

FieldTrip workshop, January 2016, Marseille

Frequency and time-frequency analysis

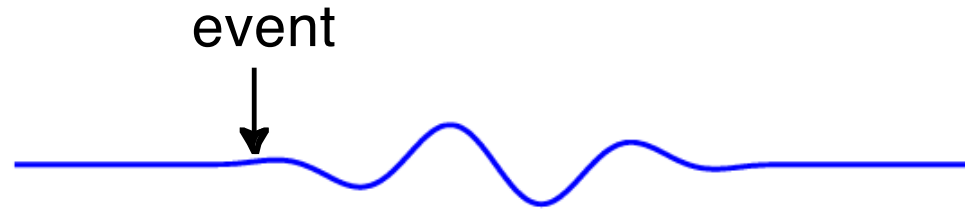
Robert Oostenveld

Donders Institute, Radboud University, Nijmegen, NL

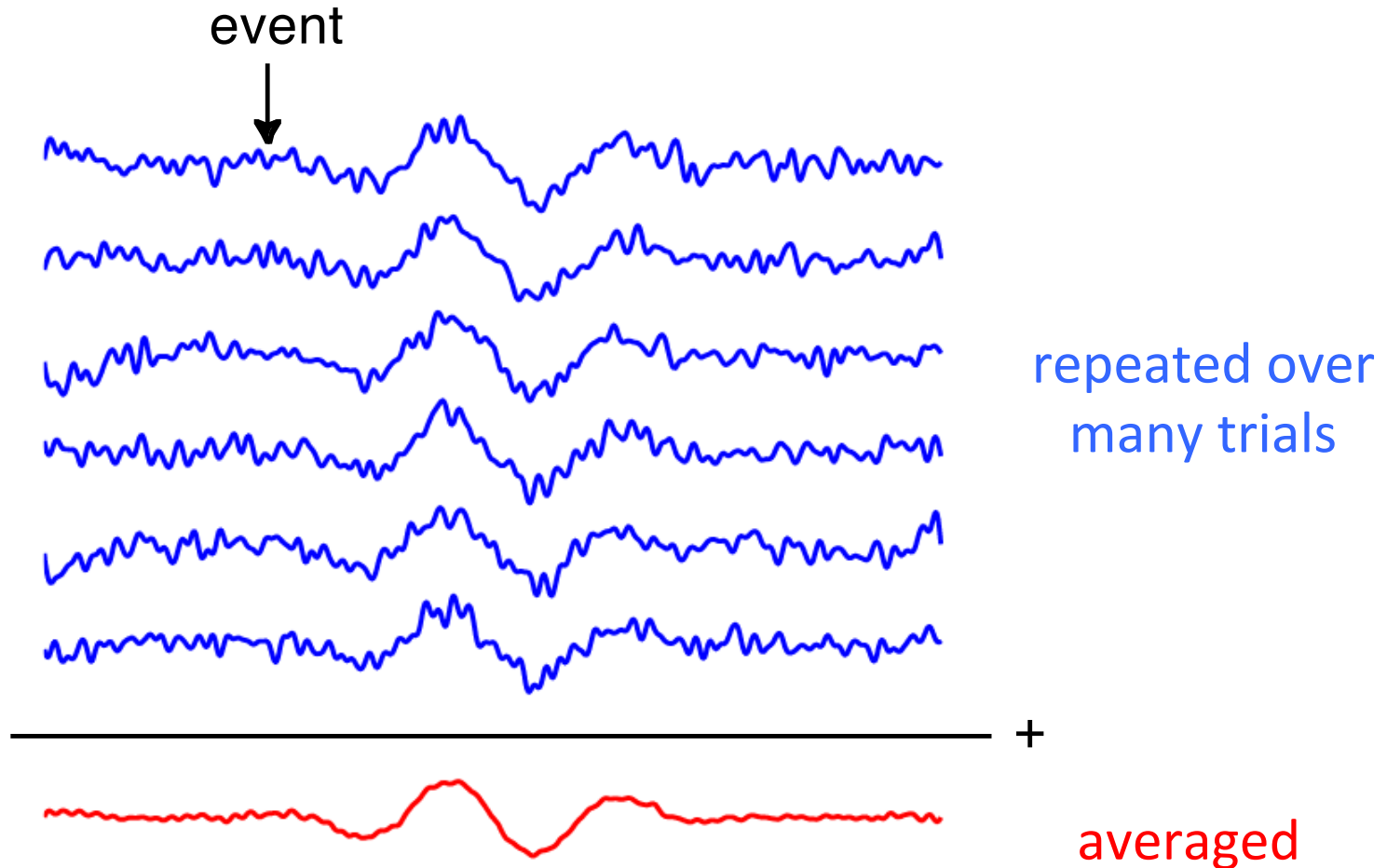
NatMEG, Karolinska Institute, Stockholm, SE



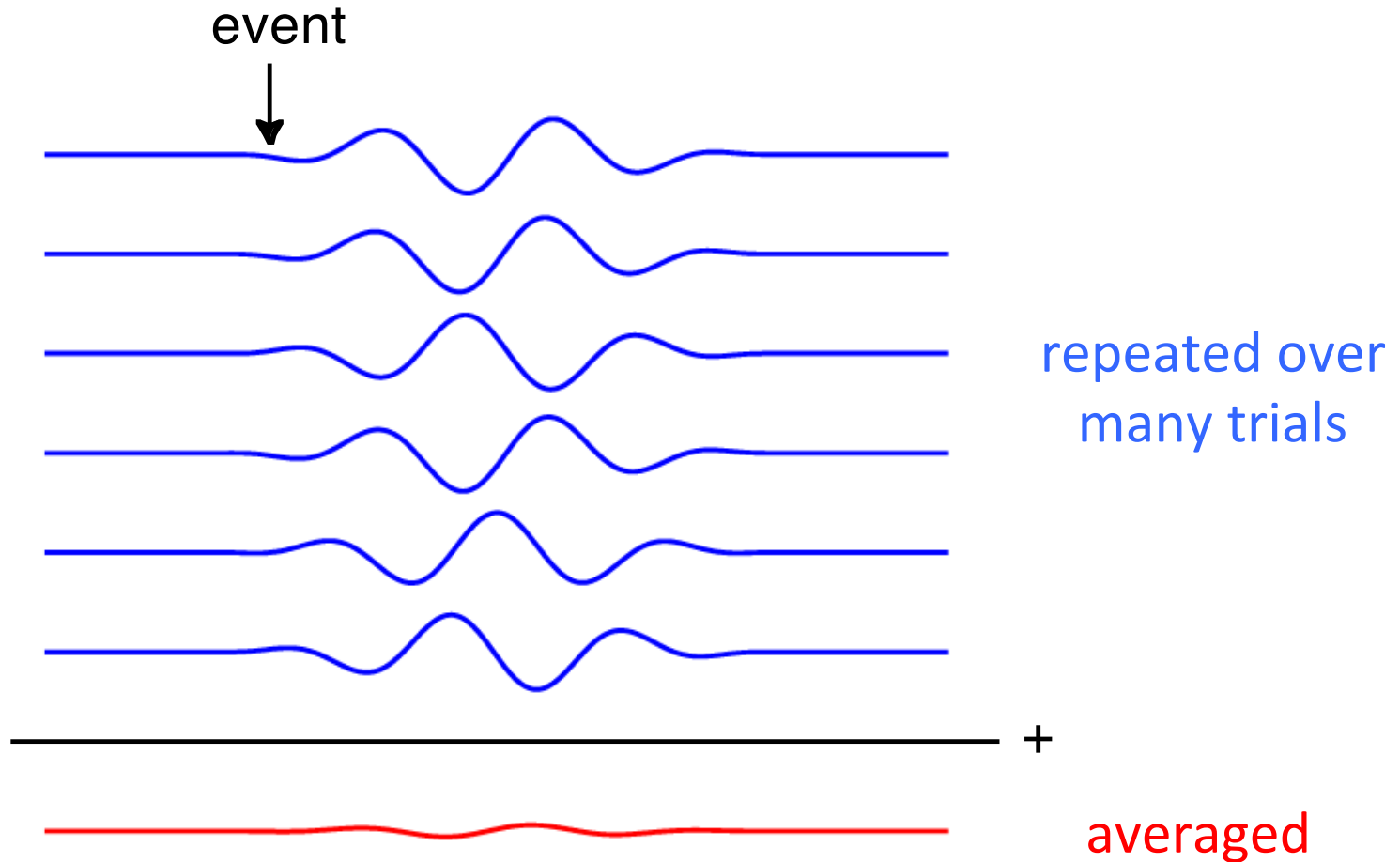
Evoked activity



Evoked activity



Induced activity



M/EEG signal characteristics considered during analysis

timecourse of activity

-> ERP

spectral characteristics

-> power spectrum

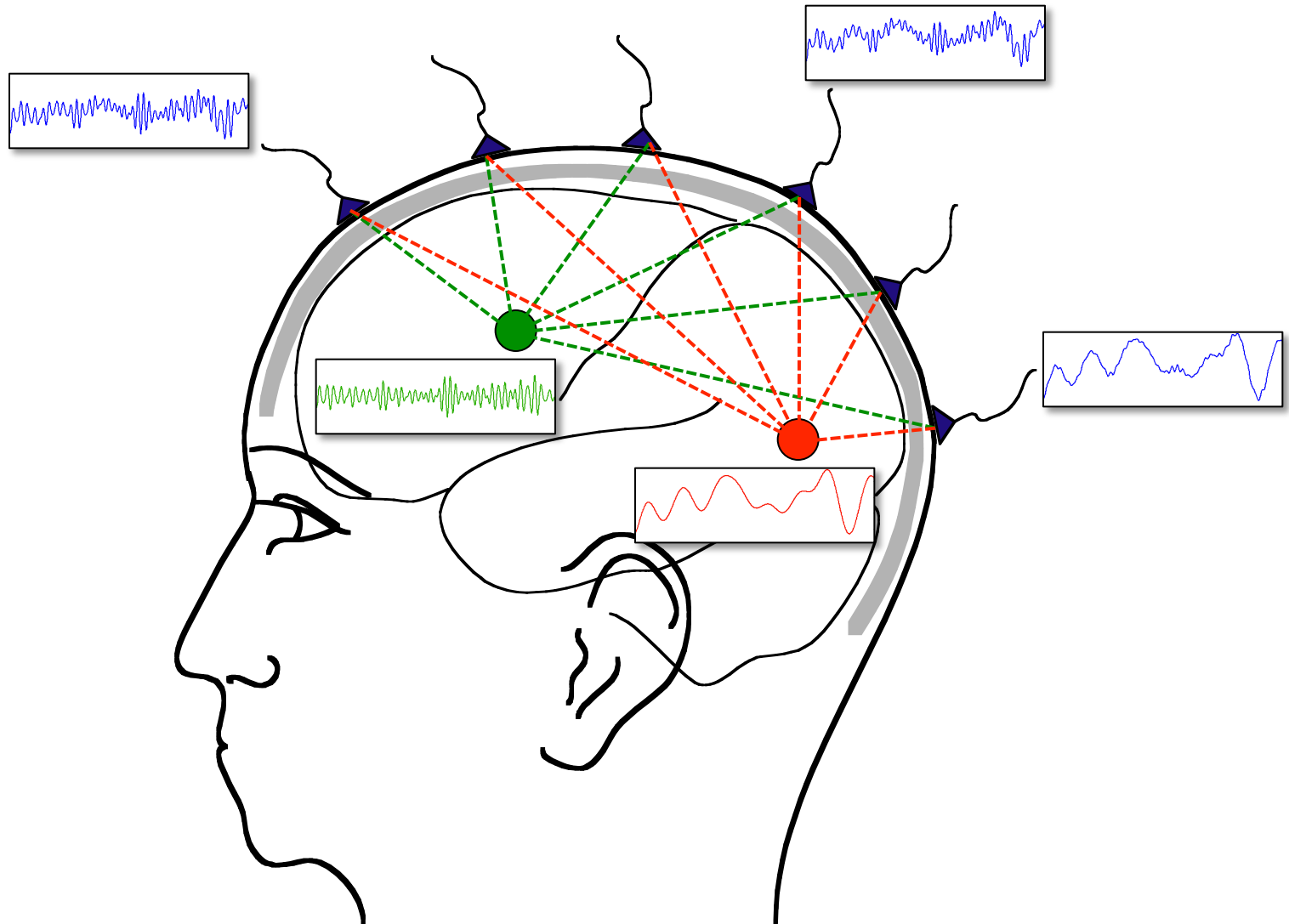
temporal changes in power

-> time-frequency response (TFR)

spatial distribution of activity over the head

-> source reconstruction

Superposition of source activity



Separating activity of different sources (and noise)

