Adapted from talk at:

Biomag satellite symposium:

"From zero to hero"

Connectivity analysis: the basics

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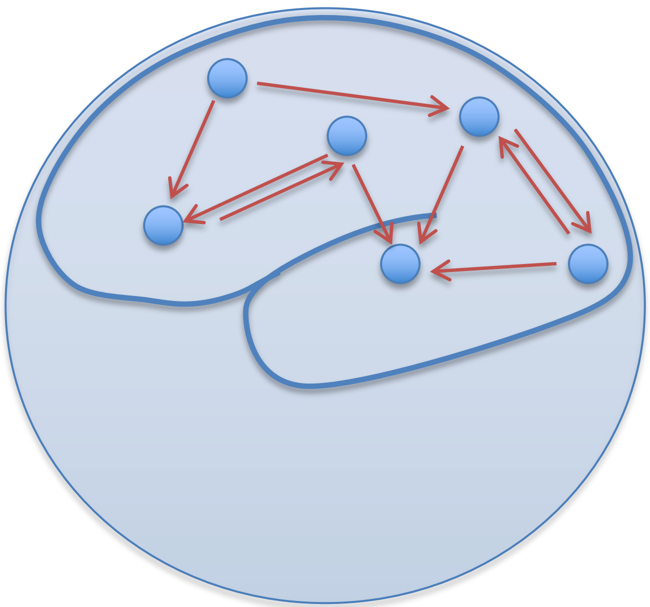
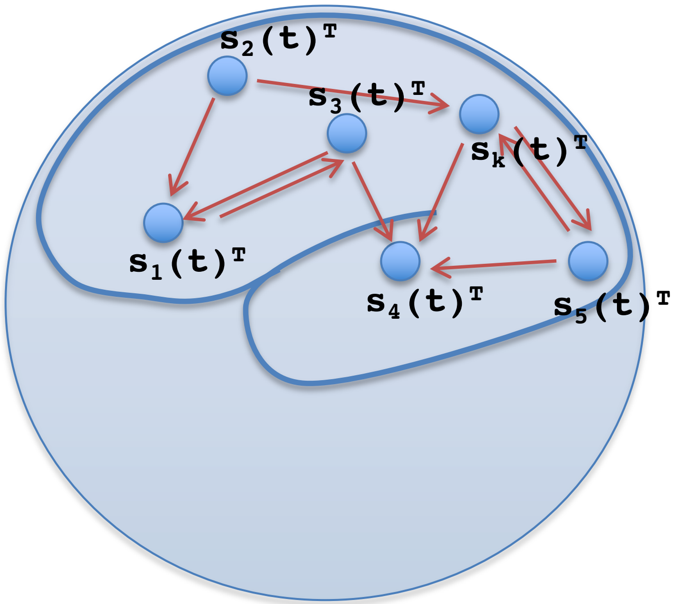
Connectivity analysis: goal



Connectivity analysis: goal



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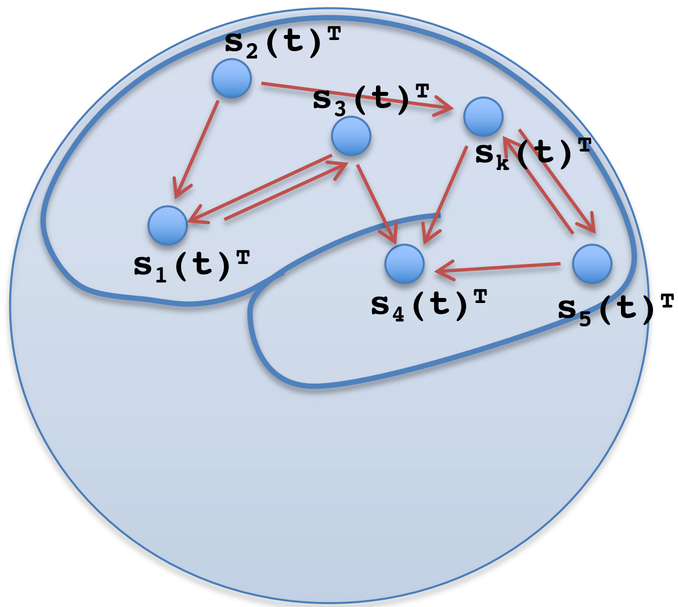
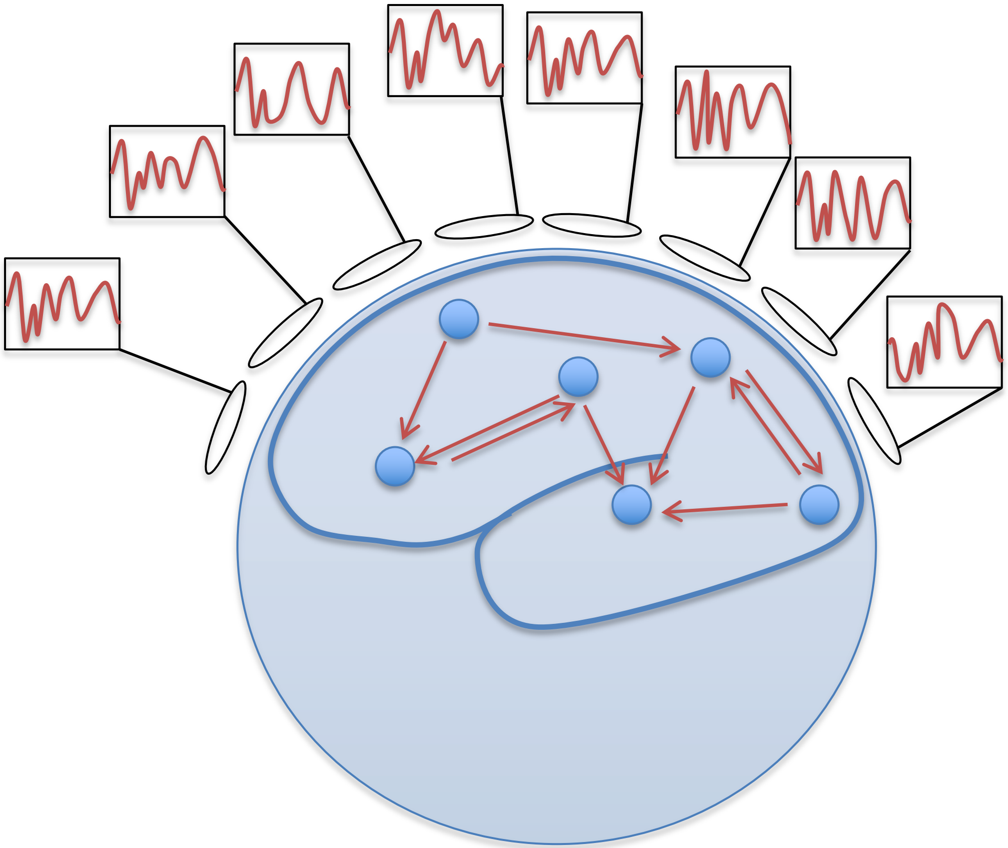


