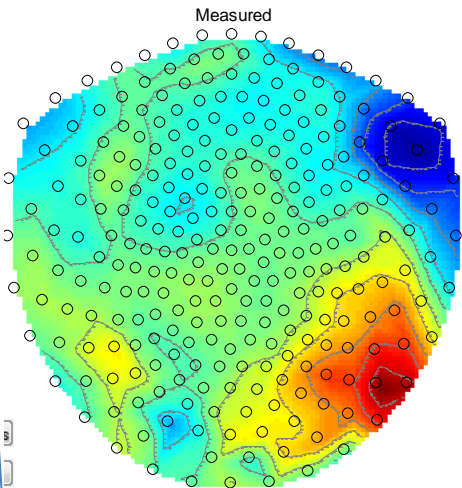


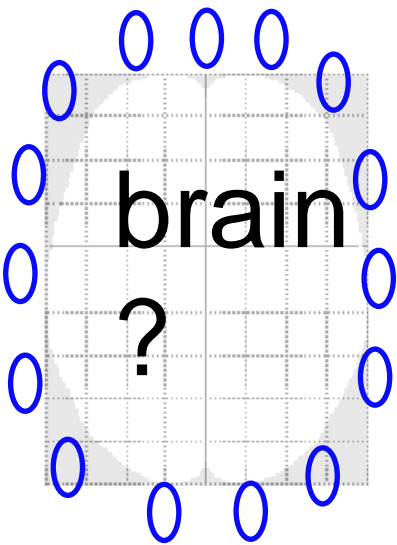
# The M/EEG inverse problem

Gareth R. Barnes

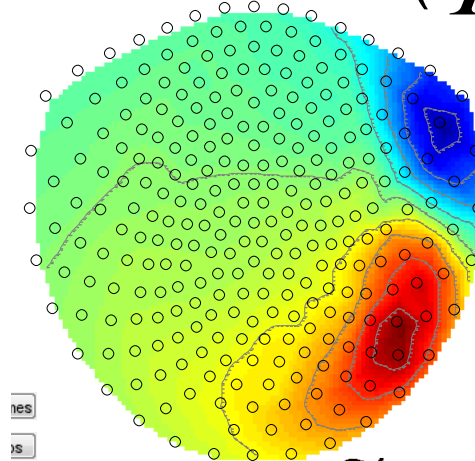
# Measurement (Y)



M/EEG sensors

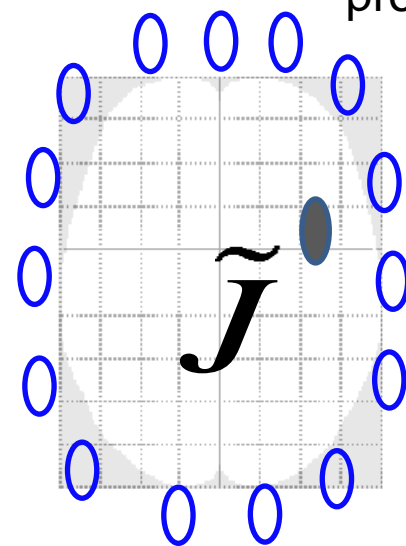


# Prediction ( $\tilde{Y}$ )



$$\tilde{Y} = L\tilde{J}$$

Forward problem



Inverse problem

$$\tilde{J} = WY$$

Prior info

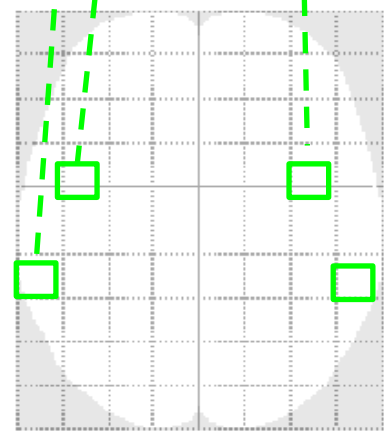
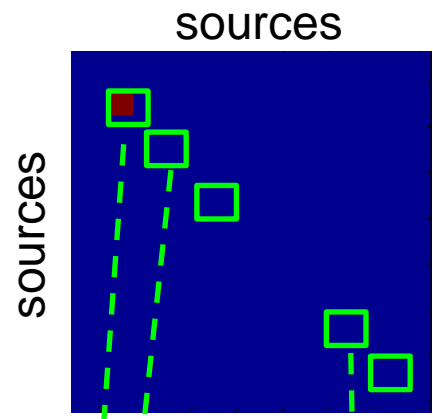
Current density Estimate