Use the temporal aspects of the data
at the channel level

- ERF latencies

- ERF difference waves

- Filtering the time-series

- Spectral decomposition

Use the spatial aspects of the data

- Volume conduction model of head

- Estimate source model parameters

Separating activity of different sources (and noise)

Use the temporal aspects of the data
at the channel level

ERF latencies

ERF difference waves

Filtering the time-series

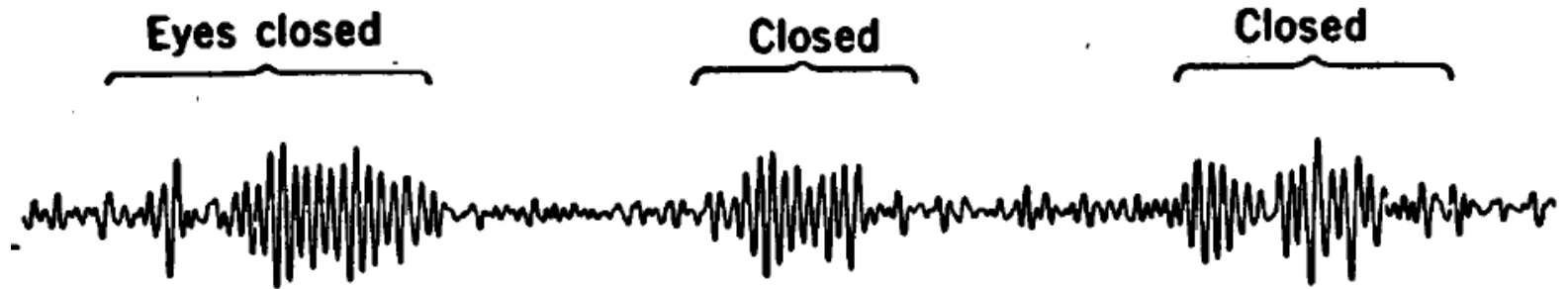
Spectral decomposition

Use the spatial aspects of the data

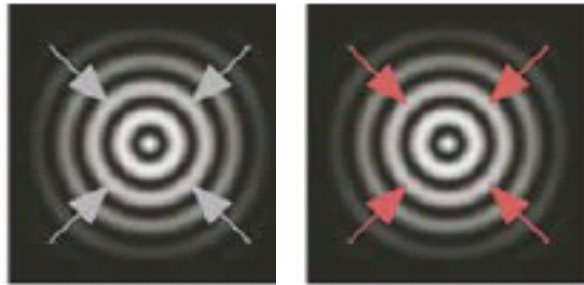
Volume conduction model of head

Estimate source model parameters

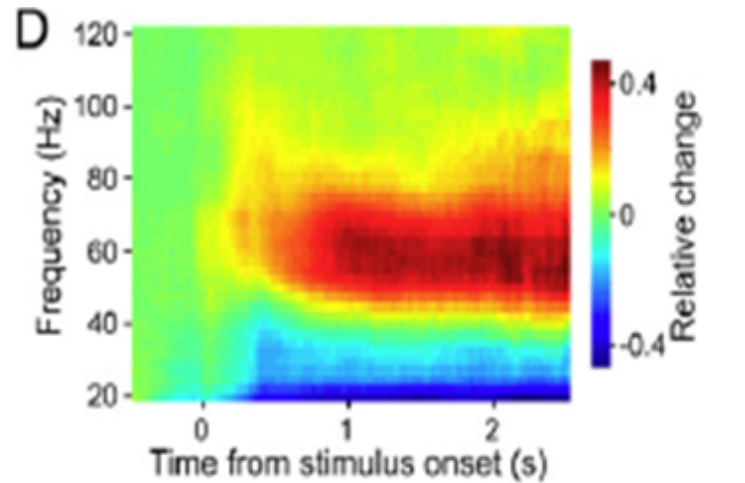
Brain signals contain oscillatory activity at multiple frequencies



Cohen, 1972



Hoogenboom et al, 2006



Outline

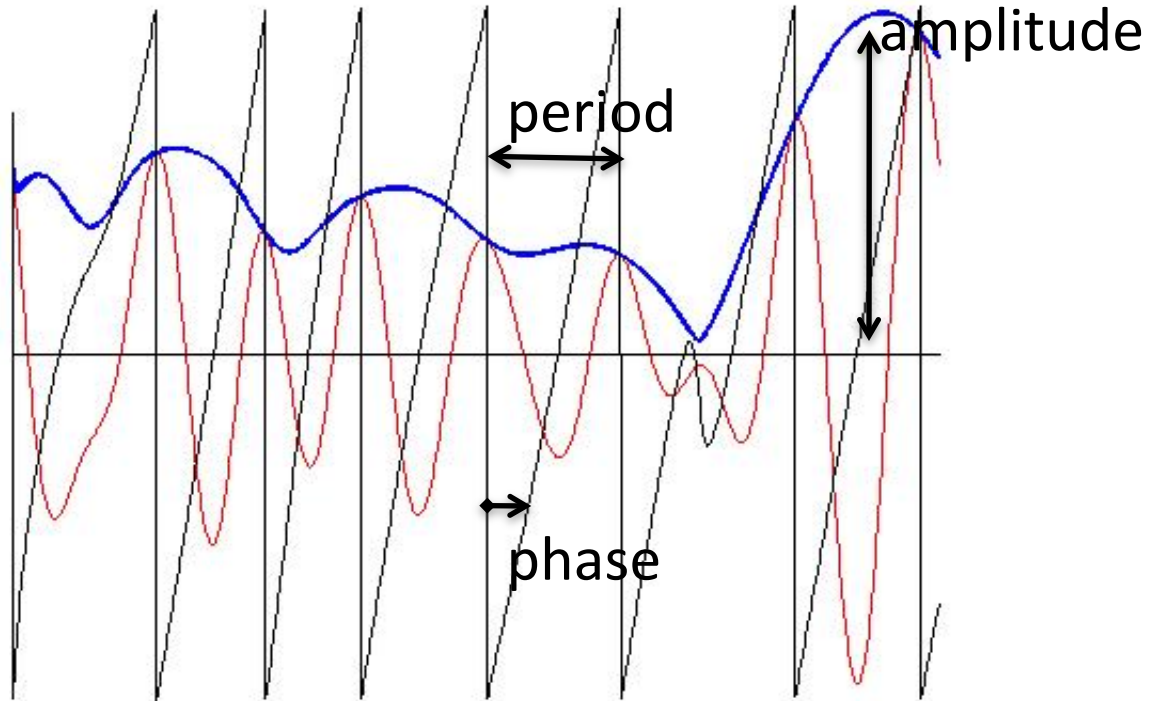
Spectral analysis: going from time to frequency domain