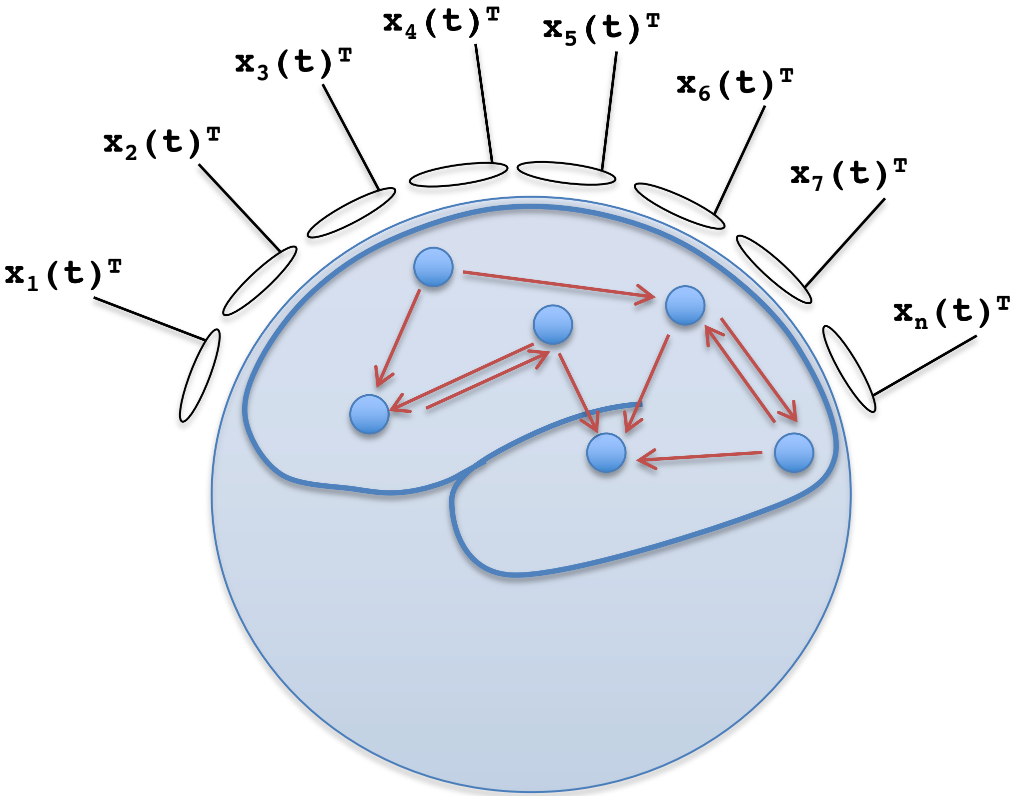
$$C = f(\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_k)$$

$$C = f(\mathbf{s}_i, \mathbf{s}_j)$$

Connectivity analysis: practice

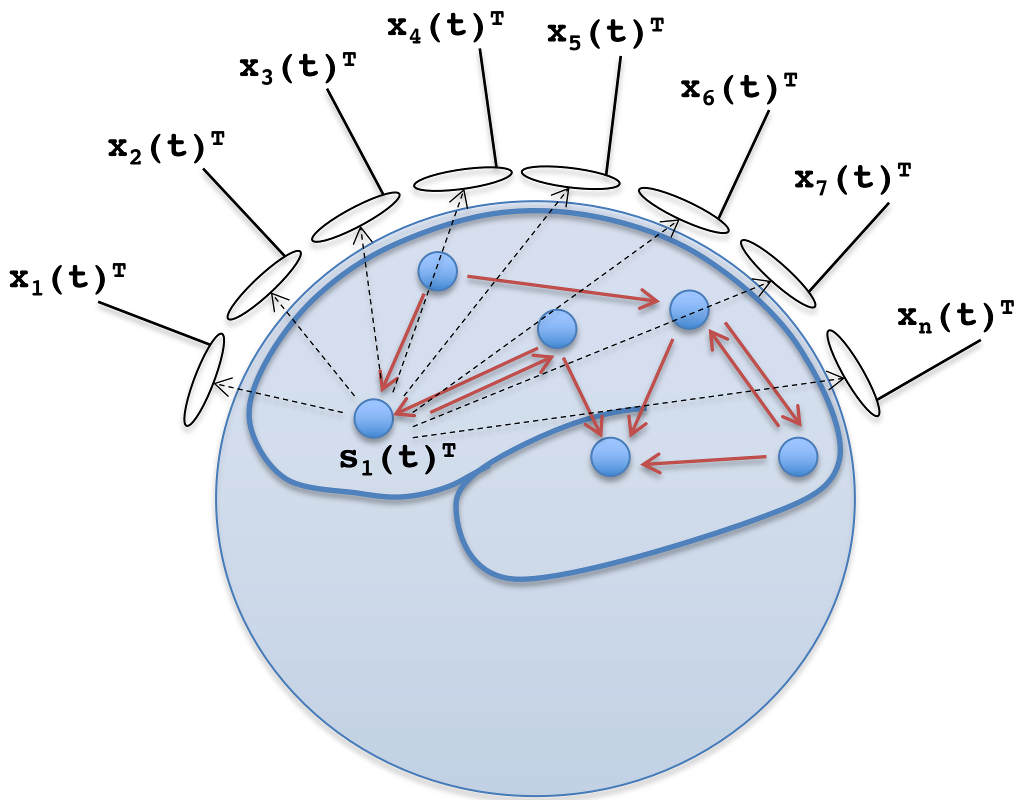


Connectivity analysis: practice



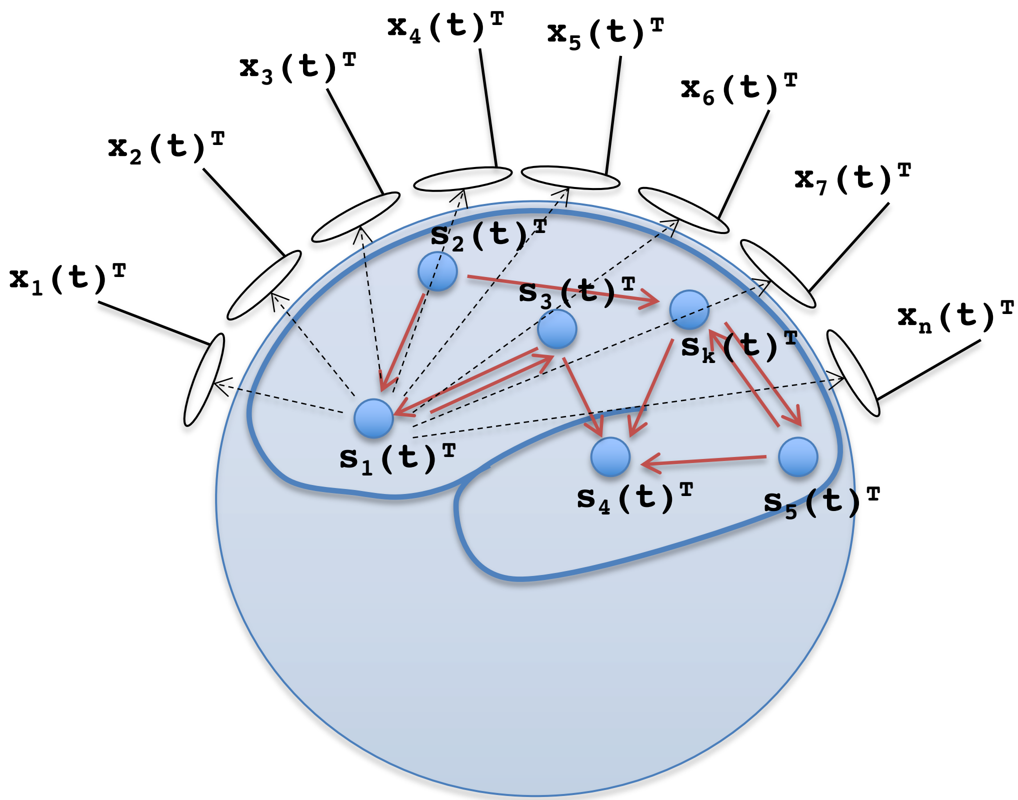
$$C = f(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_k)$$