Inversion depends on choice of source covariance matrix  
(prior information)

$$W = C L (R + L C L)^{-1}$$

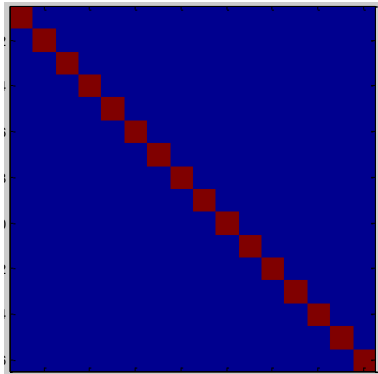
Sensor Noise (known)      Lead field (known)

Source covariance matrix,  
One diagonal element per source

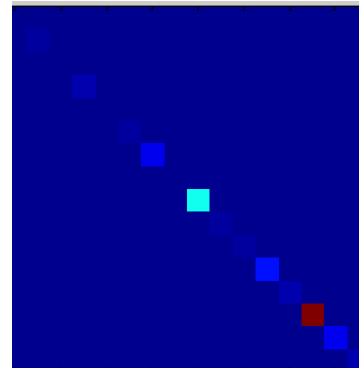


Prior information

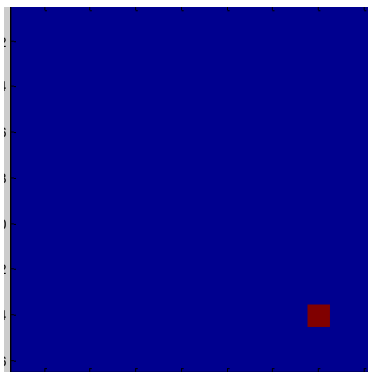