Issues with finite and discrete sampling

Spectral leakage and (multi-)tapering

Time-frequency analysis

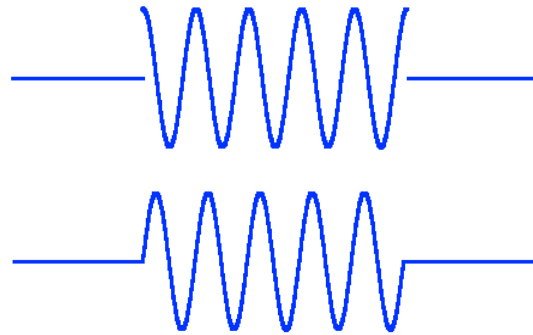
A background note on oscillations



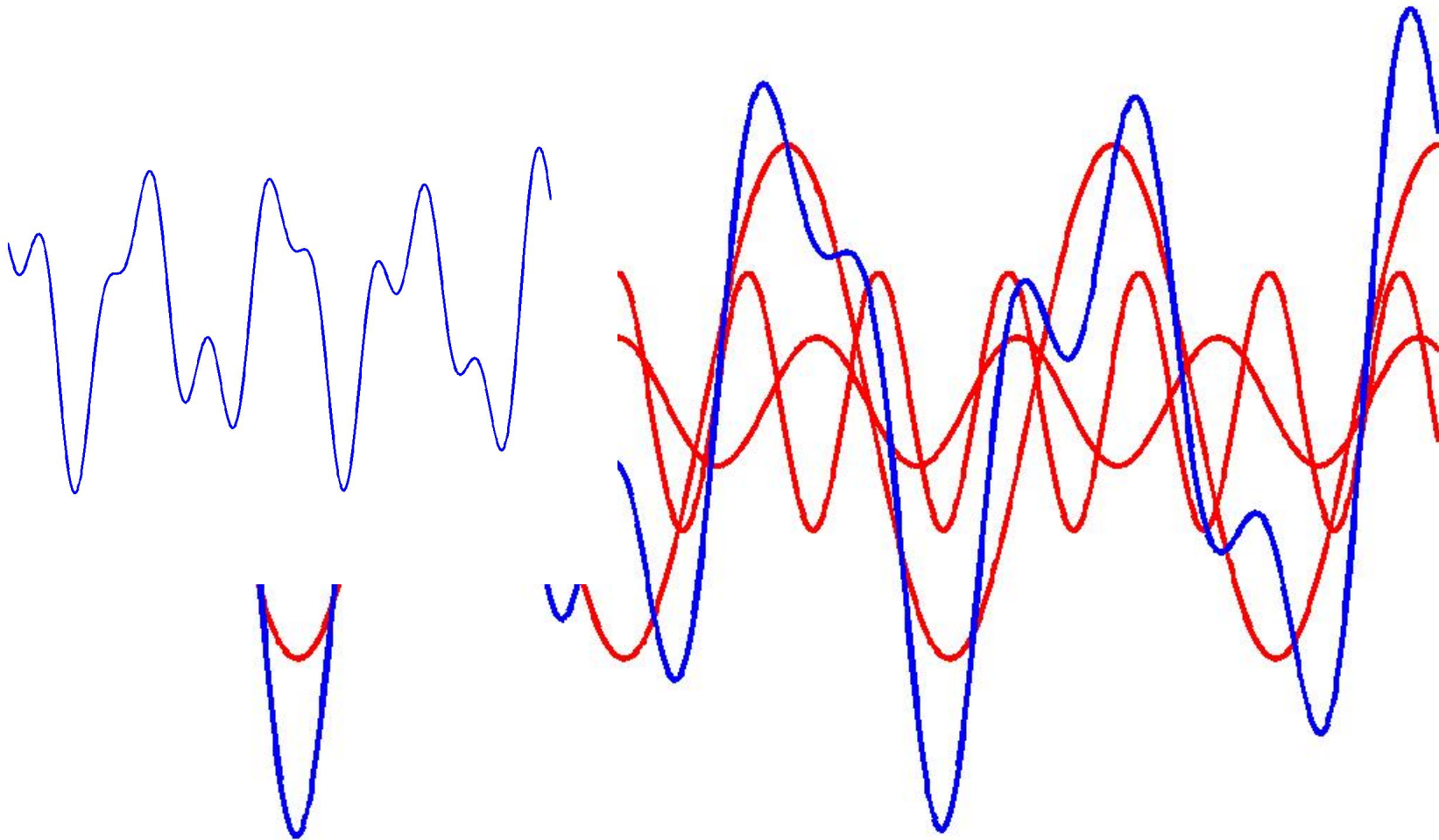
Spectral analysis

Deconstructing a time domain signal into its constituent oscillatory components, a.k.a. Fourier analysis

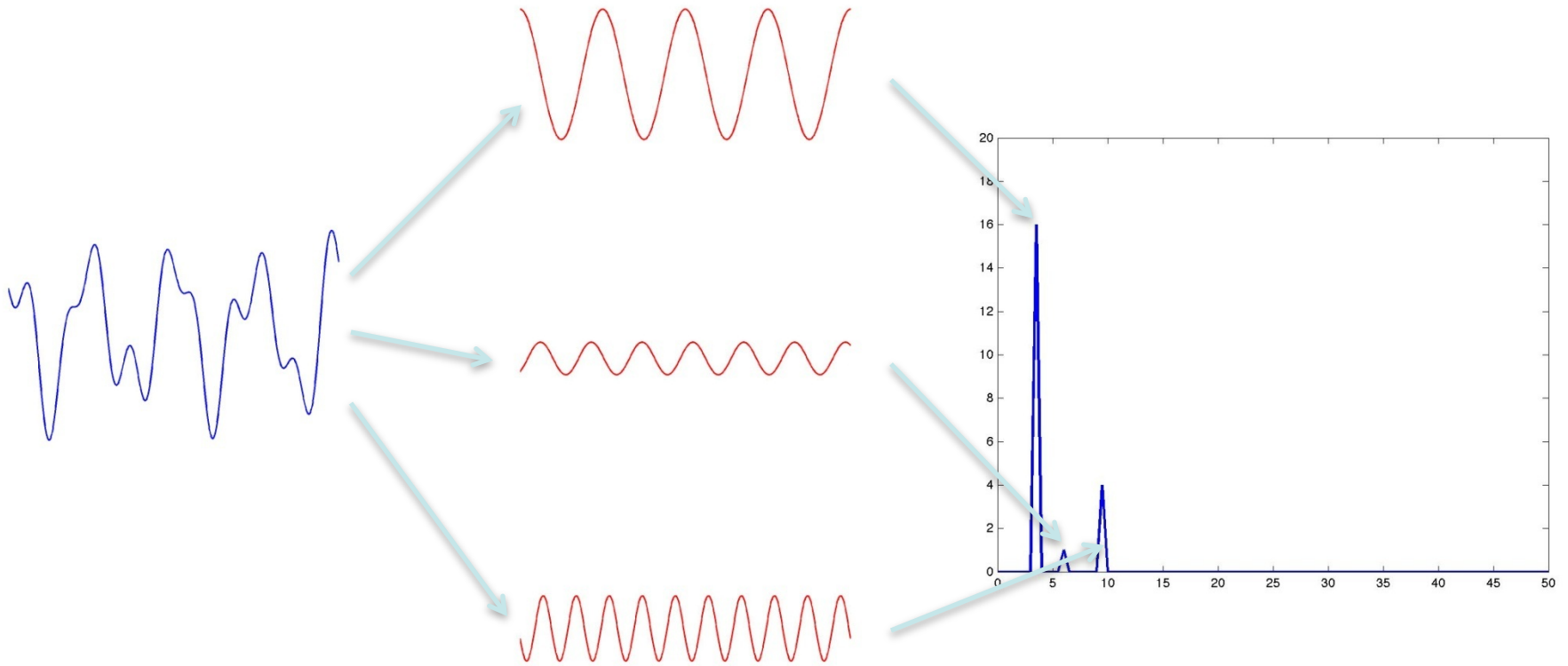
Using simple oscillatory functions: cosines and sines



Spectral decomposition: the principle



Spectral decomposition: the power spectrum



Spectral analysis

Deconstructing a time domain signal into its constituent oscillatory components, a.k.a. Fourier analysis

Using simple oscillatory functions: cosines and sines

Express signal as function of frequency, rather than time

Concept: linear regression using oscillatory basis functions

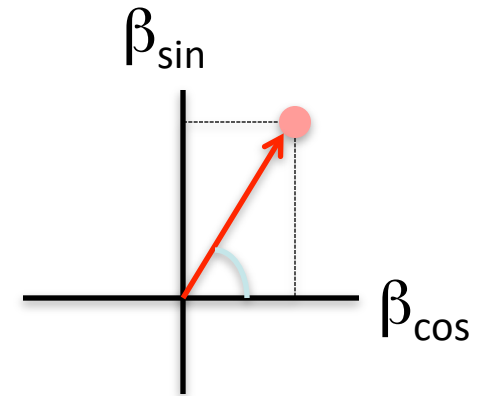
Spectral analysis ~ GLM

$$\mathbf{Y} = \beta \times \mathbf{X}$$

\mathbf{X} set of basis functions

β_i contribution of basis function i to the data.

β for cosine and sine components for a given frequency
map onto amplitude and phase estimate.



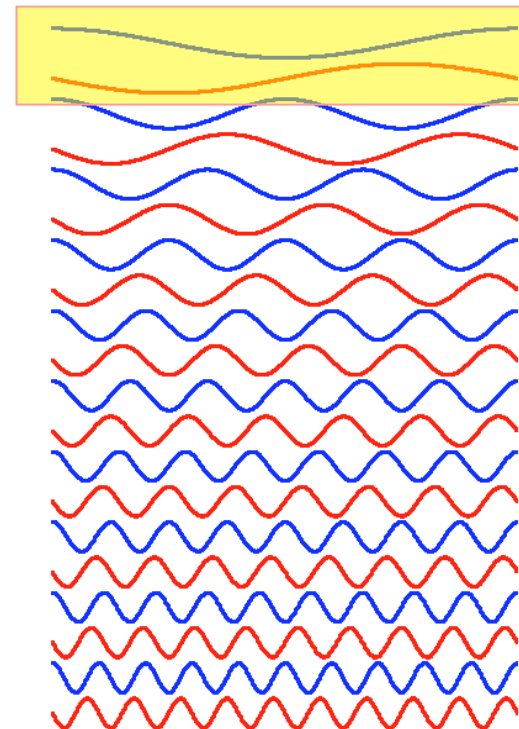
Restriction: basis functions should be ‘orthogonal’

Consequence 1: frequencies not arbitrary

-> integer amount of cycles should fit into N points.

Consequence 2: N -point signal

-> N basis functions



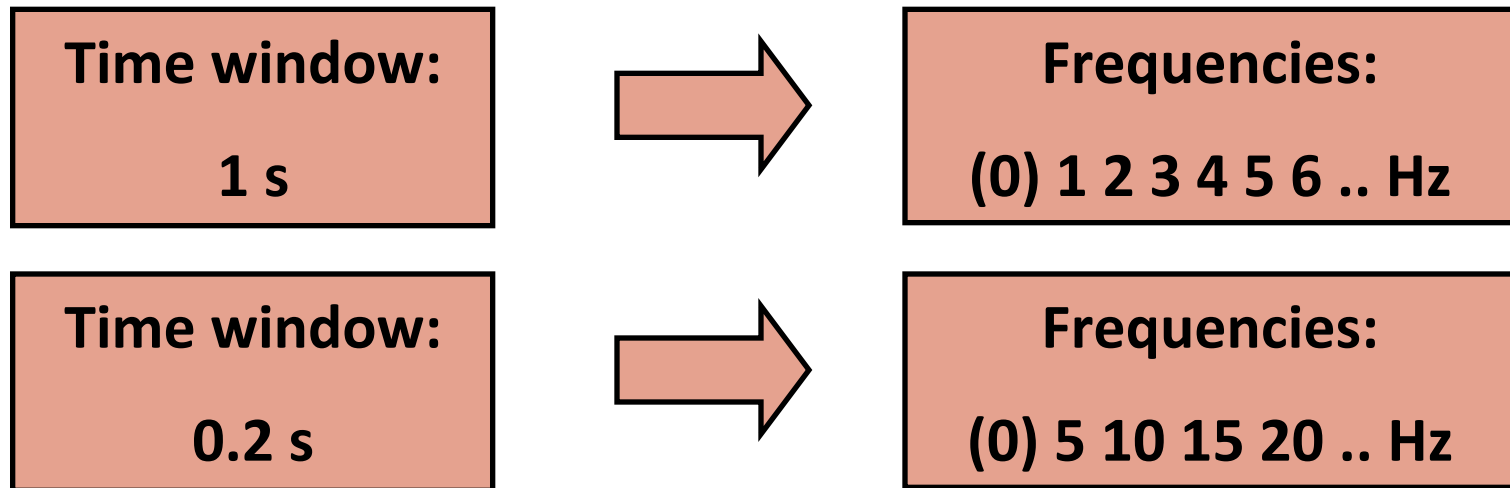
Time-frequency relation

Consequence 1: frequencies not arbitrary

-> integer amount of cycles should fit into N samples of Δt each.