Connectivity analysis: challenge



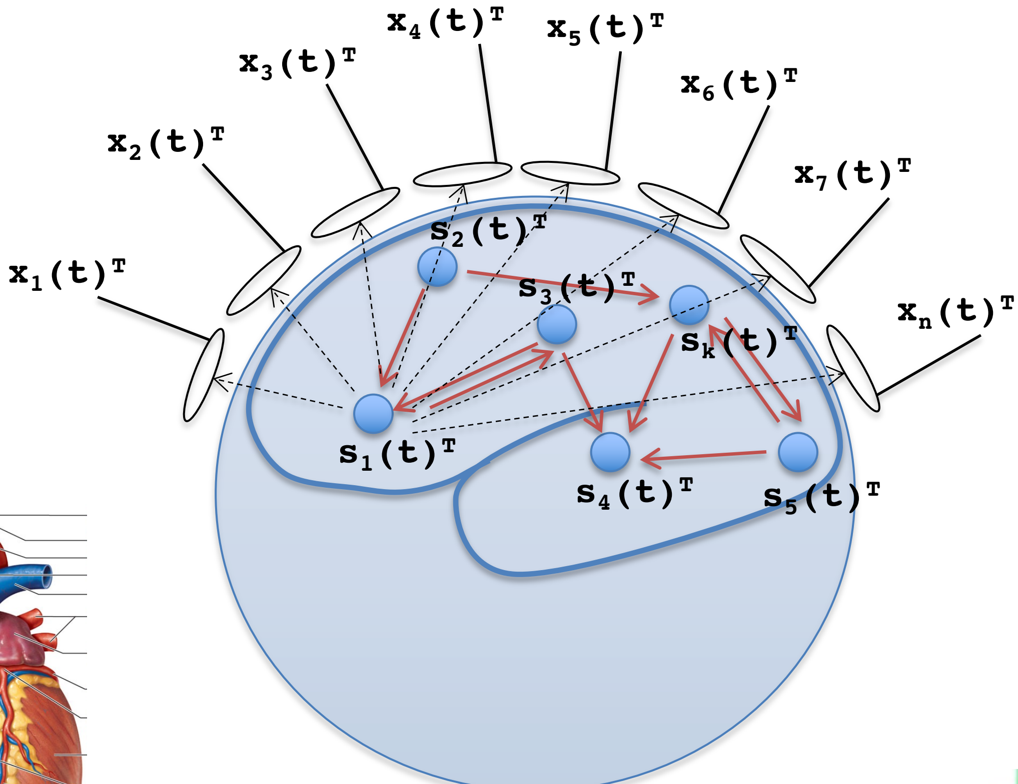
$$\mathbf{X} = [\mathbf{x}_1\ \mathbf{x}_2\ \ldots\ \mathbf{x}_n]^T = \mathbf{l}_1\mathbf{s}_1^T + \mathbf{N}$$

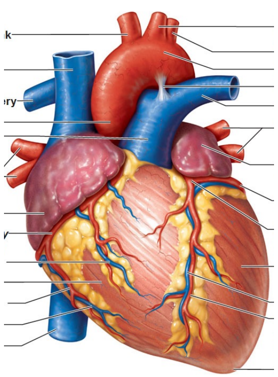
Connectivity analysis: challenge



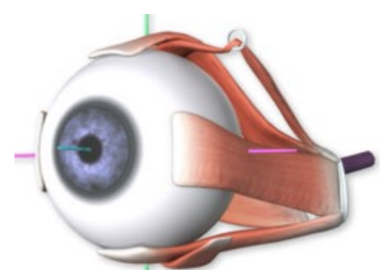
$$\mathbf{X} = \mathbf{l}_1\mathbf{s}_1^T + \mathbf{l}_2\mathbf{s}_2^T + \ldots + \mathbf{l}_k\mathbf{s}_k^T + \mathbf{N}$$

Connectivity analysis: challenge





$$\mathbf{X} = \mathbf{LS} + \mathbf{N}$$



This talk

$$\mathtt{C} = f(\mathbf{y}_i, \mathbf{y}_j)$$

- Choice of *f*
- Choice of y_i and y_j
- What to keep in mind when interpreting C

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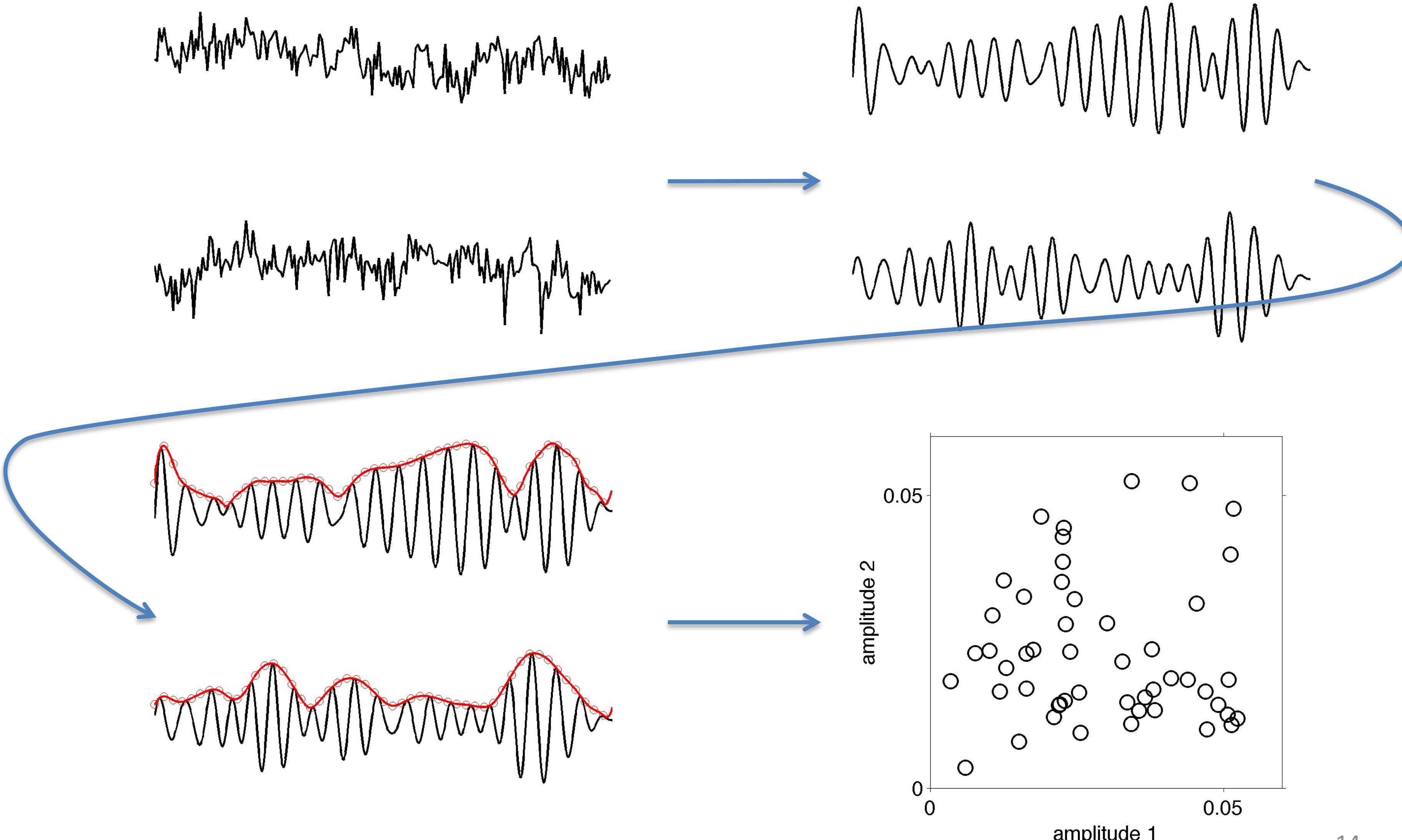
Connectivity metrics

- Many a metric on the market
- Functional versus effective connectivity
- Time domain versus frequency domain
- Frequency domain: using amplitude information versus using phase (+amplitude) information