# Some popular priors



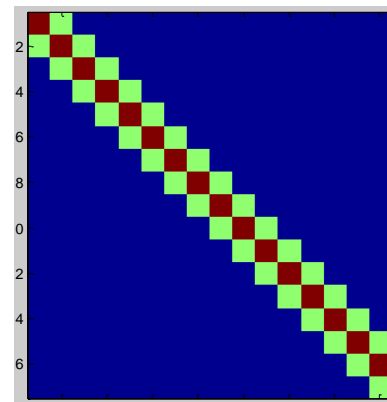
Minimum norm



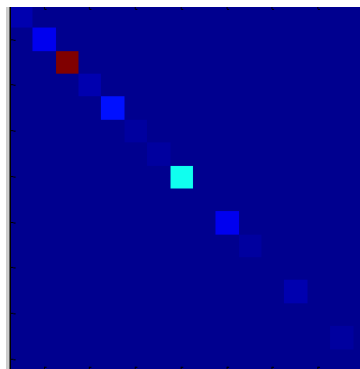
SAM, DICS  
Beamformer



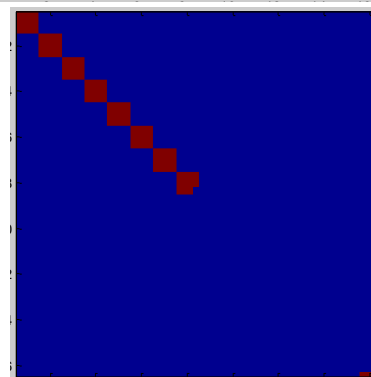
Dipole fit



LORETA



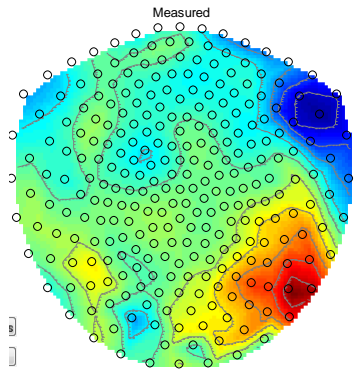
fMRI biased  
dSPM



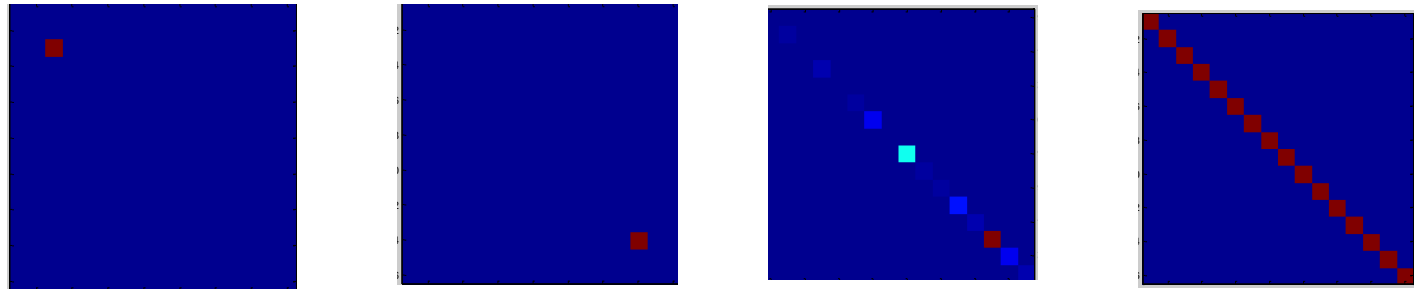
?

Y (measured field)

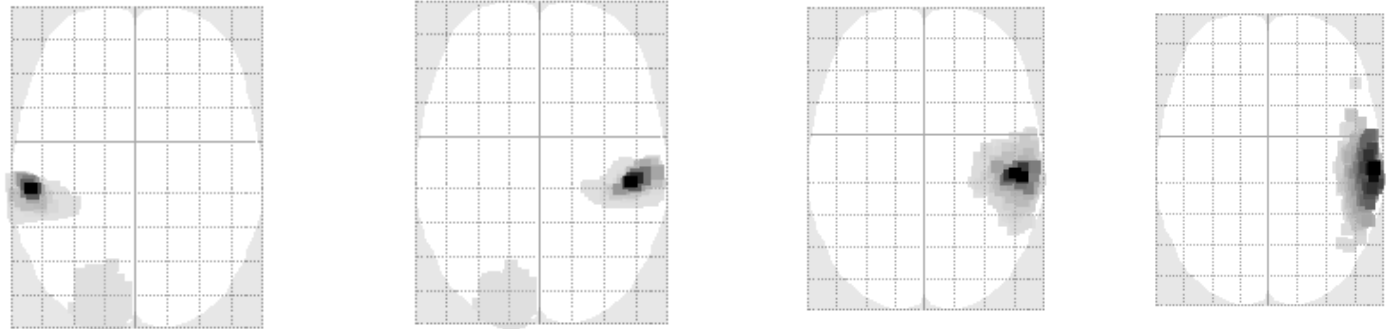
# How do we choose between priors ?