The frequency resolution is determined by the total length of the data segments (T)

Rayleigh frequency = $1/T = \Delta f =$ frequency resolution



Time-frequency relation

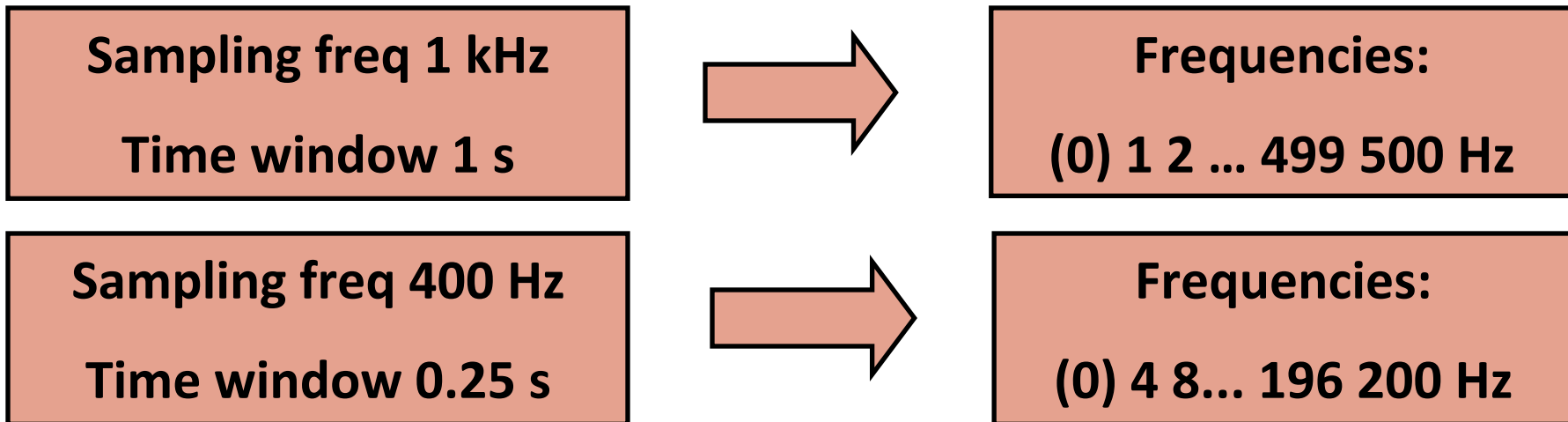
Consequence 2: N-point signal

-> N basis functions

N basis functions -> N/2 frequencies

The highest frequency that can be resolved depends
on the sampling frequency F

Nyquist frequency = $F/2$



Spectral analysis

Deconstructing a time domain signal into its constituent oscillatory components, a.k.a. Fourier analysis

Using simple oscillatory functions: cosines and sines

Express signal as function of frequency, rather than time

Concept: linear regression using oscillatory basis functions

Each oscillatory component has an amplitude and phase

Discrete and finite sampling constrains the frequency axis

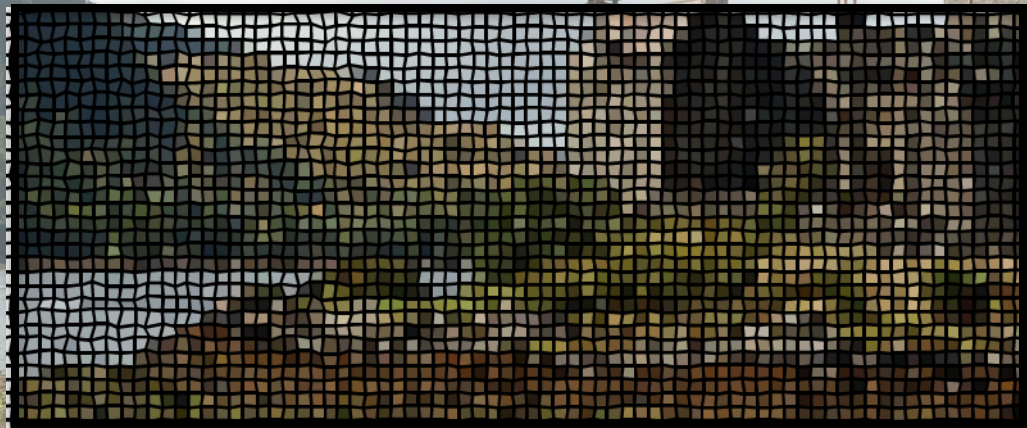
Goal and challenges

- Estimate the true oscillations from the observed data
- Limited time available for Fourier transform
- You are looking at the activity through a time restricted window



Goal and challenges

- Estimate the true oscillations from the observed data
- Limited time available for Fourier transform
- You are looking at the activity through a time restricted window

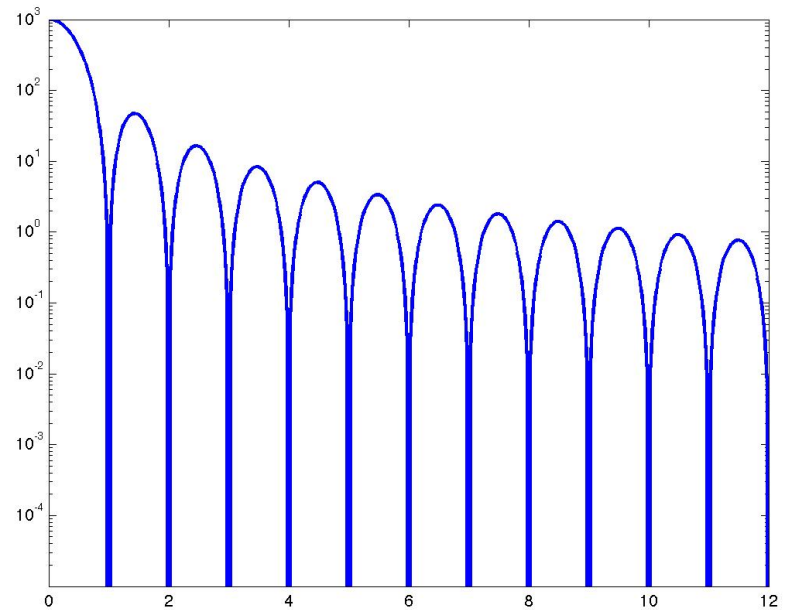
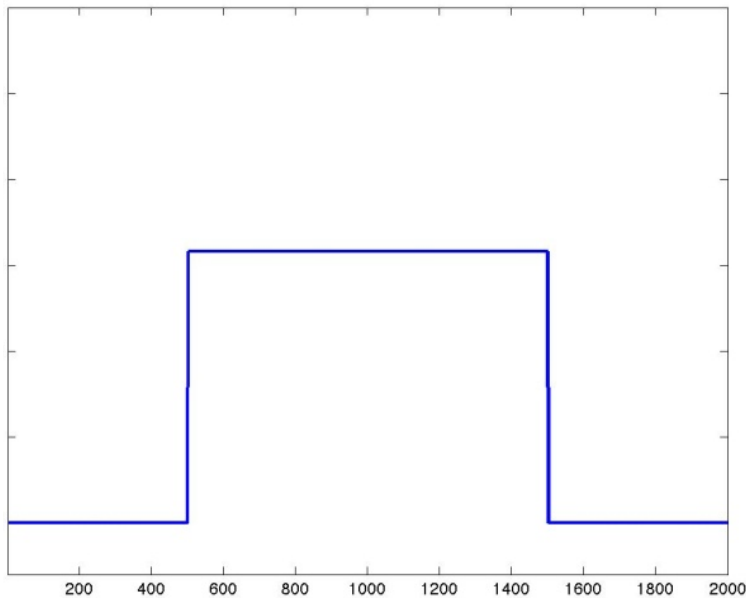


- This implicitly means that the data are ‘tapered’ with a boxcar
- Furthermore, data are discretely sampled