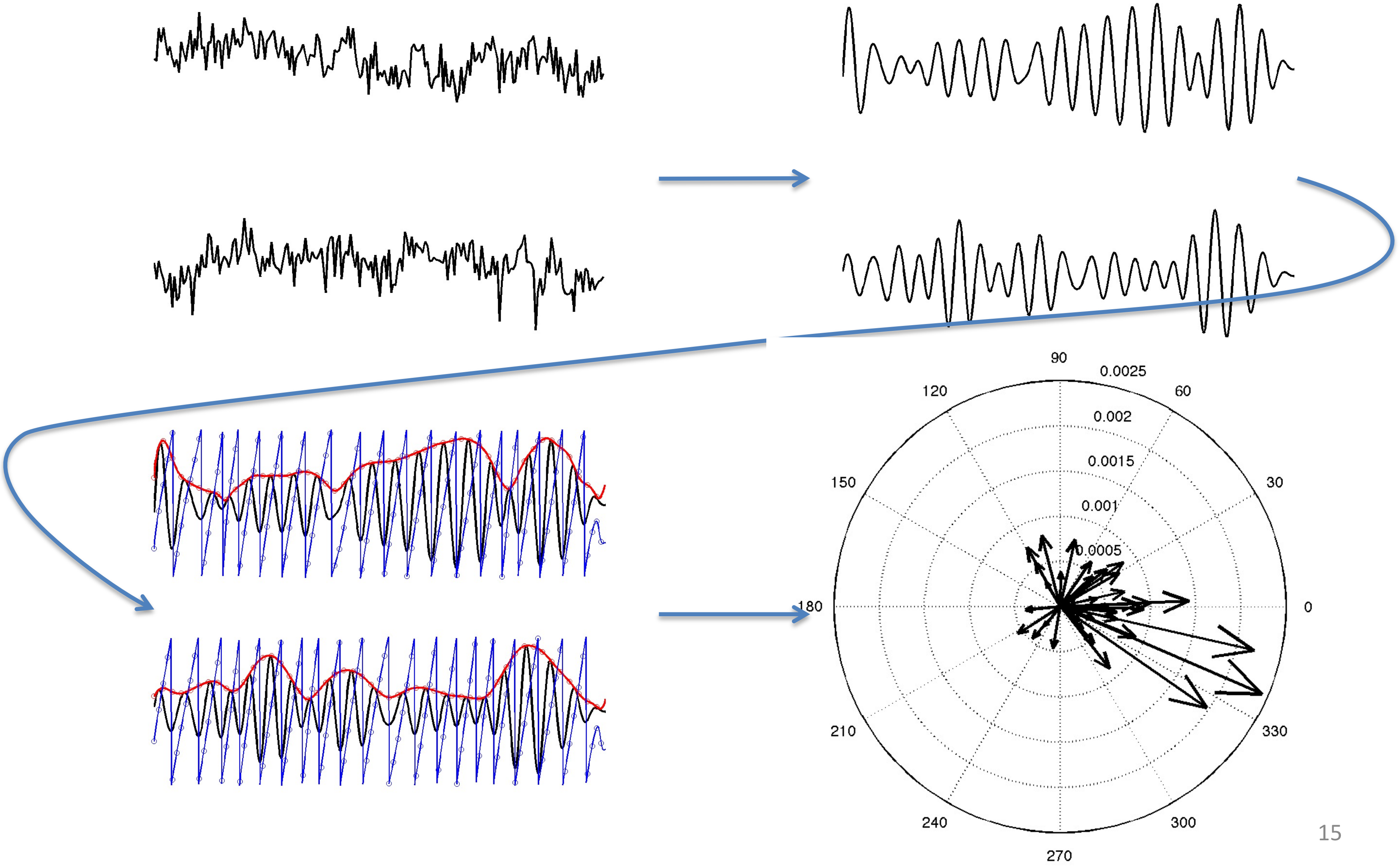
Spectrally-resolved C-metrics

- amplitude/power (envelope) correlation
- phase difference consistency measures
- spectrally-resolved Granger causality
- cross-frequency interactions (not discussed today)

Amplitude/power (envelope) correlation

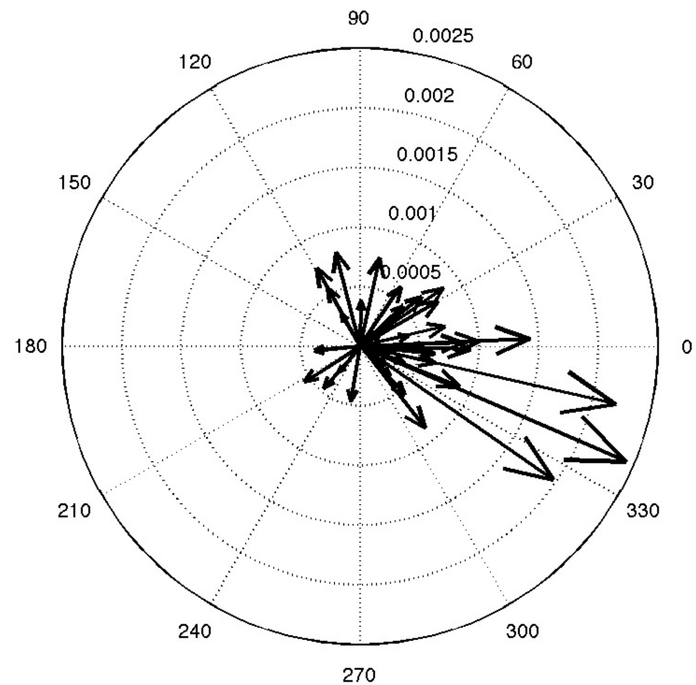


Phase difference consistency measures



Phase difference consistency measures

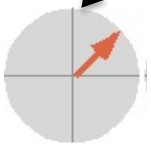
- Coherence
- Phase locking value
- Imaginary part of coherency
- Phase slope index
- Phase lag index
- Weighted phase lag index



Coherence

$$x_1 x_2^* = A_1 e^{i\varphi_1} \cdot A_2 e^{-i\varphi_2} = A_1 A_2 e^{i(\varphi_1 - \varphi_2)}$$

single trial cross-spectral density



Coherence & co

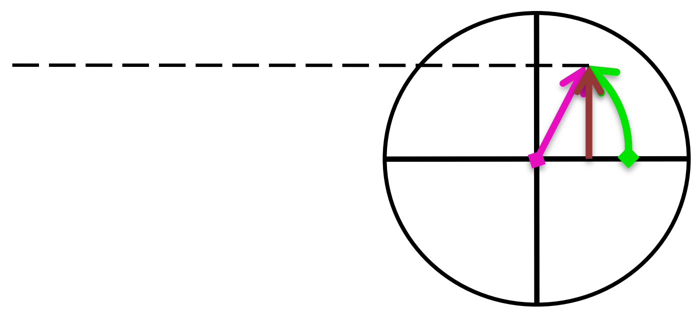
$$\text{Coherence} = \left| \frac{1/N \sum A_1 A_2 e^{i(\varphi_1 - \varphi_2)}}{\sqrt{(1/N \sum A_1^2)(1/N \sum A_2^2)}} \right|$$

$$\text{PLV} = \left| \frac{1/N \sum 1 \text{x} 1 \text{x} e^{i(\varphi_1 - \varphi_2)}}{\sqrt{(1/N \sum 1^2)(1/N \sum 1^2)}} \right|$$

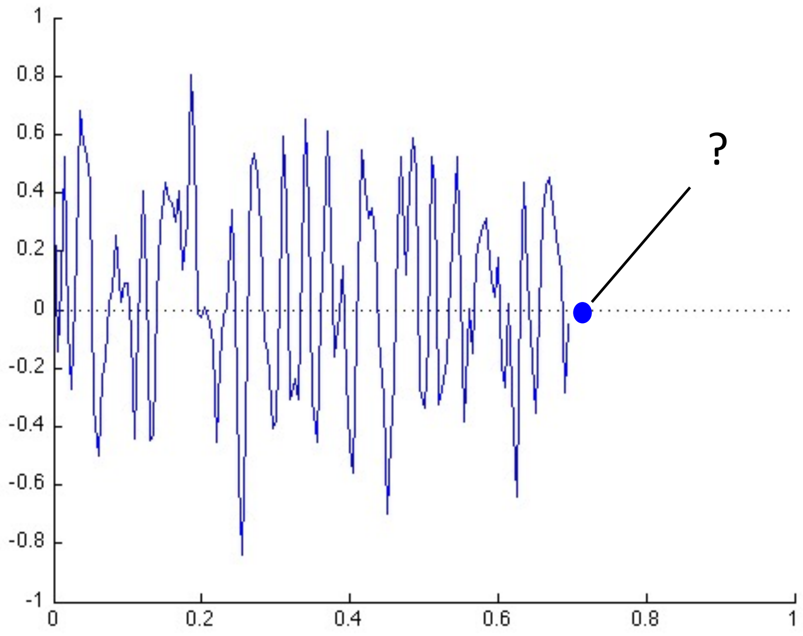
Coherence & co

$$\text{Coherency} = \frac{1/N \sum A_1 A_2 e^{i(\varphi_1 - \varphi_2)}}{\sqrt{(1/N \sum A_1^2)(1/N \sum A_2^2)}} = C e^{i\Delta\varphi}$$