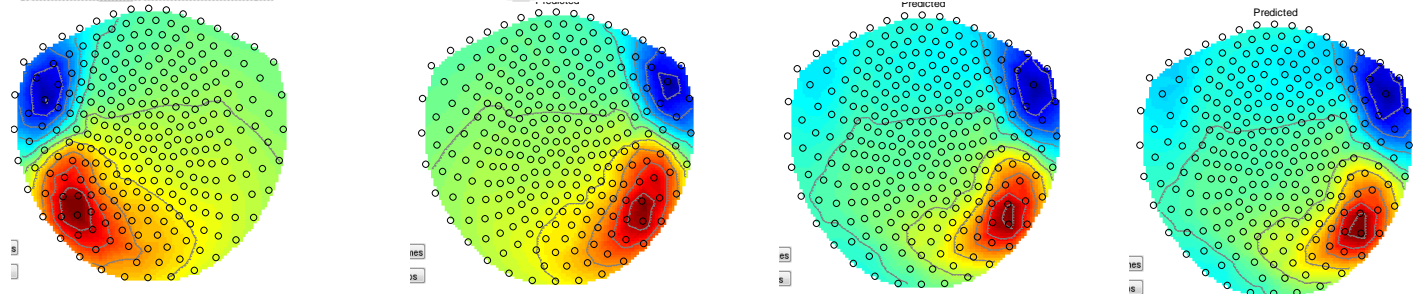
Prior



$\tilde{J}$



$\tilde{Y}$



Variance explained

11 %

96%

97%

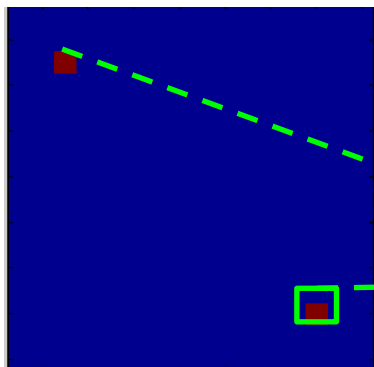
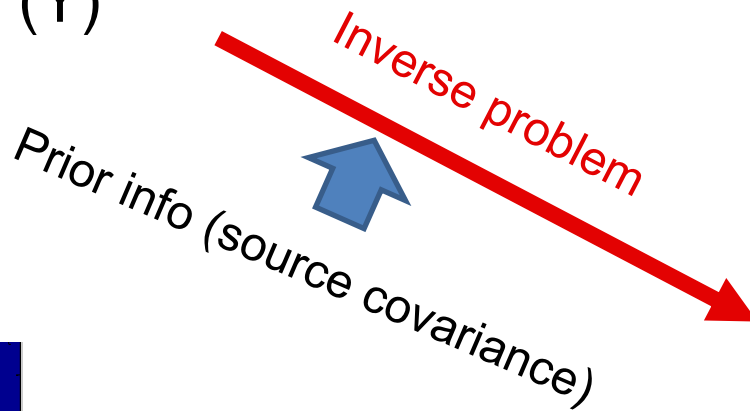
98%

# Use prior info (possible ingredients)

$\tilde{Y}$



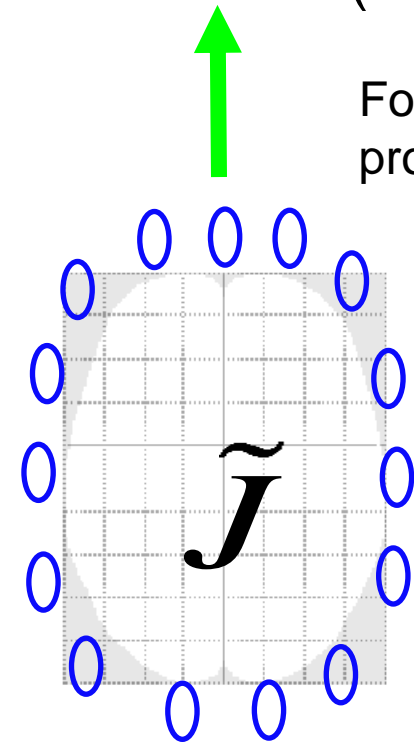
Measurement ( $Y$ )



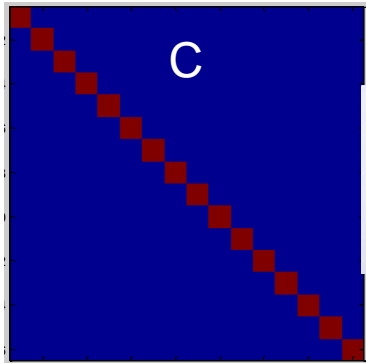
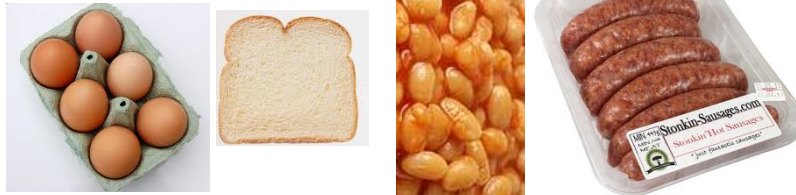
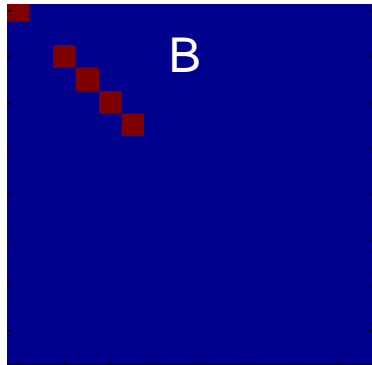
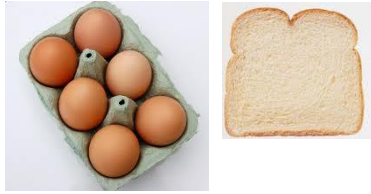
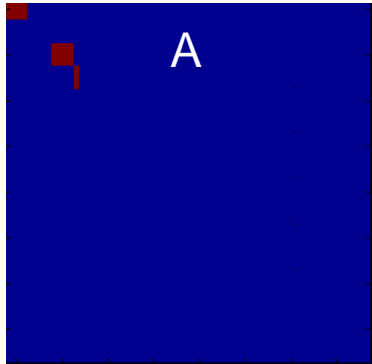
Diagonal elements correspond to ingredients

Prediction ( )

Forward problem



# Possible priors



Which is most likely prior (which prior has highest evidence) ?