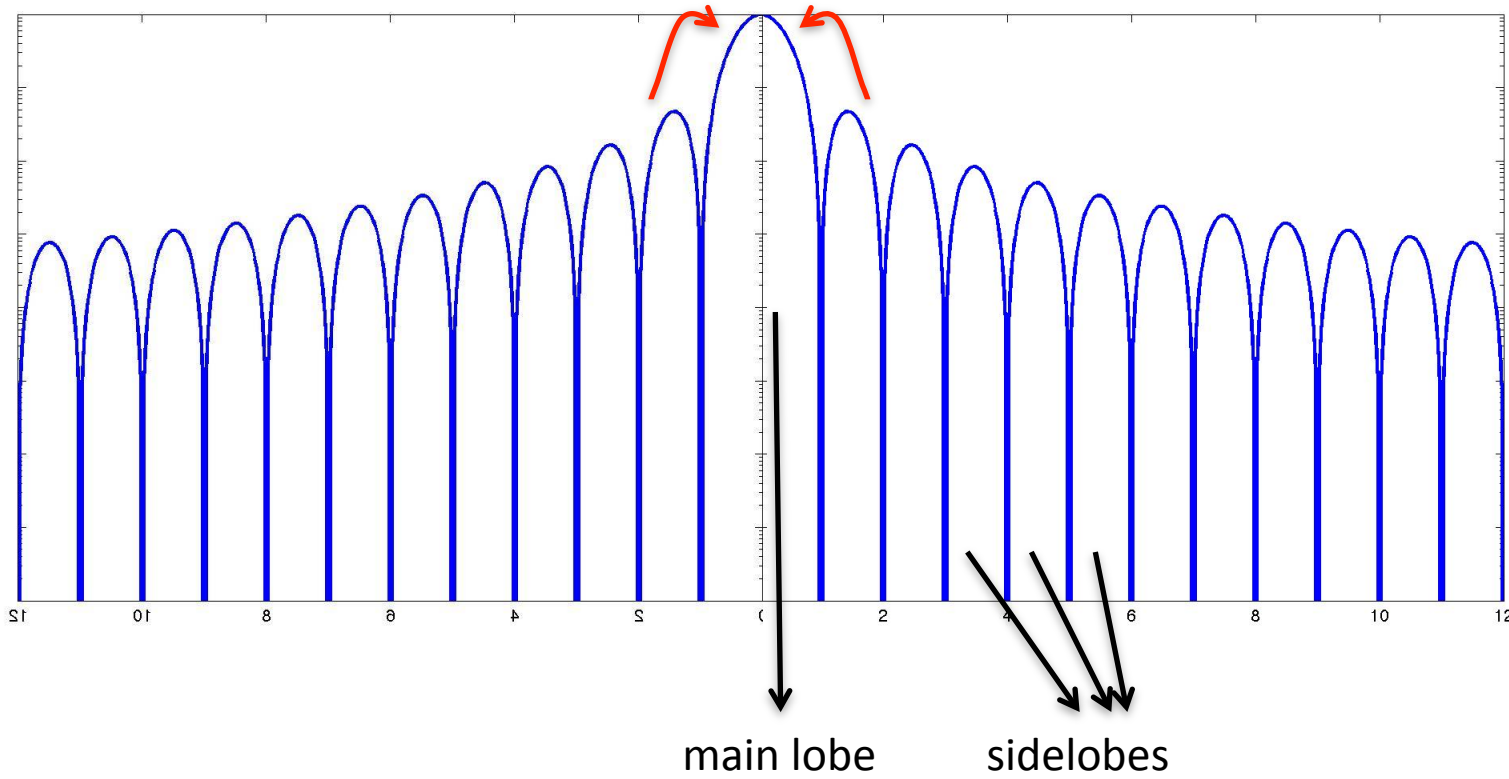


Spectral leakage and tapering

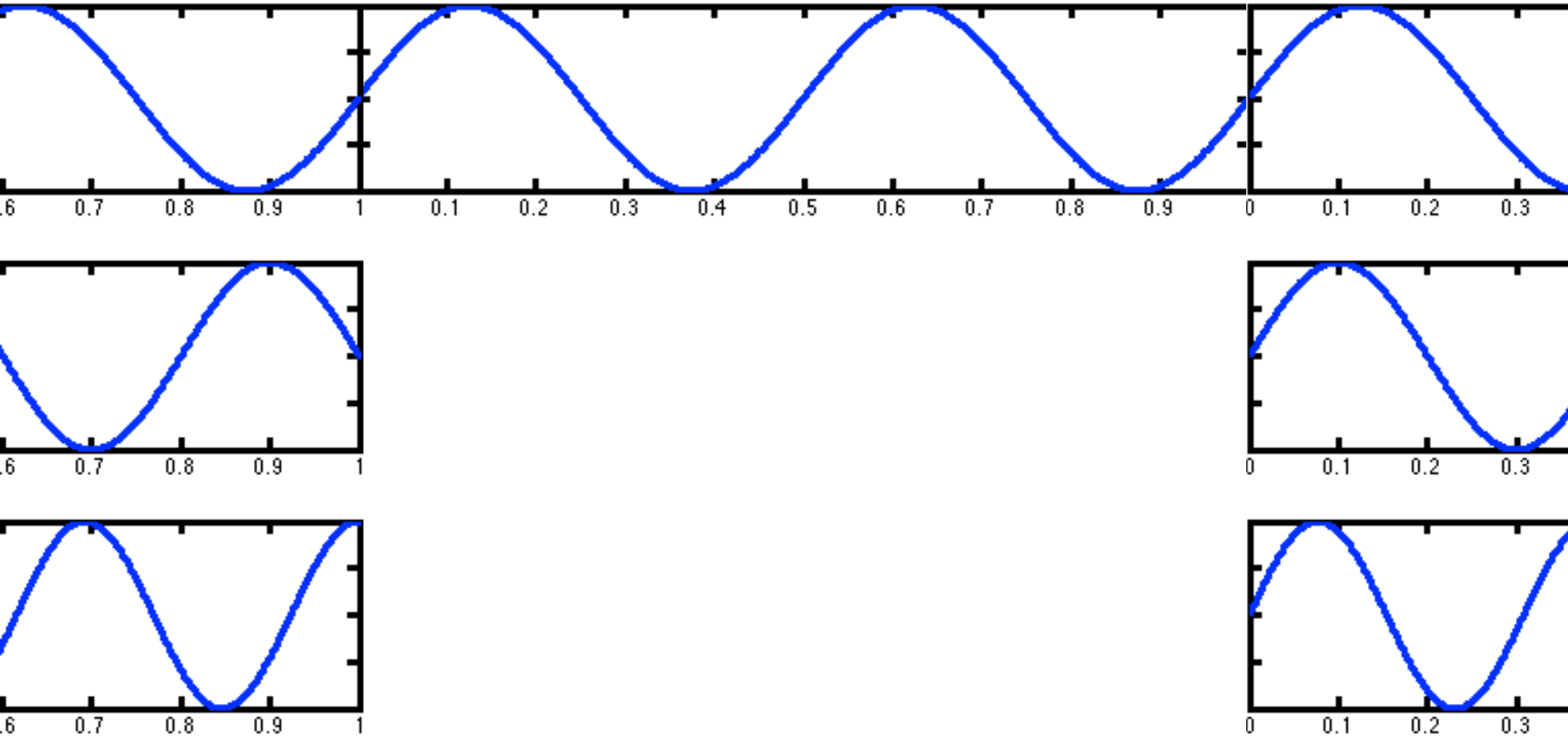
- True oscillations in data at frequencies **not sampled** with Fourier transform **spread their energy** to the sampled frequencies
- Not tapering is equal to applying a “boxcar” taper
- Each type of taper has a specific leakage profile