Imaginary part of coherency



Predicting the future
the concept of Granger causality



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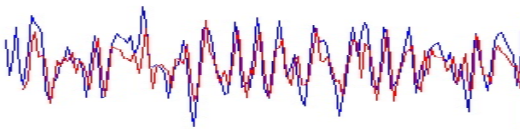

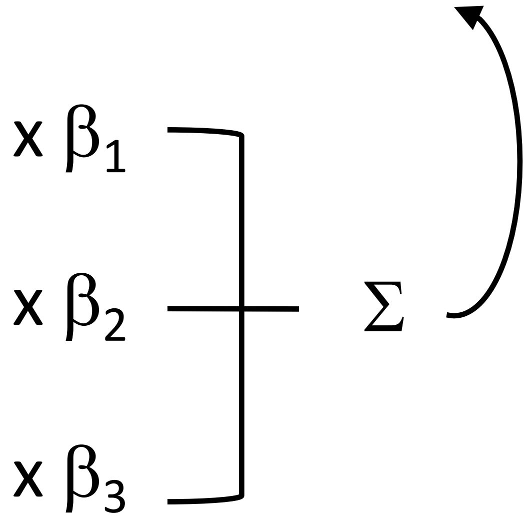

$$X(t) = \sum \beta_\tau X(t-\tau) + \eta$$

Granger causality

$X(t) = \sum \beta_{\tau 1} X(t-\tau) + \eta_1$

$Y(t) = \sum \beta_{\tau 2} Y(t-\tau) + \eta_2$

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$$X(t) = \sum \beta_{\tau 11} X(t-\tau) + \sum \beta_{\tau 21} Y(t-\tau) + \varepsilon_1$$

$$Y(t) = \sum \beta_{\tau 12} X(t-\tau) + \sum \beta_{\tau 22} Y(t-\tau) + \varepsilon_2$$

Granger causality

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Granger causality

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$$F_{Y \rightarrow X} = \ln\left(\frac{var(\eta_1)}{var(\varepsilon_1)} \right)$$

$$F_{X \rightarrow Y} = \ln\left(\frac{var(\eta_2)}{var(\varepsilon_2)} \right)$$

Granger causality

- Fourier transformation of autoregressive coefficients gives spectral transfer matrix
- Spectrally-resolved Granger causality
- Partial directed coherence
- Directed transfer function

This talk

$$C = f(\mathbf{y}_i, \mathbf{y}_j)$$

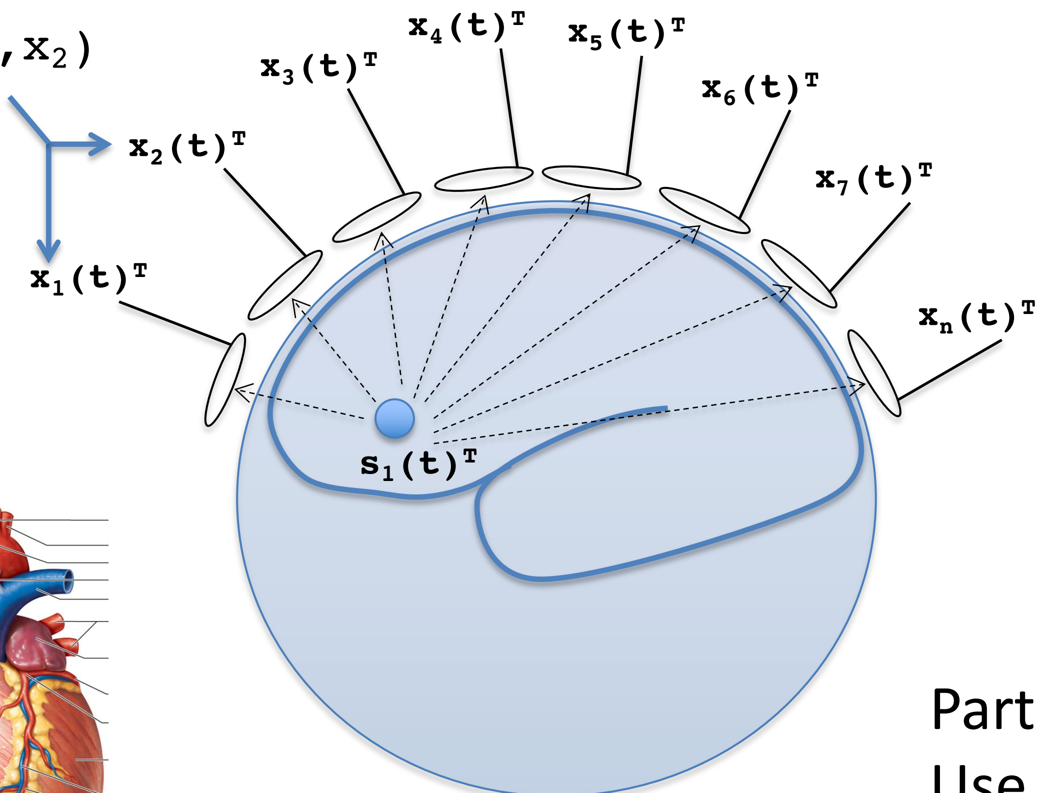
- Choice of *f*
- **Choice of $\mathbf{y}_i$ and $\mathbf{y}_j$**
- What to keep in mind when interpreting C

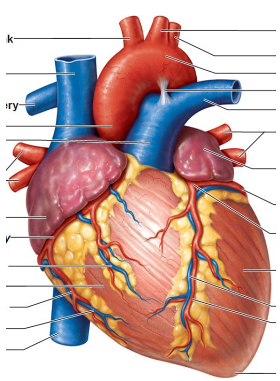
Which signals to compare?

- Sensor level signals

- Source level signals

Sensor-level connectivity: bad idea



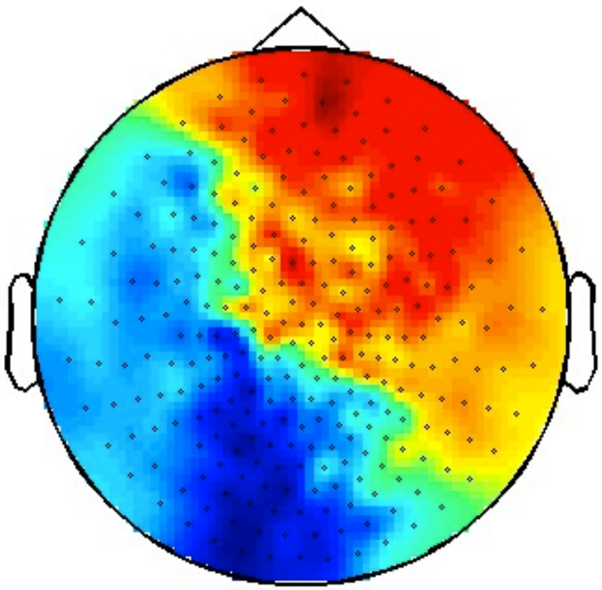


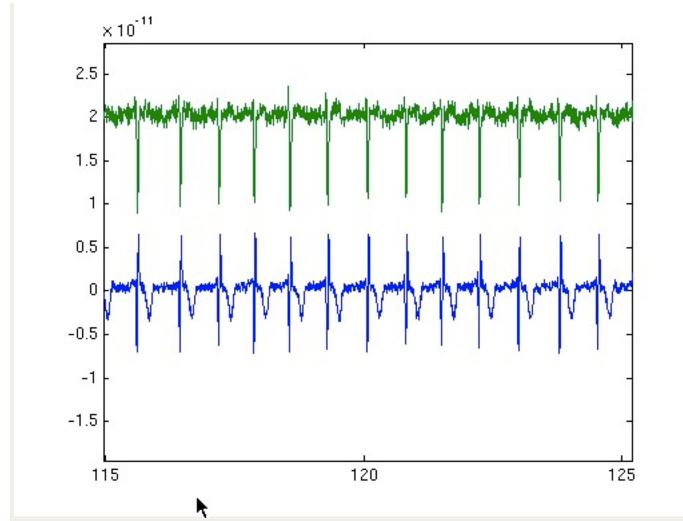
Partial remedy:

Use C-metric such as

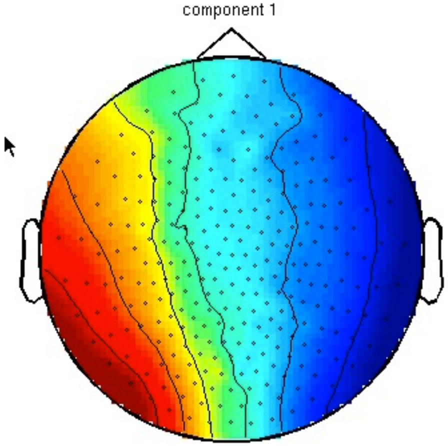
- Imag(coherency)
- Phase Lag Index
- Phase Slope Index

WPLI suggests fronto-occipital directed interaction (alpha band)

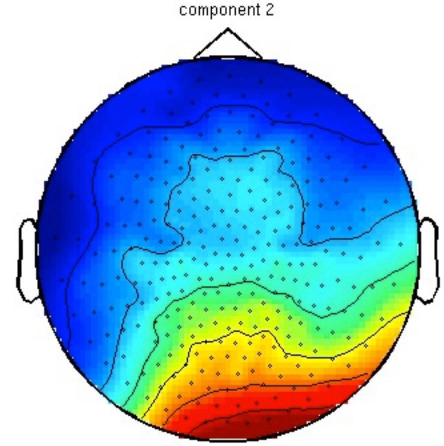




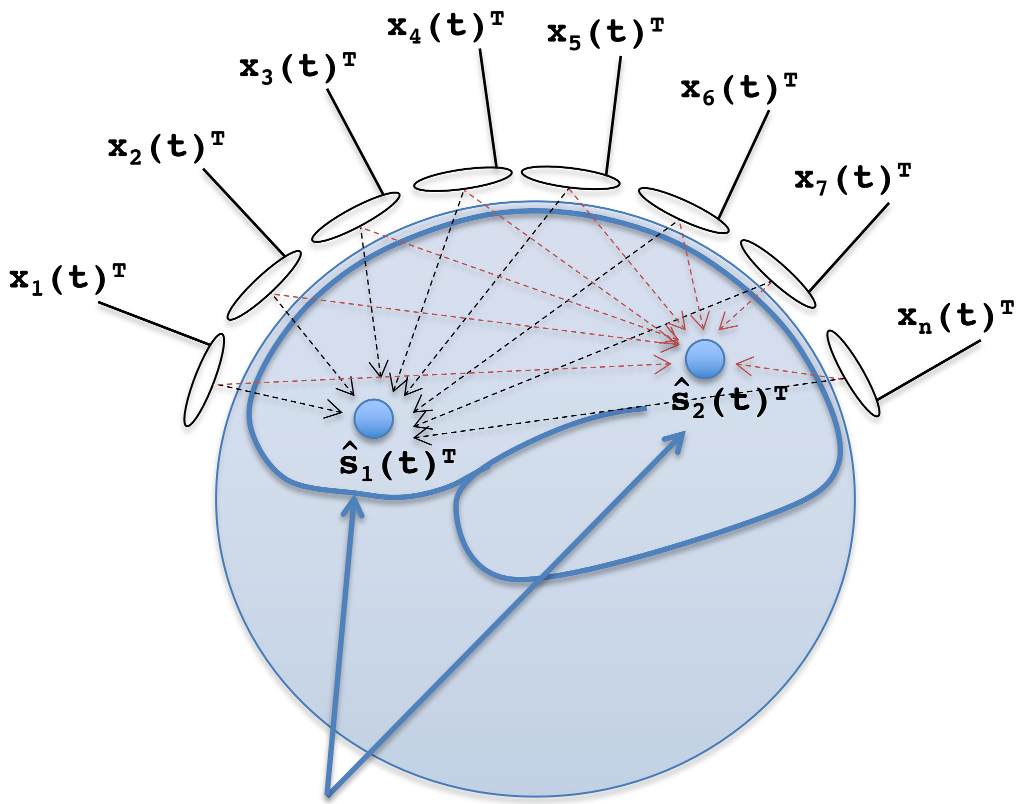
component 1



component 2



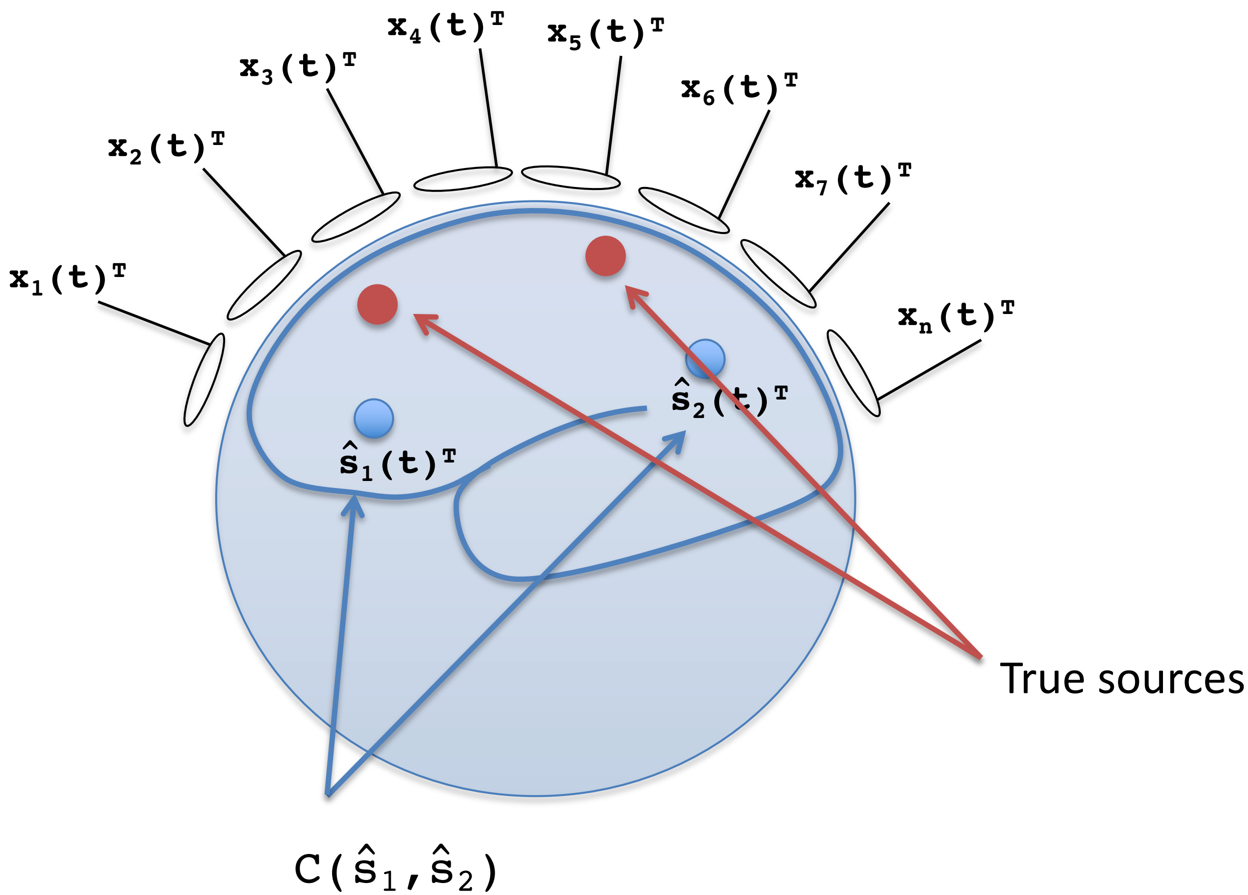
Source-level connectivity: better idea



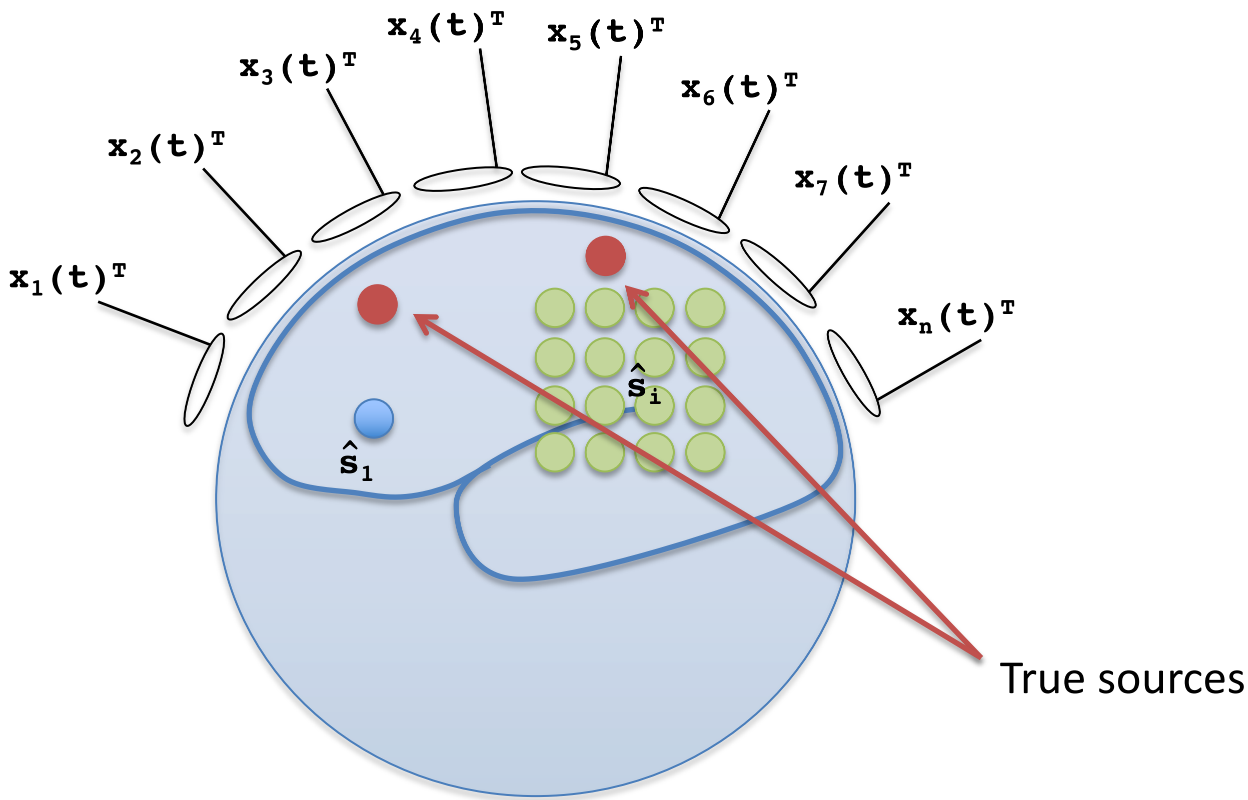
$C(\hat{s}_1, \hat{s}_2)$

Note: there will still be signal leakage

Source-level connectivity: where to look?

