_1$$

$$Y(t) = \sum \beta_{\tau 2} Y(t-\tau) + \eta_2$$

$$X(t) = \sum \beta_{\tau 11} X(t-\tau) + \sum \beta_{\tau 21} Y(t-\tau) + \varepsilon_1$$

$$Y(t) = \sum \beta_{\tau 12} X(t-\tau) + \sum \beta_{\tau 22} Y(t-\tau) + \varepsilon_2$$

$$F_{Y \to X} = \ln\left(\frac{var(\eta_1)}{var(\varepsilon_1)} \right)$$

$$F_{X \to Y} = \ln\left(\frac{var(\eta_2)}{var(\varepsilon_2)} \right)$$

Analogy between Granger and 'plain' regression

$X(t) = \sum \beta_{\tau 1} X(t-\tau) + \eta_1$

$Y(t) = \sum \beta_{\tau 2} Y(t-\tau) + \eta_2$

$X(t) = \sum \beta_{\tau 11} X(t-\tau) + \sum \beta_{\tau 21} Y(t-\tau) + \varepsilon_1$

$Y(t) = \sum \beta_{\tau 12} X(t-\tau) + \sum \beta_{\tau 22} Y(t-\tau) + \varepsilon_2$

$\text{data} = \sum \beta_\kappa X_\kappa + \eta$

$\text{data} = \sum \beta'_\kappa X_\kappa + \beta'_{\kappa+1} X_{\kappa+1} + \varepsilon$

$$F_{Y \to X} = \ln\left(\frac{\mathrm{var}(\eta_1)}{\mathrm{var}(\varepsilon_1)}\right)$$

$$F \sim \frac{\mathrm{var}(\eta)}{\mathrm{var}(\varepsilon)}$$

…only the inference is different

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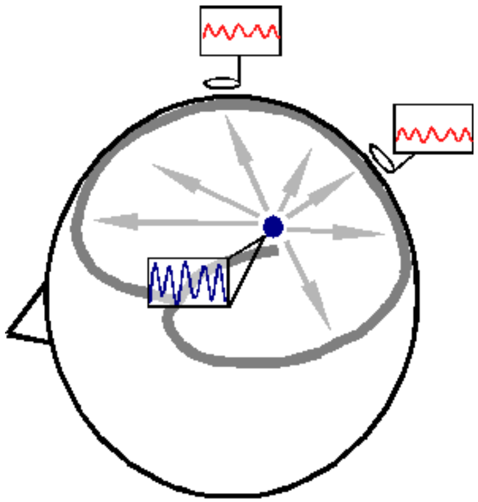
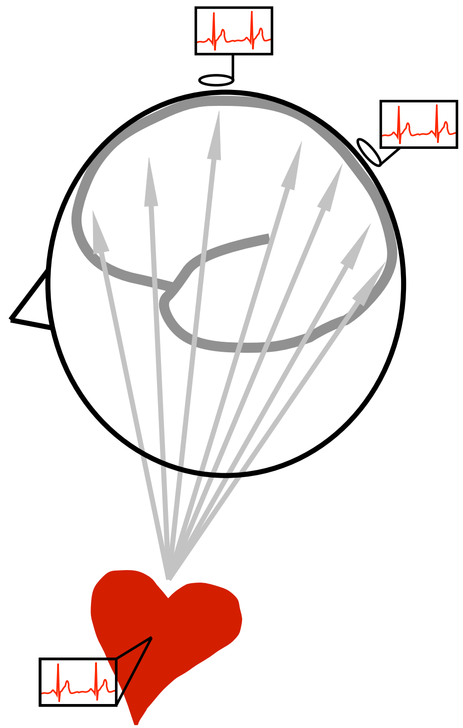
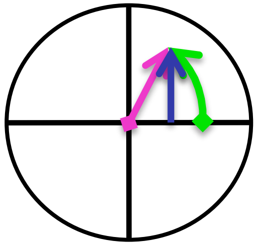
Connectivity analysis in electro-physiological data: … and issues