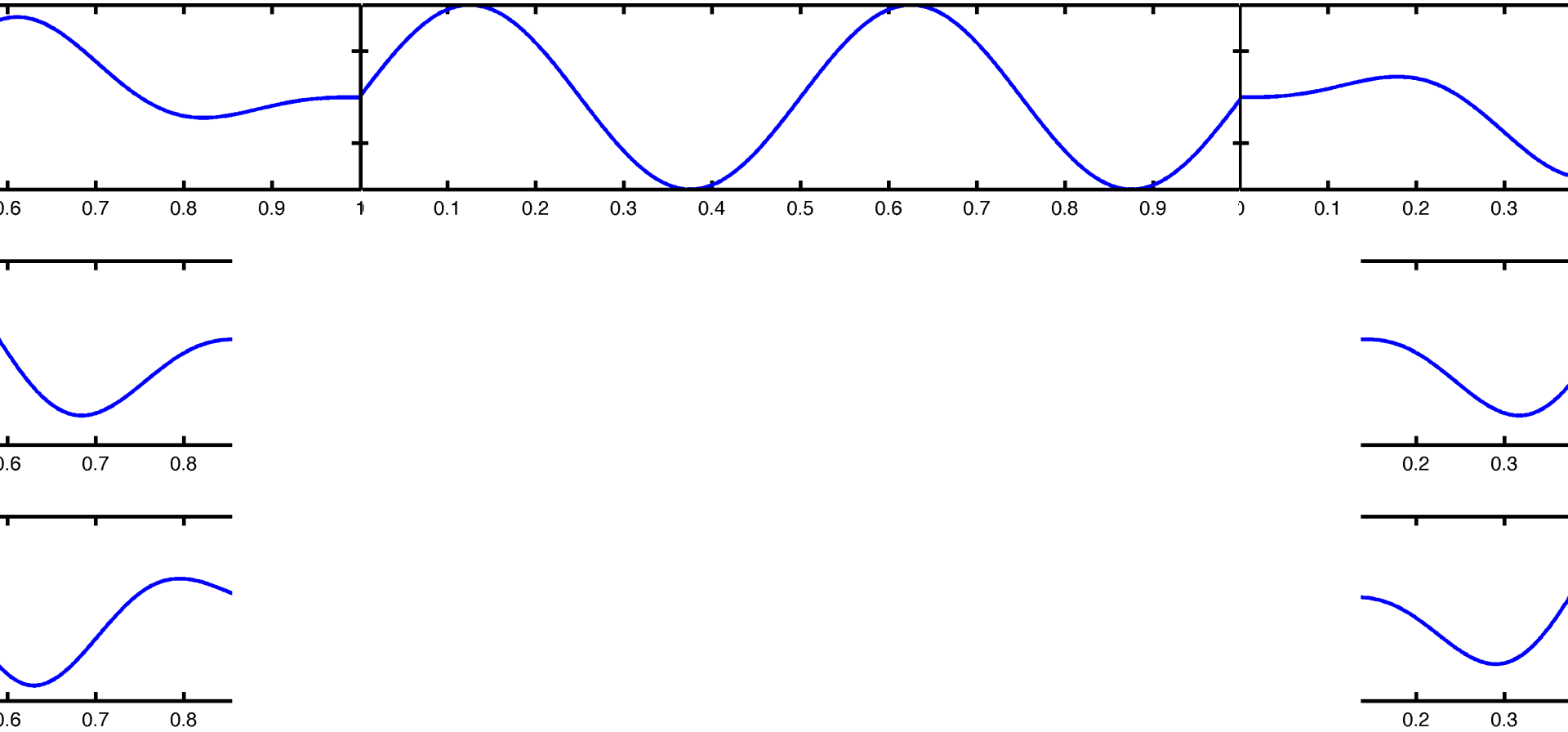
Spectral leakage



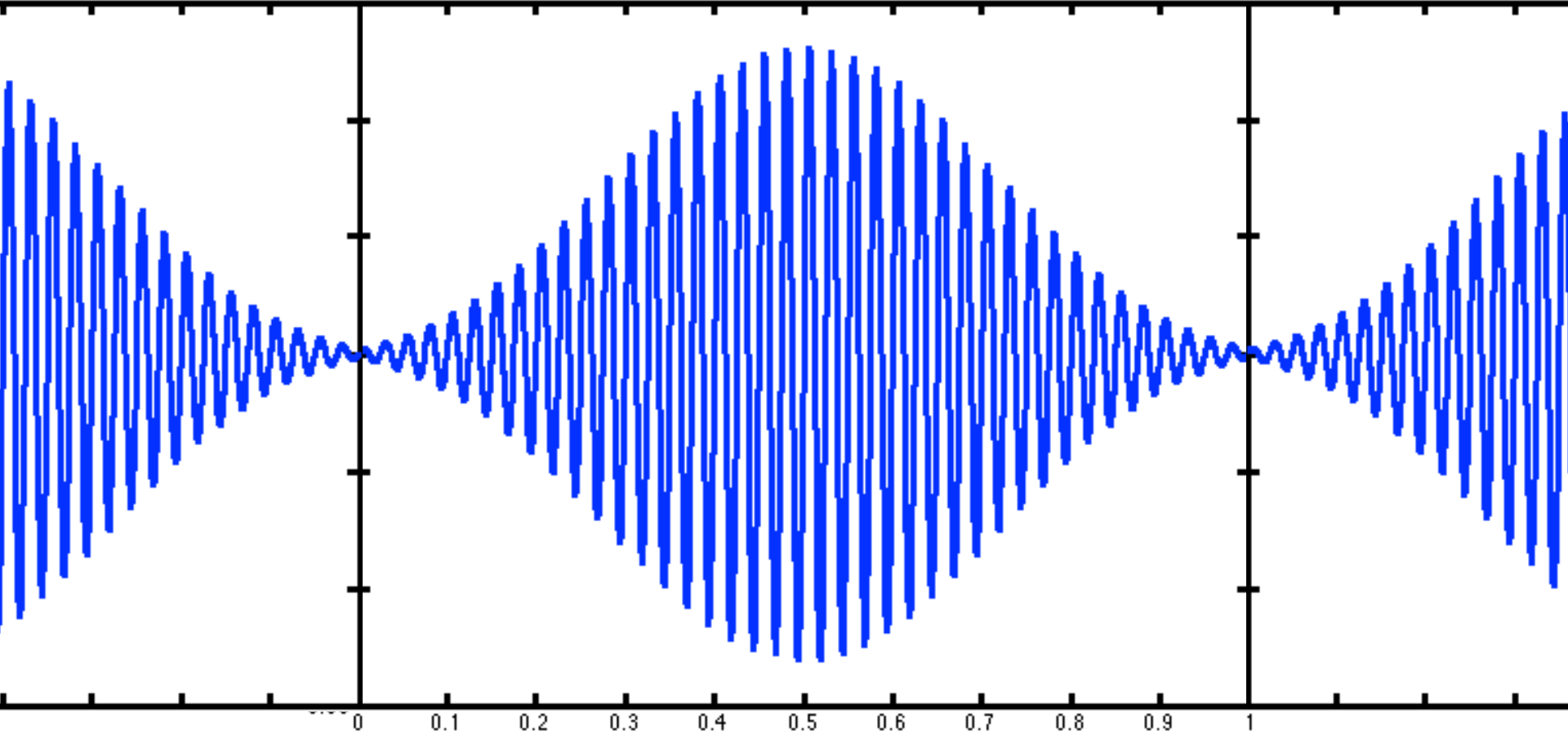
Tapering in spectral analysis



Tapering in spectral analysis

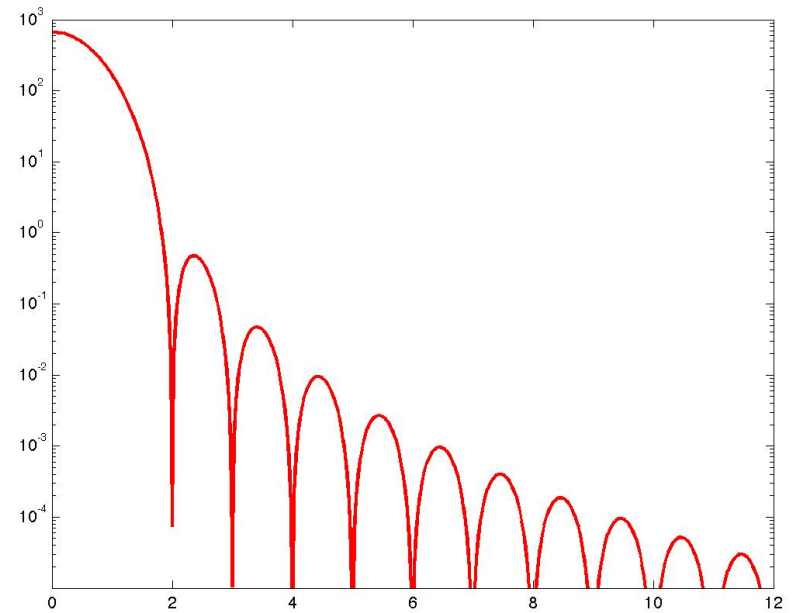
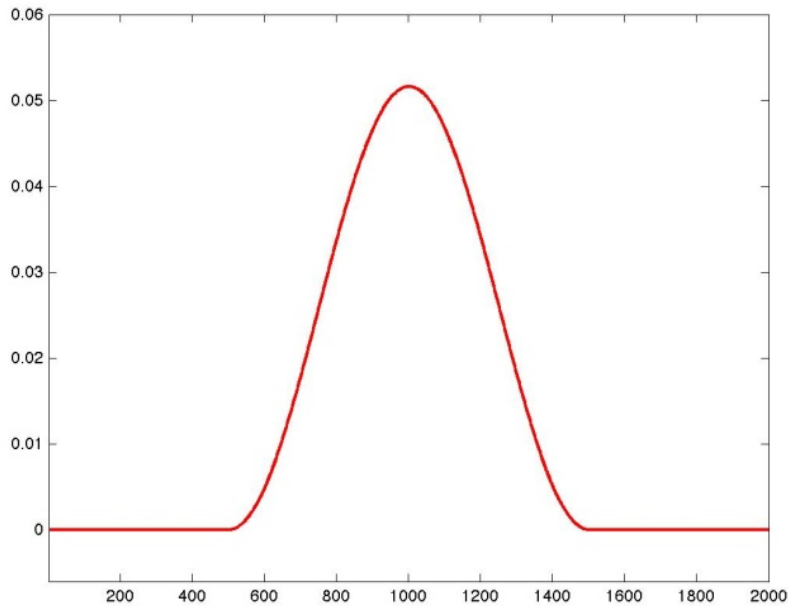


Tapering in spectral analysis



Spectral leakage and tapering

- True oscillations in data at frequencies not sampled with Fourier transform spread their energy to the sampled frequencies
- Not tapering is equal to applying a boxcar taper
- Each type of taper has a specific leakage profile



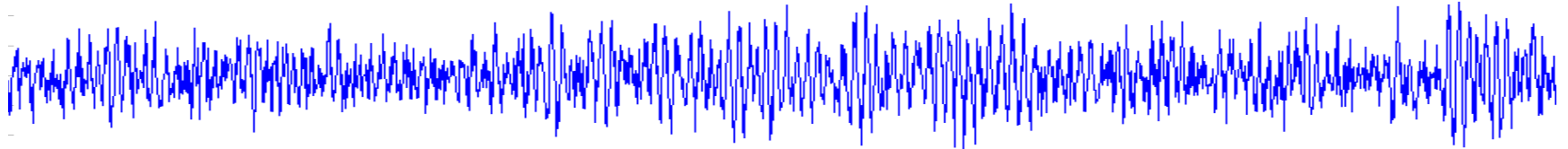
Multitapers

Make use of more than one taper and combine their properties

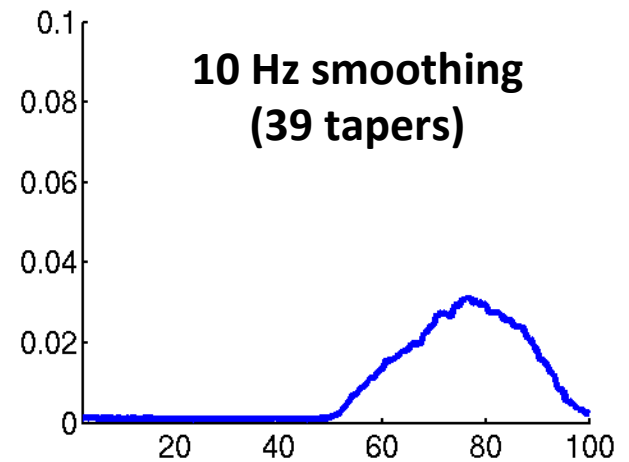
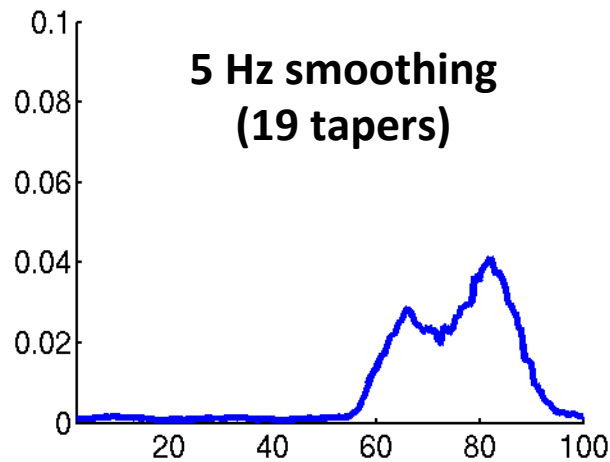
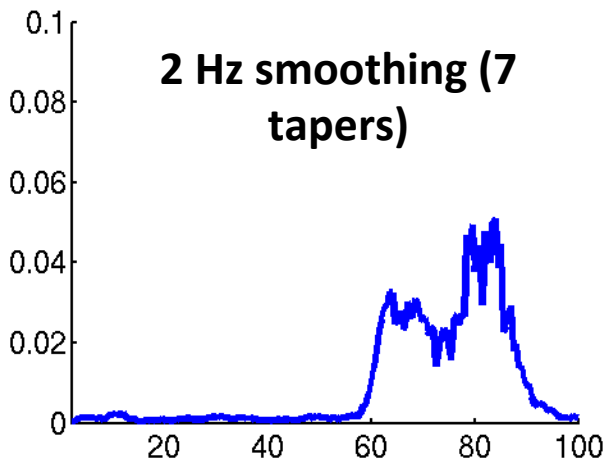
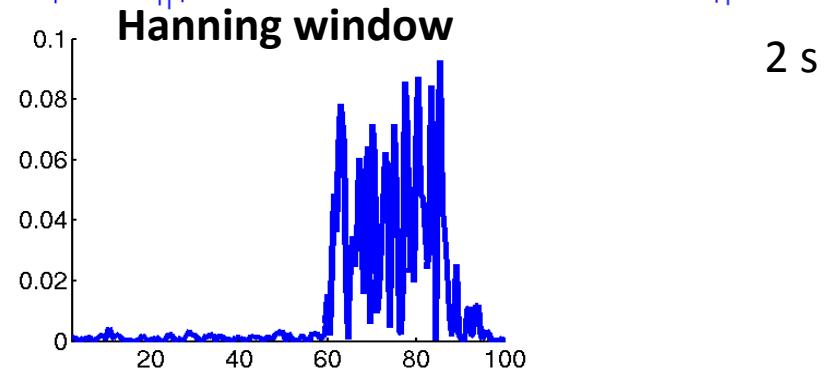
Used for smoothing in the frequency domain

Instead of “smoothing” one can also say “controlled leakage”

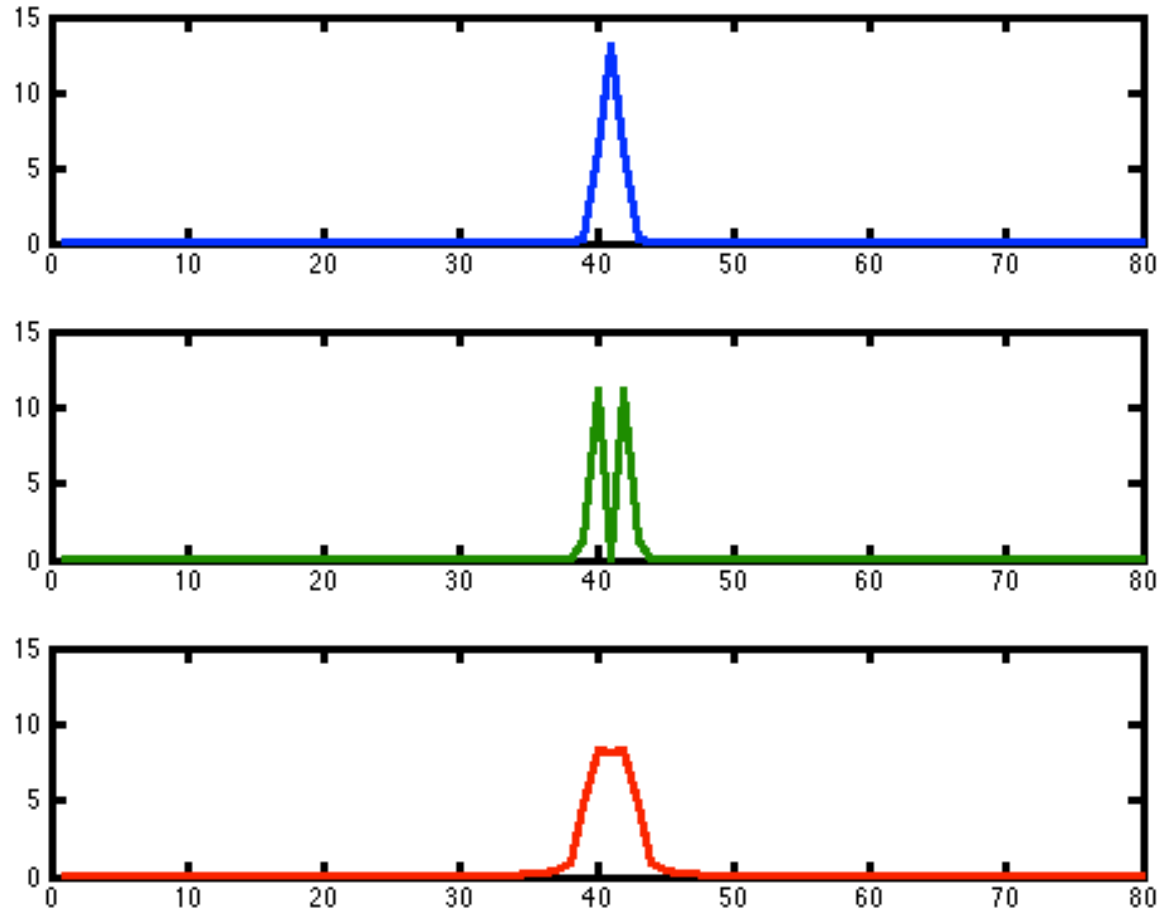
Multitapered spectral analysis



**broadband activity between
60-90 Hz**



Multitapered spectral analysis



Multitapers

Multitapers are useful for reliable estimation of high frequency components

Low frequency components are better estimated using a single (Hanning) taper

```
%estimate low frequencies
```

```
cfg = [];  
cfg.method = 'mtmfft';  
cfg.foilim = [1 30];  
cfg.taper = 'hanning';  
.  
.  
.  
freq=ft_freqanalysis(cfg, data);
```

```
%estimate high frequencies
```

```
cfg = [];  
cfg.method = 'mtmfft';  
cfg.foilim = [30 120];  
cfg.taper = 'dpss';  
cfg.tapsmofrq = 8;  
.  
.  
freq=ft_freqanalysis(cfg, data);
```