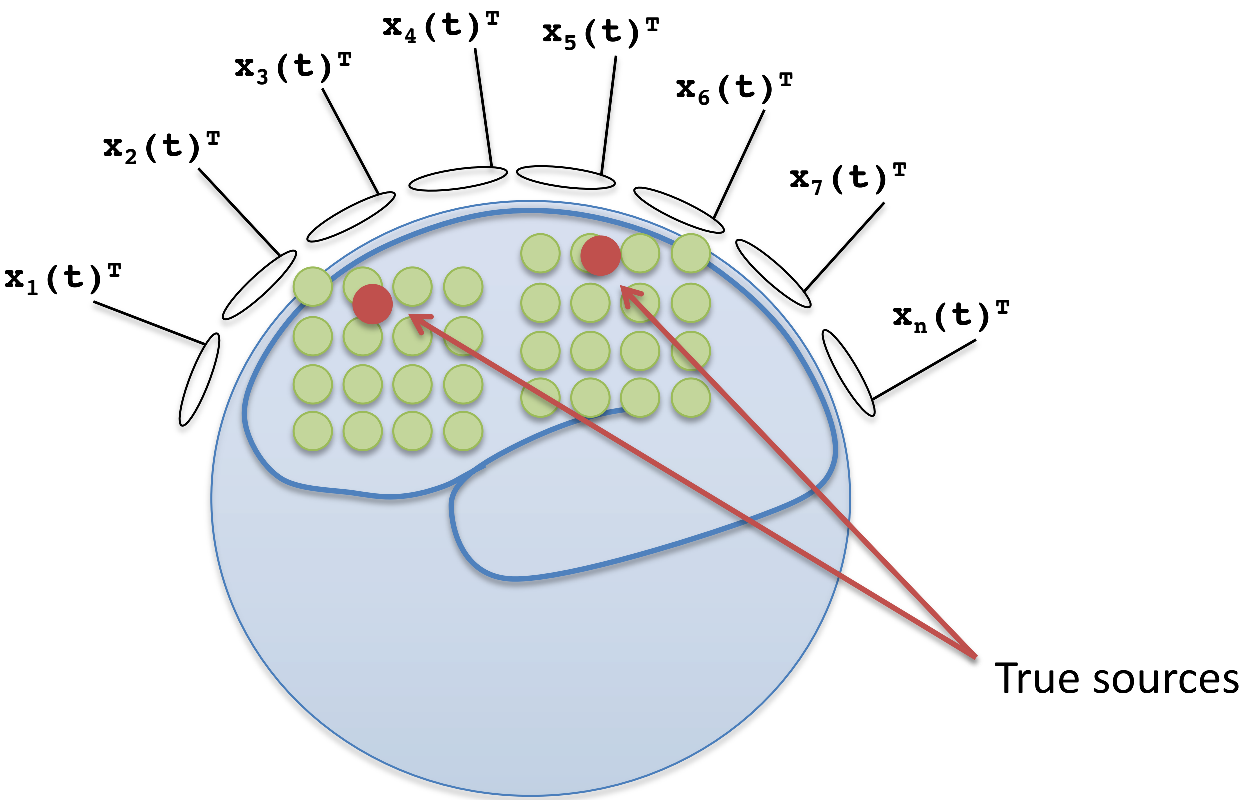


Source-level connectivity: seed-based approach



$C(\hat{s}_1, \hat{s}_i)$

Source-level connectivity: all-to-all approach



$C(\hat{s}_i, \hat{s}_j)$

This talk

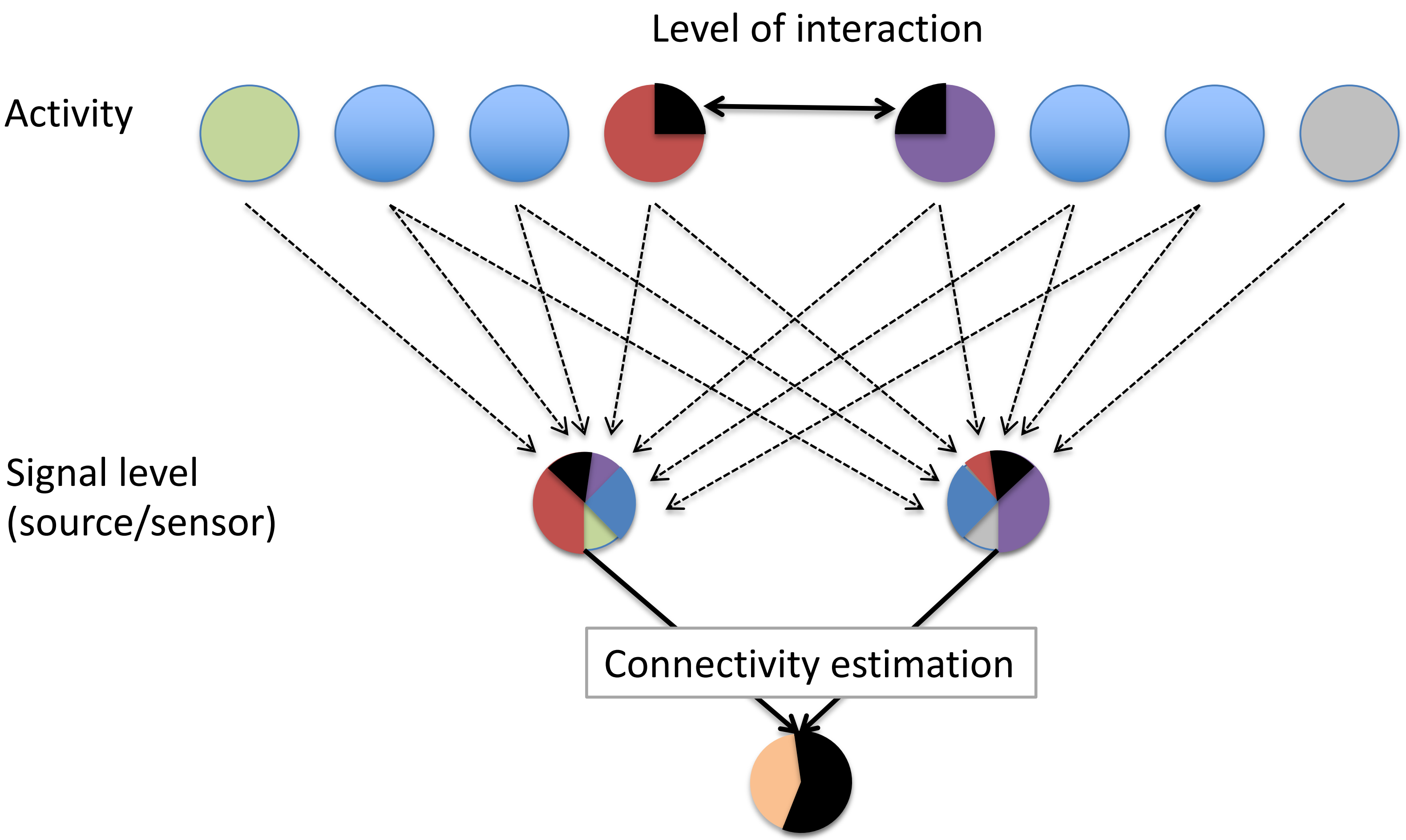
$$C = f(\mathbf{y}_i, \mathbf{y}_j)$$

- Choice of *f*
- Choice of y_i and y_j
- **What to keep in mind when interpreting C**

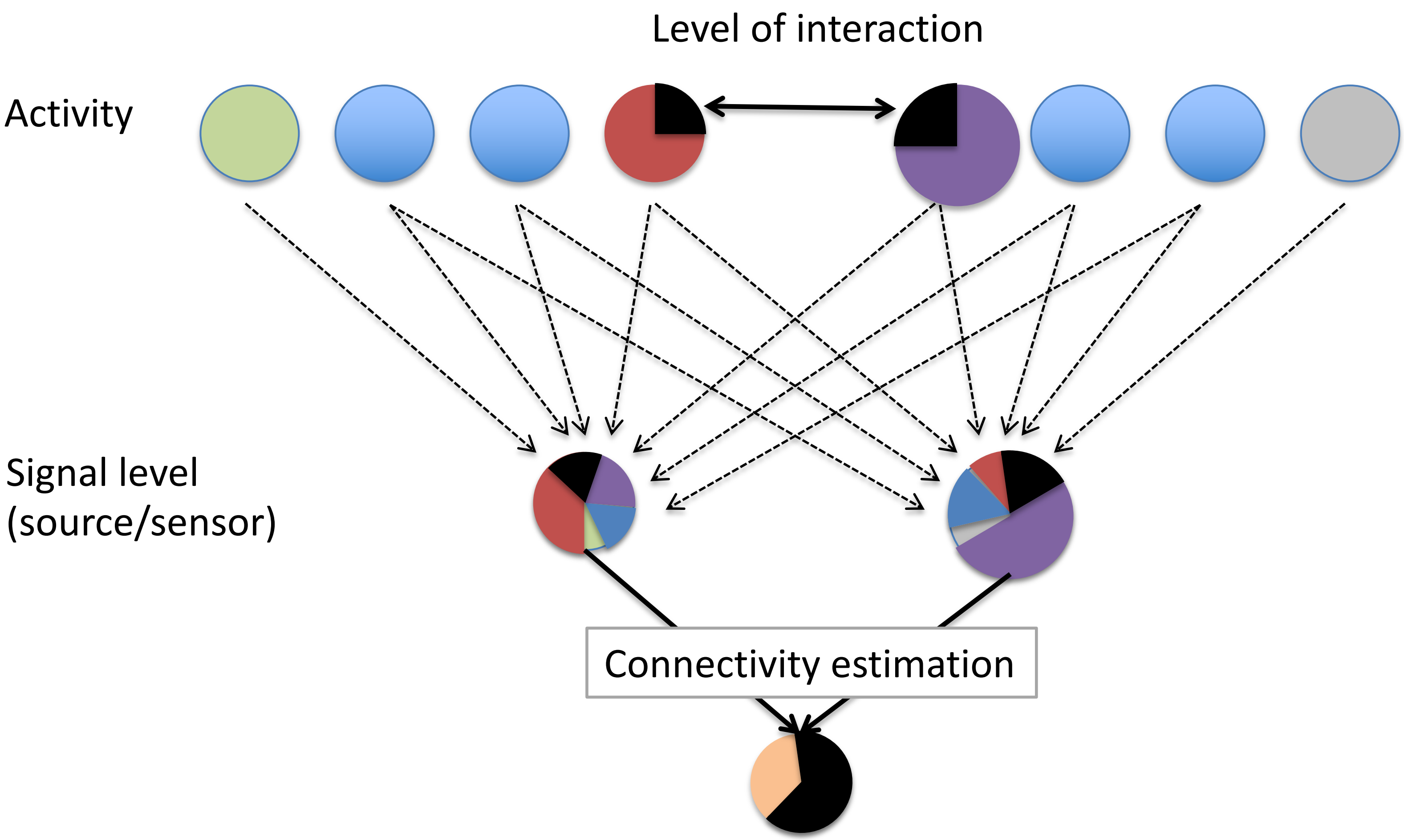
Interpretation: comparison

- Account for ‘spuriosity’ in the estimates