$\tilde{Y}$



Measurement (Y)



Prediction ( )

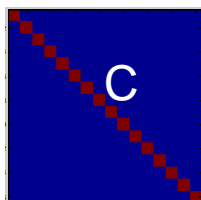
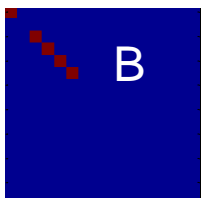
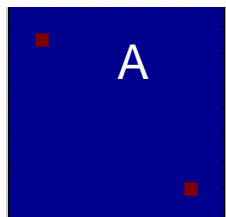
Forward problem



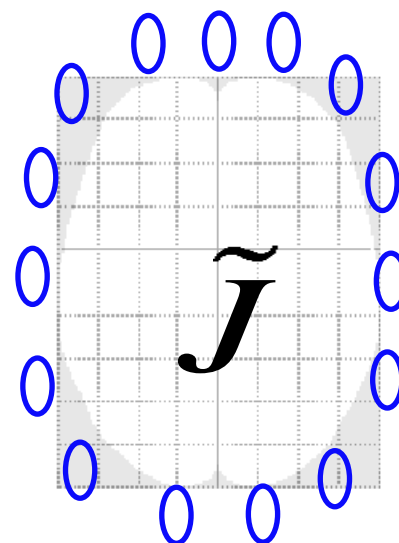
Inverse problem



Prior info (source covariance)

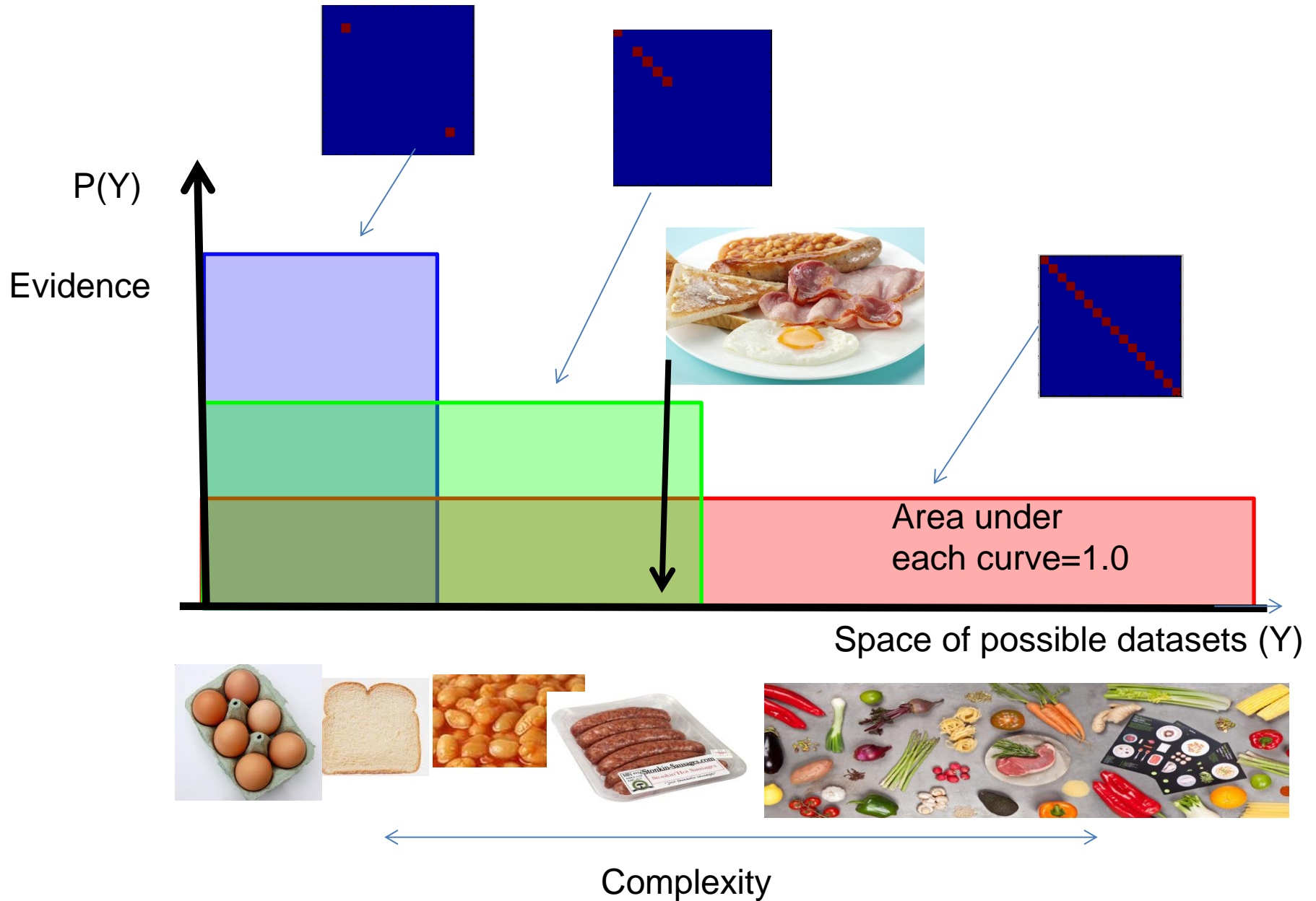


?