Interim summary

Spectral analysis

Decompose signal into its constituent
oscillatory components

Focused on 'stationary' power

Tapers

Boxcar, Hanning, Gaussian

Multitapers

Control spectral leakage/smoothing

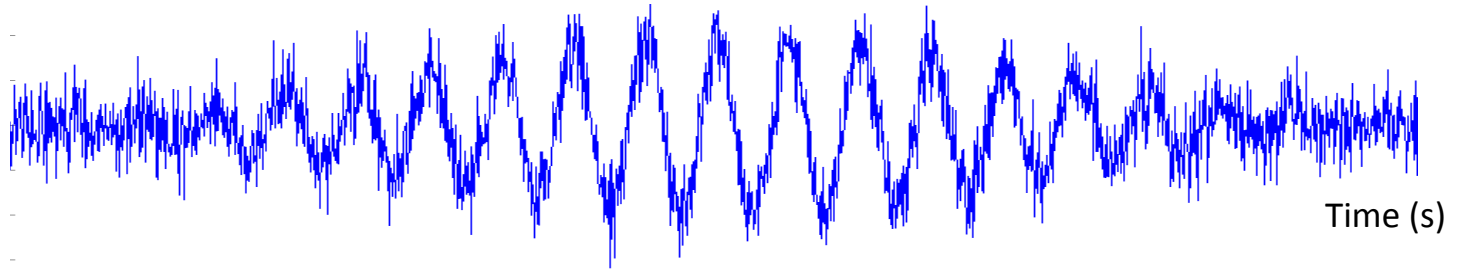
Time-frequency analysis

Typically, brain signals are not ‘stationary’

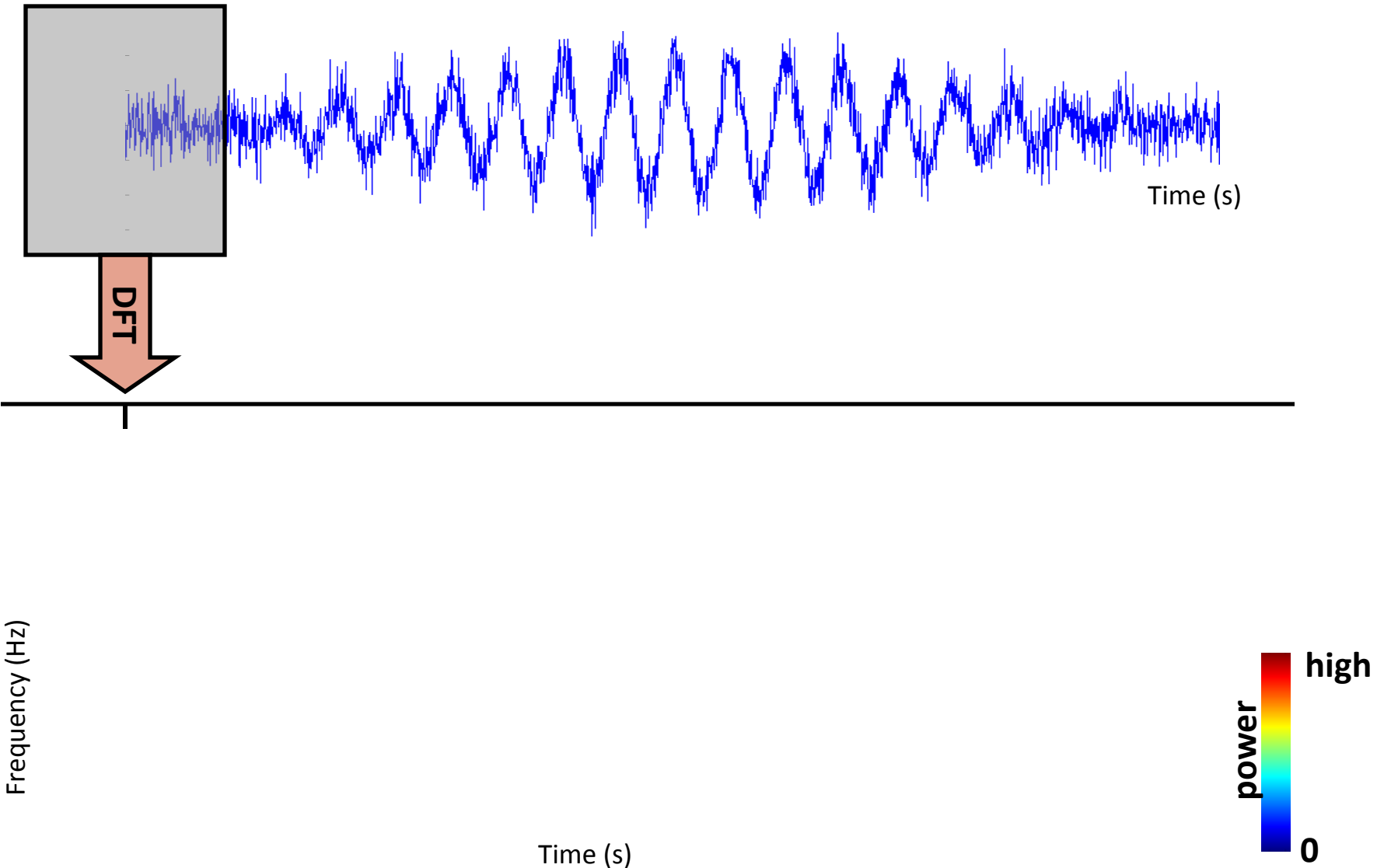
- Divide the measured signal in shorter time segments and apply Fourier analysis to each signal segment
- Everything we saw so far with respect to frequency resolution applies here as well

```
cfg = [];  
cfg.method = 'mtmconvol';  
.  
.  
.  
freq = ft_freqanalysis(cfg, data);
```