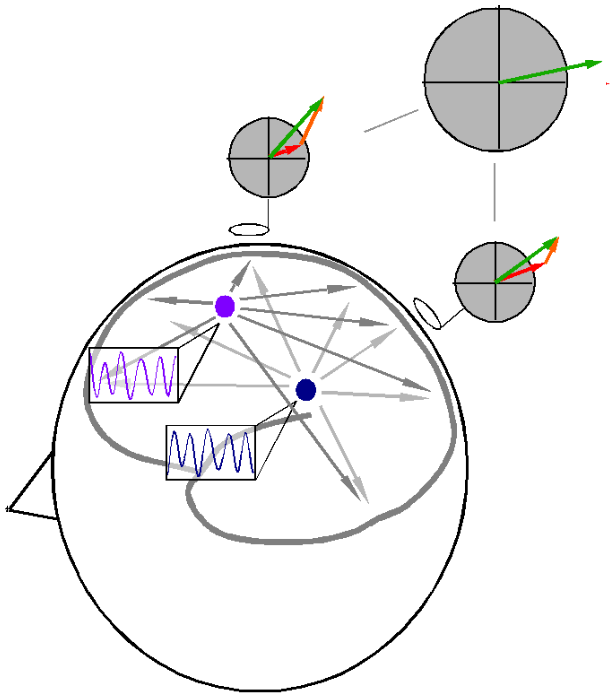
Practial issues: Electromagnetic field spread





Practical issues: imaginary part of coherency

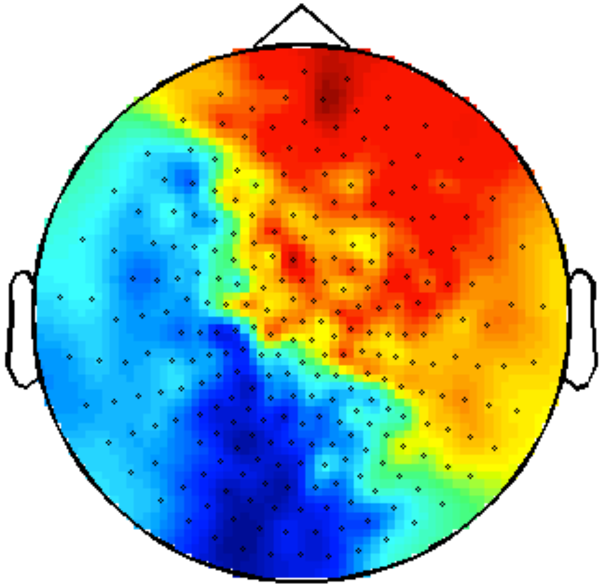


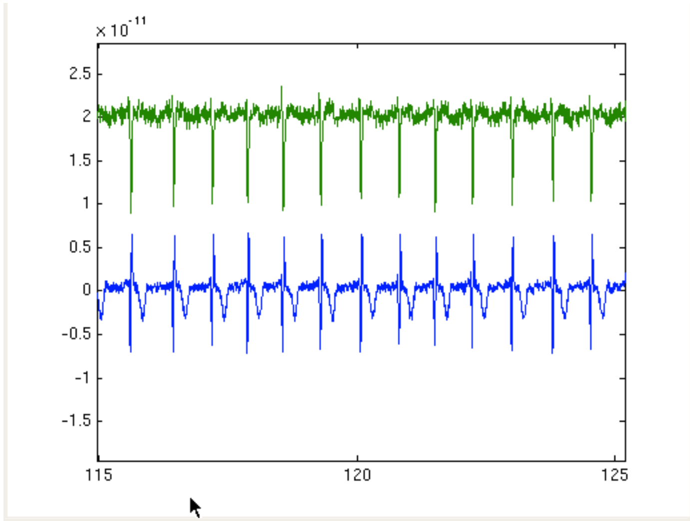


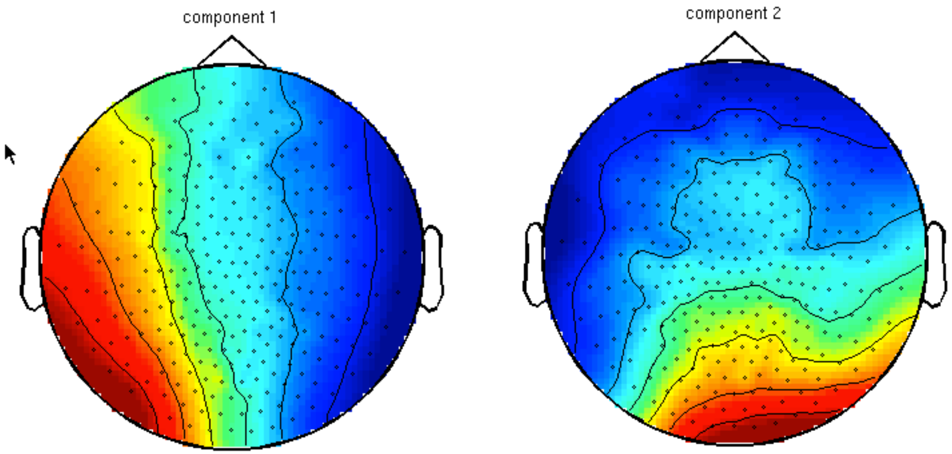
Im(coherency) ≠ 0

MEG connectivity

WPLI suggests fronto-occipital directed interaction (alpha band)