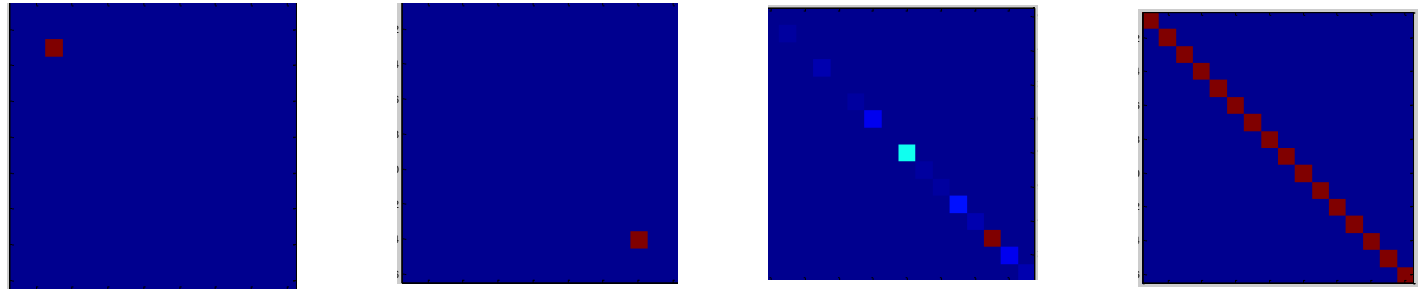


# Consider 3 generative models

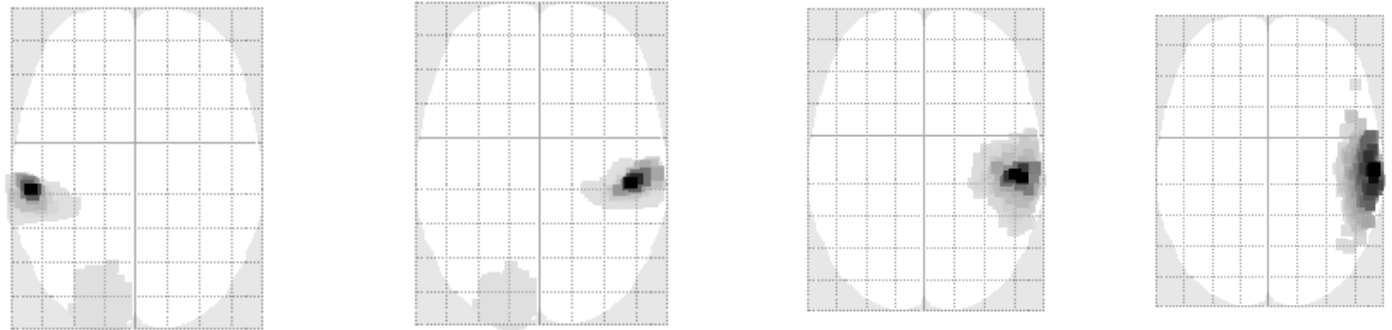


# How do we choose between priors ?

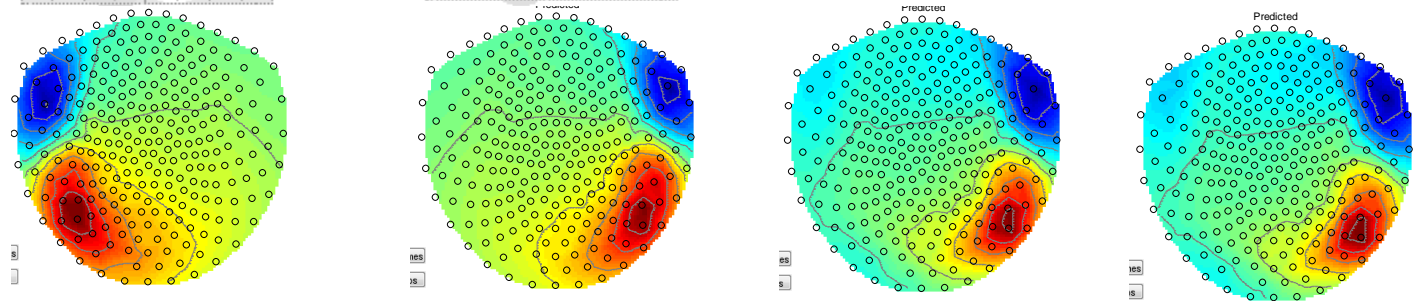
Prior



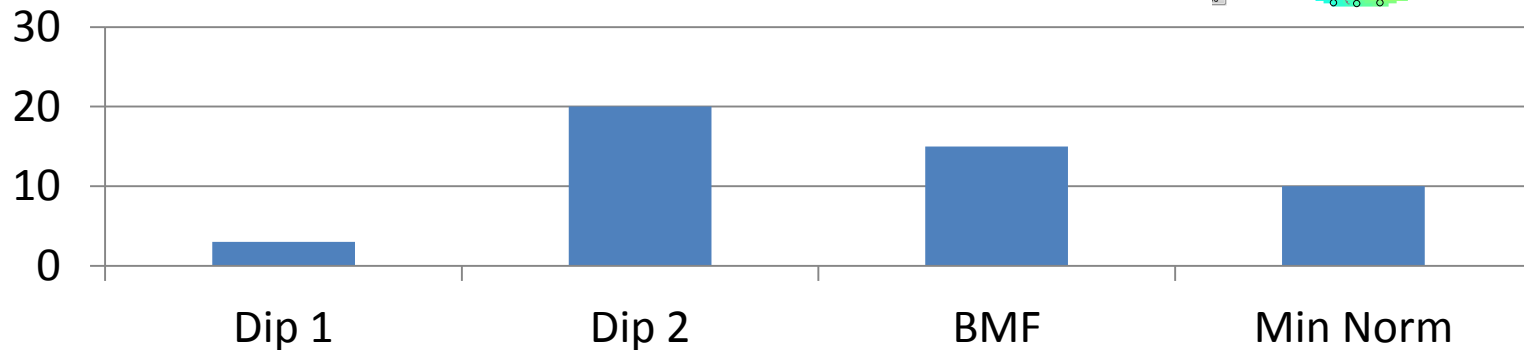