- Across groups of participants
- Across experimental conditions
- Against surrogate data

Interpretation



Interpretation



Summary

- Choice of *f*:

  Many C-metrics to choose from.

- Choice of y_i and y_j:

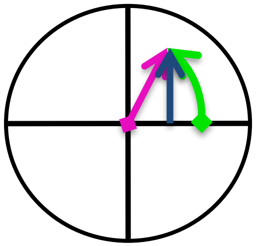
  Do connectivity analysis at the source level.

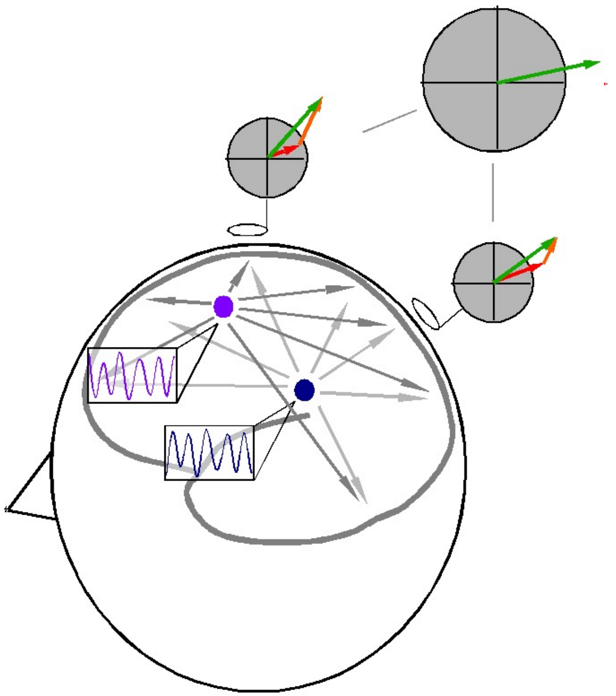
- What to keep in mind when interpreting C:

  Be careful with interpretation: SNR confound

Thanks for listening

Remedial C-metrics





Im(coherency) ≠ 0