Compute connectivity at the source level

Brain ordinates

Inverse operator

x

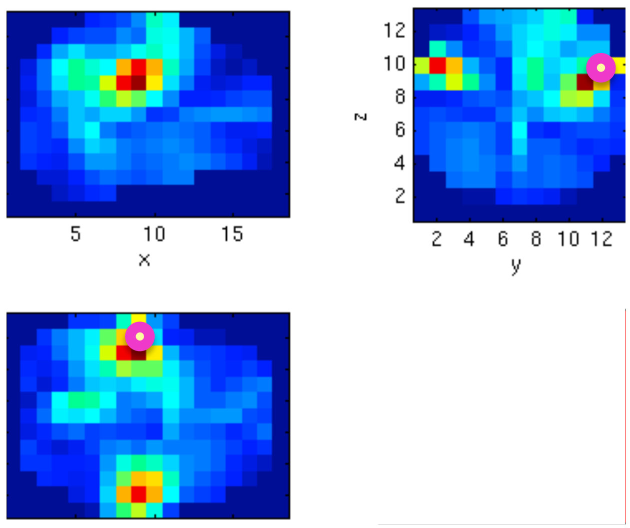
Channels

Observations

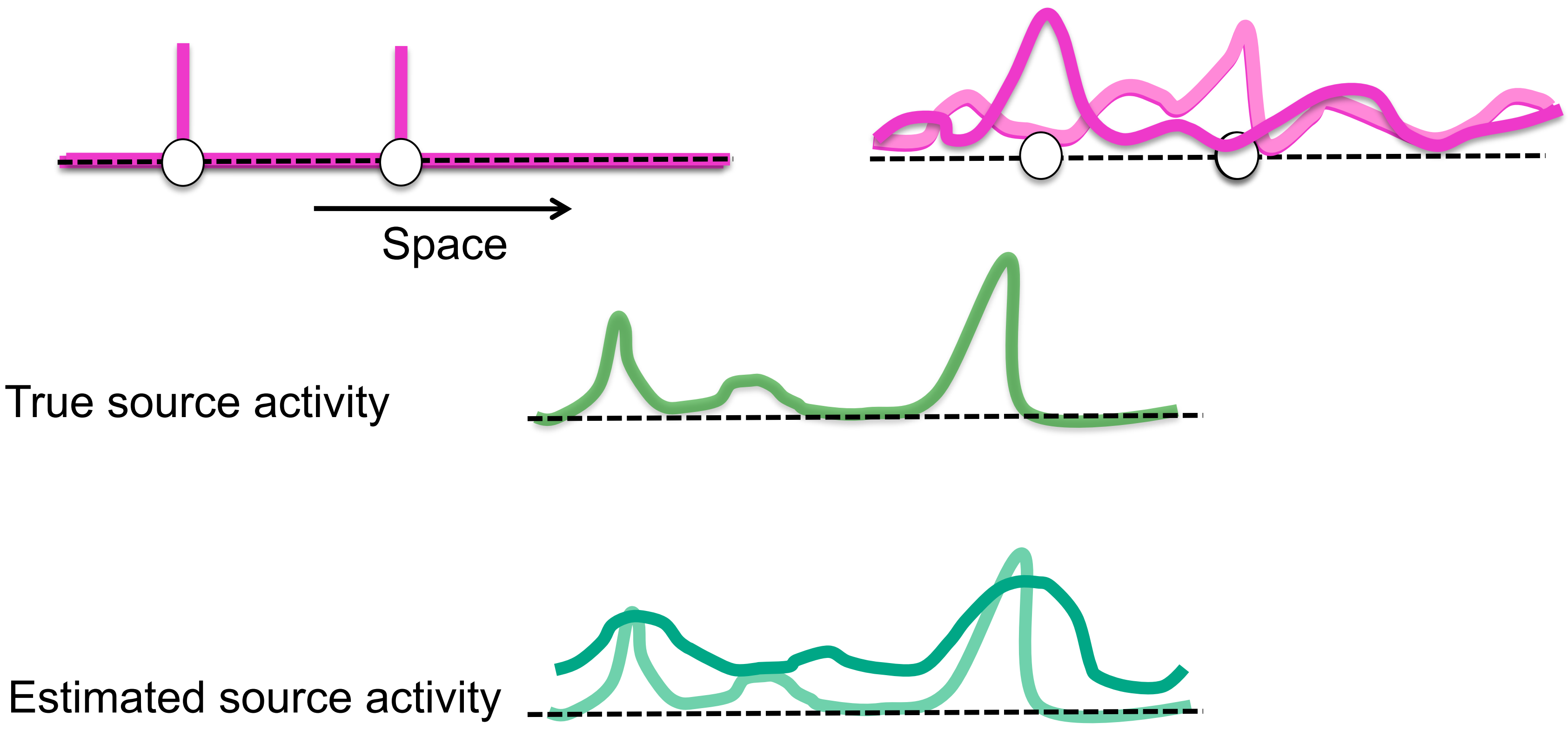
Sensor data

Source data

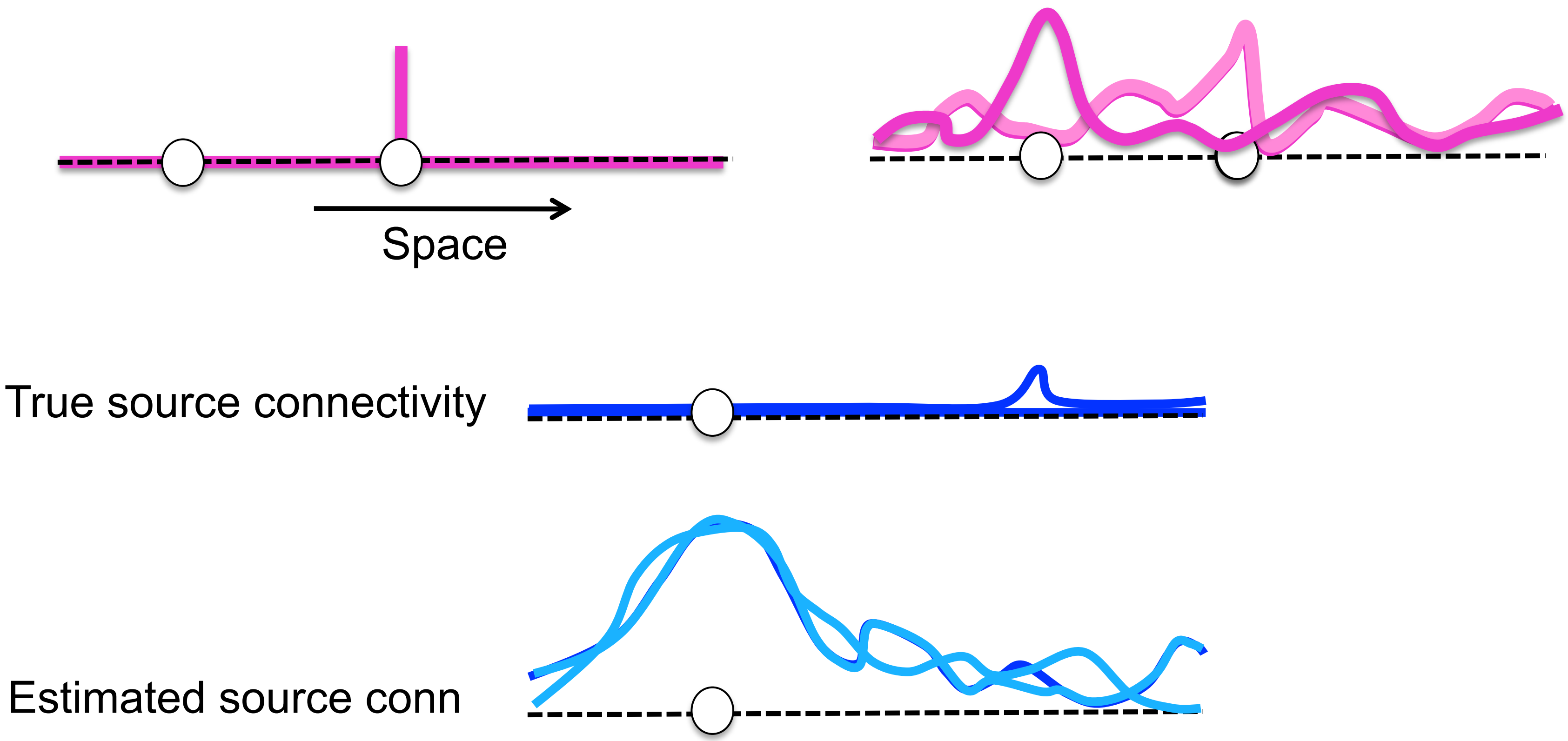
Connectivity estimate



Features of spatial filters



Features of spatial filters: spurious connectivity due to spatial leakage of ‘noise’



Concluding remarks

- Connectivity analysis is cool
- Many measures on the market
- Interpretation of results should be done with care