$\tilde{J}$



$\tilde{Y}$



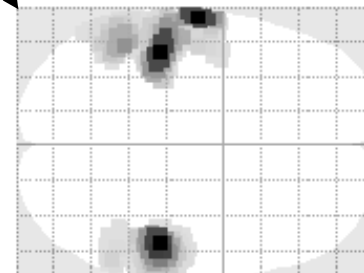
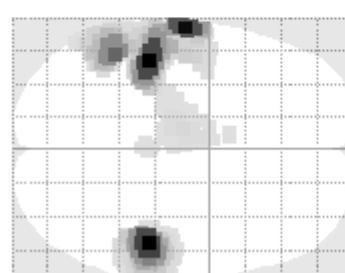
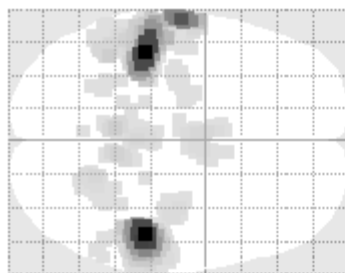
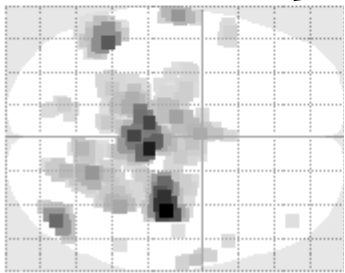
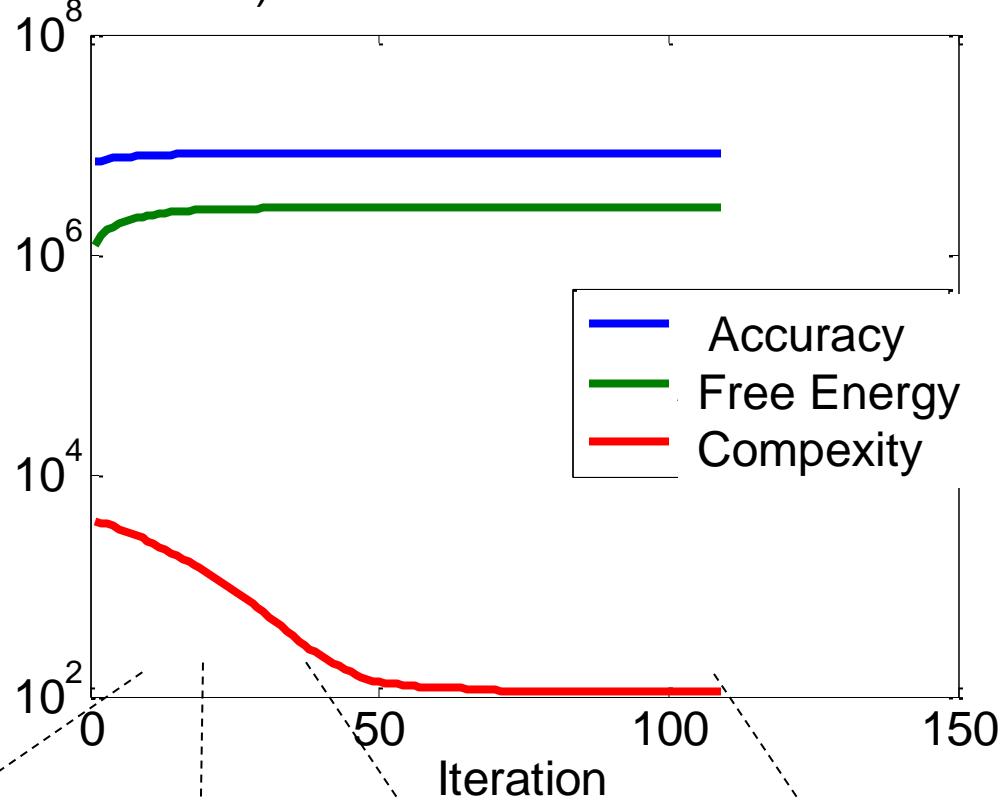
Log model evidence





# Multiple Sparse priors

So now construct the priors to maximise model evidence  
(minimise cross validation error).



# Conclusion

- MEG inverse problem can be solved easily with some prior knowledge.
- All prior knowledge encapsulated in a source covariance matrix.
- Can test between priors (or develop new priors) using Bayesian framework.

# Thank you

- Karl Friston
- Jose David Lopez
- Vladimir Litvak
- Guillaume Flandin
- Will Penny
- Jean Daunizeau
- Christophe Phillips
- Rik Henson
- Jason Taylor
- Luzia Troebinger
- Chris Mathys
- Saskia Helbling

And all SPM developers

## References

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- J. Mosher, S. Baillet, R.M. Leahy
- **Equivalence of linear approaches in bioelectromagnetic inverse solutions**
- IEEE Workshop on Statistical Signal Processing (2003), pp. 294–297
  
- [Friston et al., 2008](#)
- K. Friston, L. Harrison, J. Daunizeau, S. Kiebel, C. Phillips, N. Trujillo-Barreto, R. Henson, G. Flandin, J. Mattout
- **Multiple sparse priors for the M/EEG inverse problem**
- NeuroImage, 39 (2008), pp. 1104–1120
  
- [Wipf and Nagarajan, 2009](#)
- D. Wipf, S. Nagarajan
- **A unified Bayesian framework for MEG/EEG source imaging**
- NeuroImage, 44 (2009), pp. 947–966

LONGER VERSION OF THIS TALK:

- <http://www.nottingham.ac.uk/conference/fac-sci/physics/meg-uk-conference/workshop.aspx>

# Analytical approximation to model evidence

- Free energy= accuracy- complexity

$$F = -\frac{N_n}{2} \text{tr}(\Sigma_Y \Sigma^{-1}) - \frac{N_n}{2} \log|\Sigma| - \frac{N_n N_c}{2} \log 2\pi \\ - \frac{1}{2} (\hat{\lambda} - \nu)^T \Pi (\hat{\lambda} - \nu) + \frac{1}{2} \log|\Sigma_\lambda \Pi|$$

$$F = - \left[ \begin{array}{c} \text{Model} \\ \text{error} \end{array} \right] - \left[ \begin{array}{c} \text{Size of model} \\ \text{covariance} \end{array} \right] - \left[ \begin{array}{c} \text{Num of data} \\ \text{samples} \end{array} \right] \\ - \left[ \begin{array}{c} \text{Error in} \\ \text{hyperparameters} \end{array} \right] + \left[ \begin{array}{c} \text{Error in covariance} \\ \text{of hyperparameters} \end{array} \right].$$