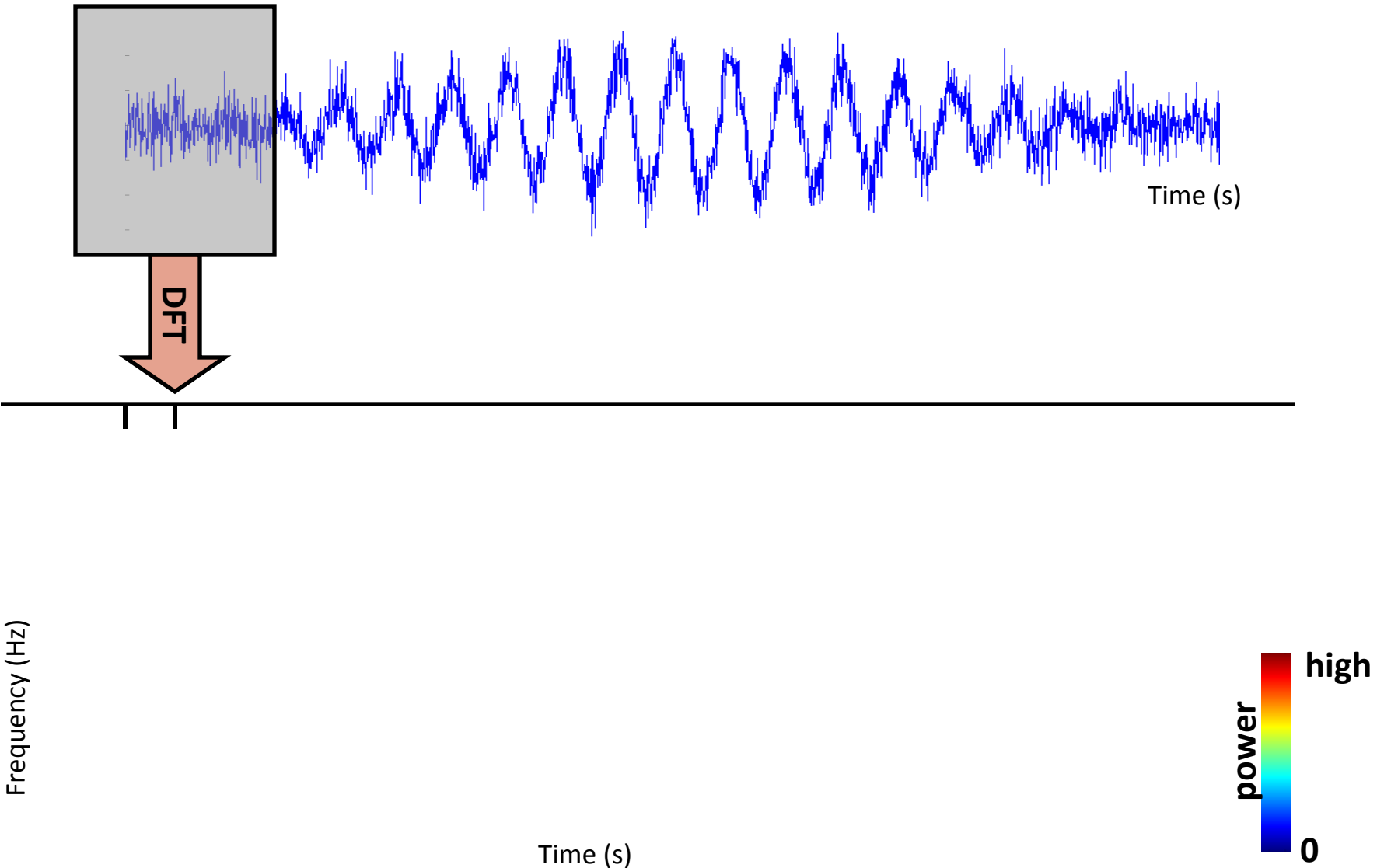
Time frequency analysis



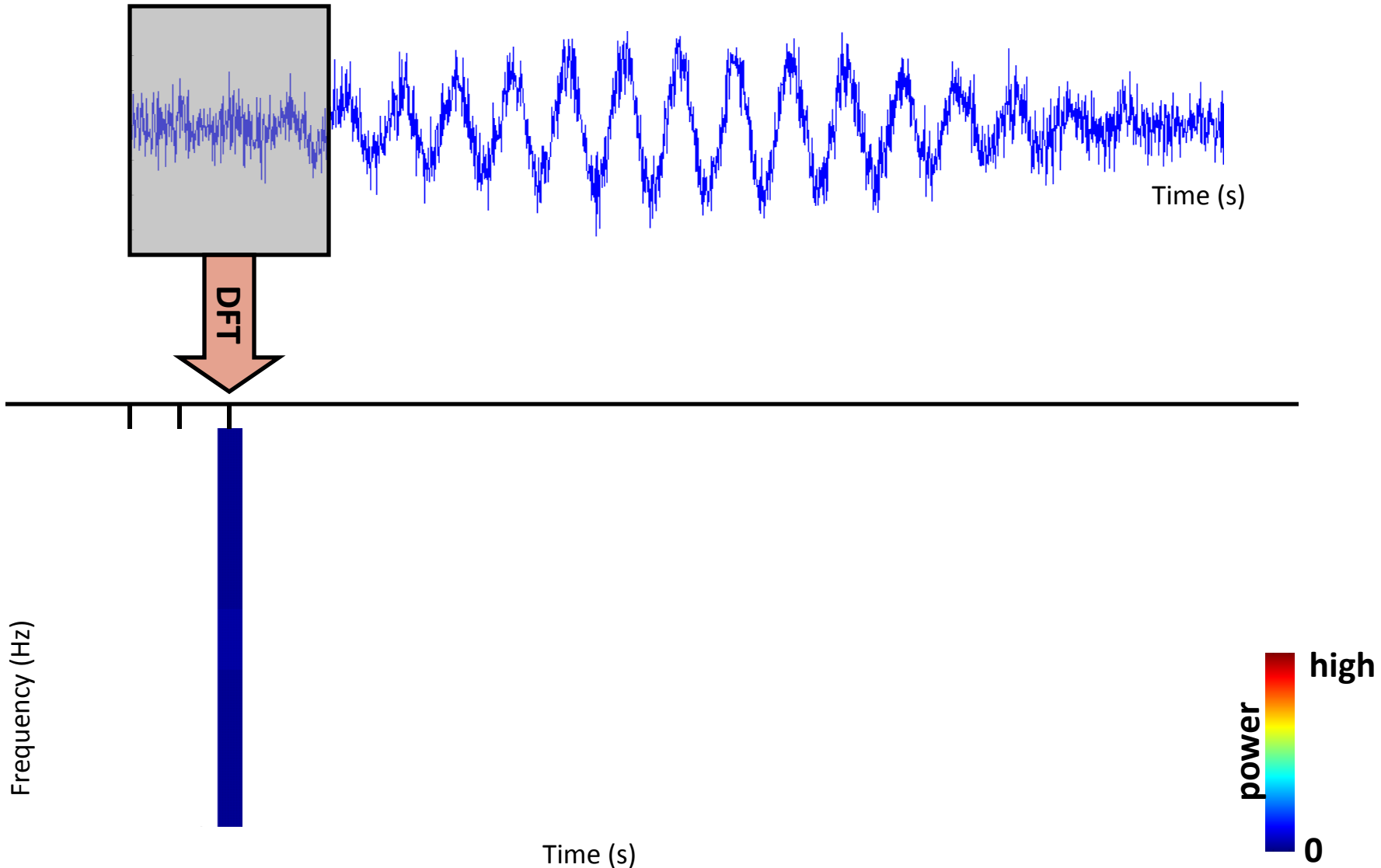
Time frequency analysis



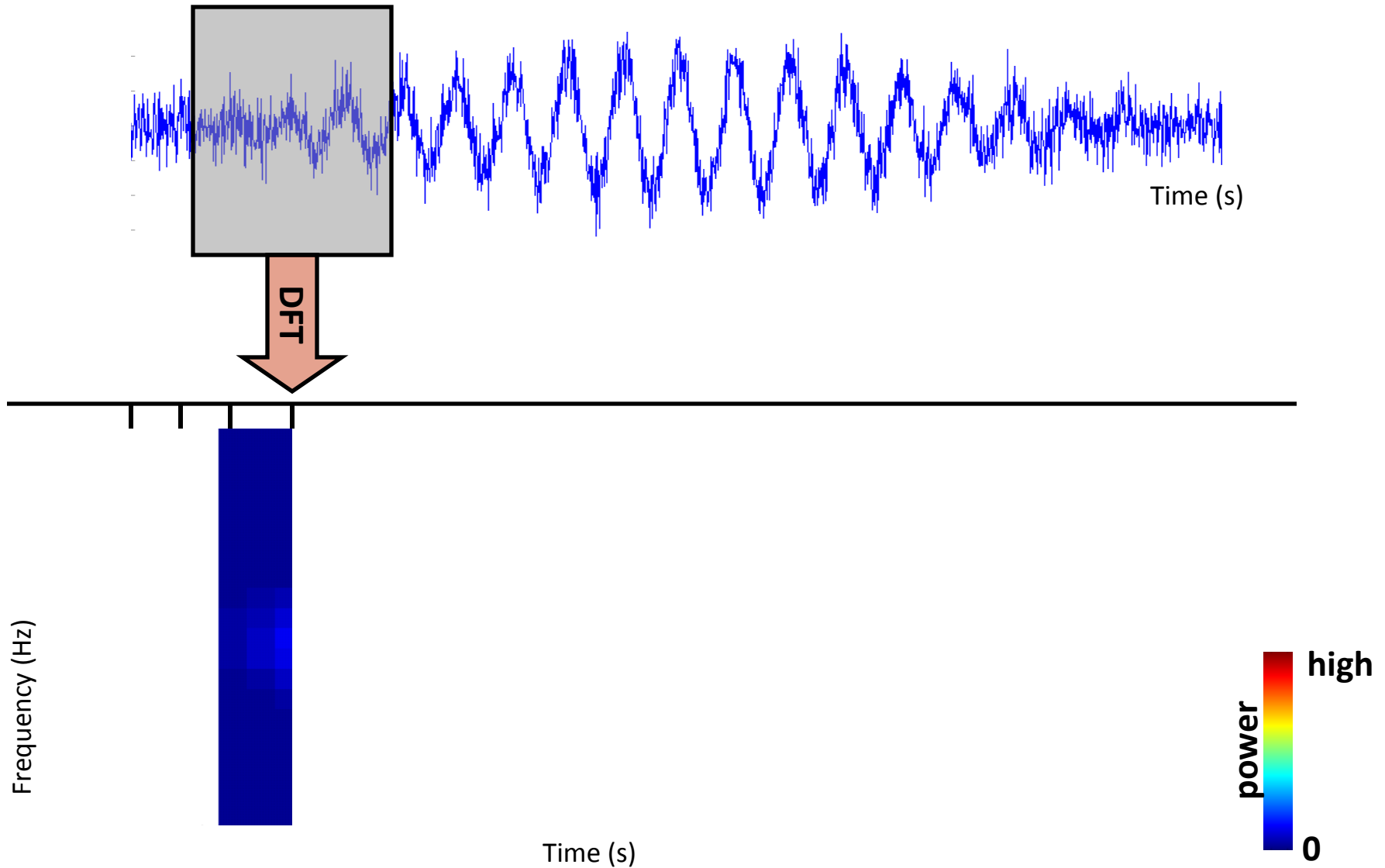
Time frequency analysis



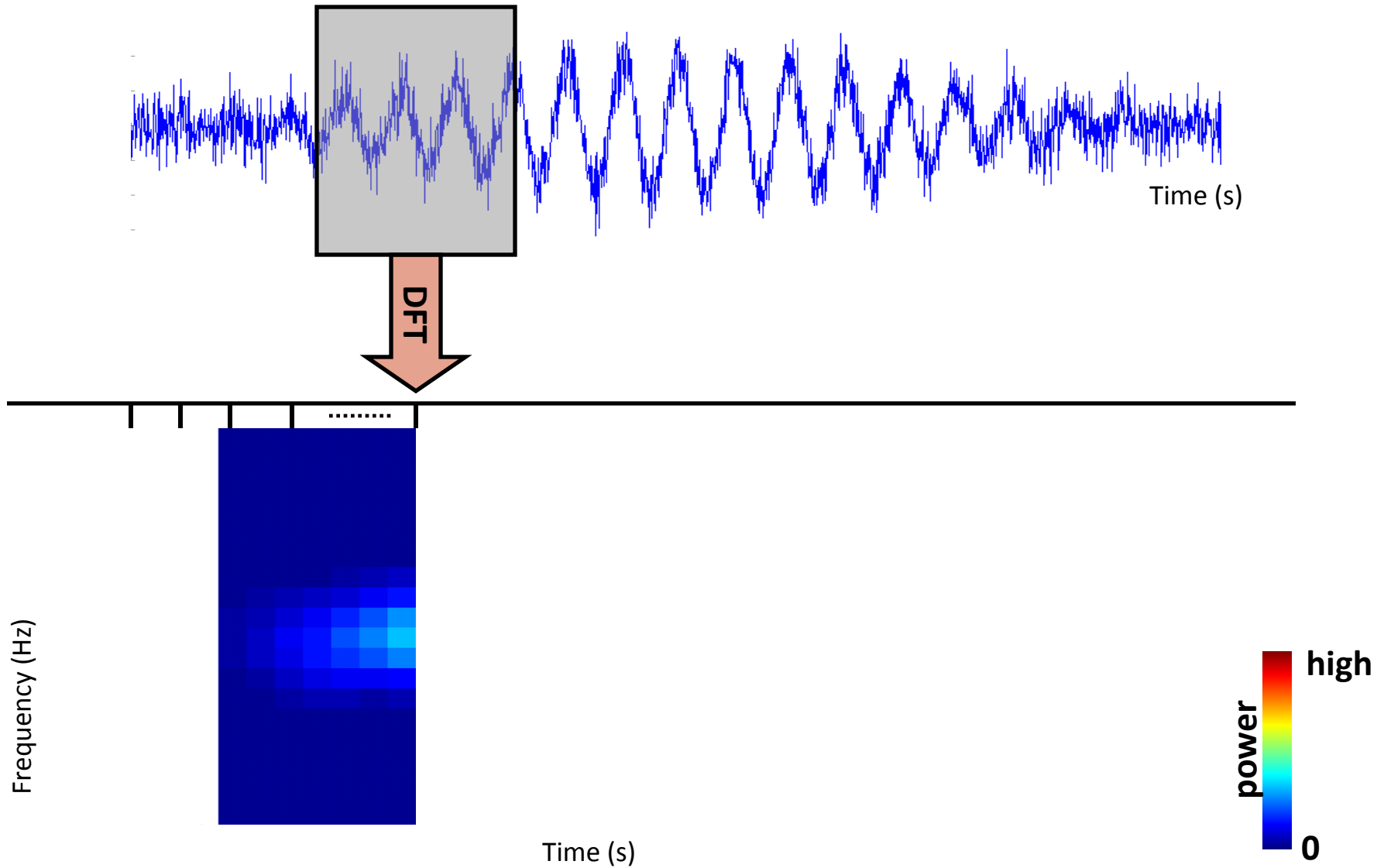
Time frequency analysis



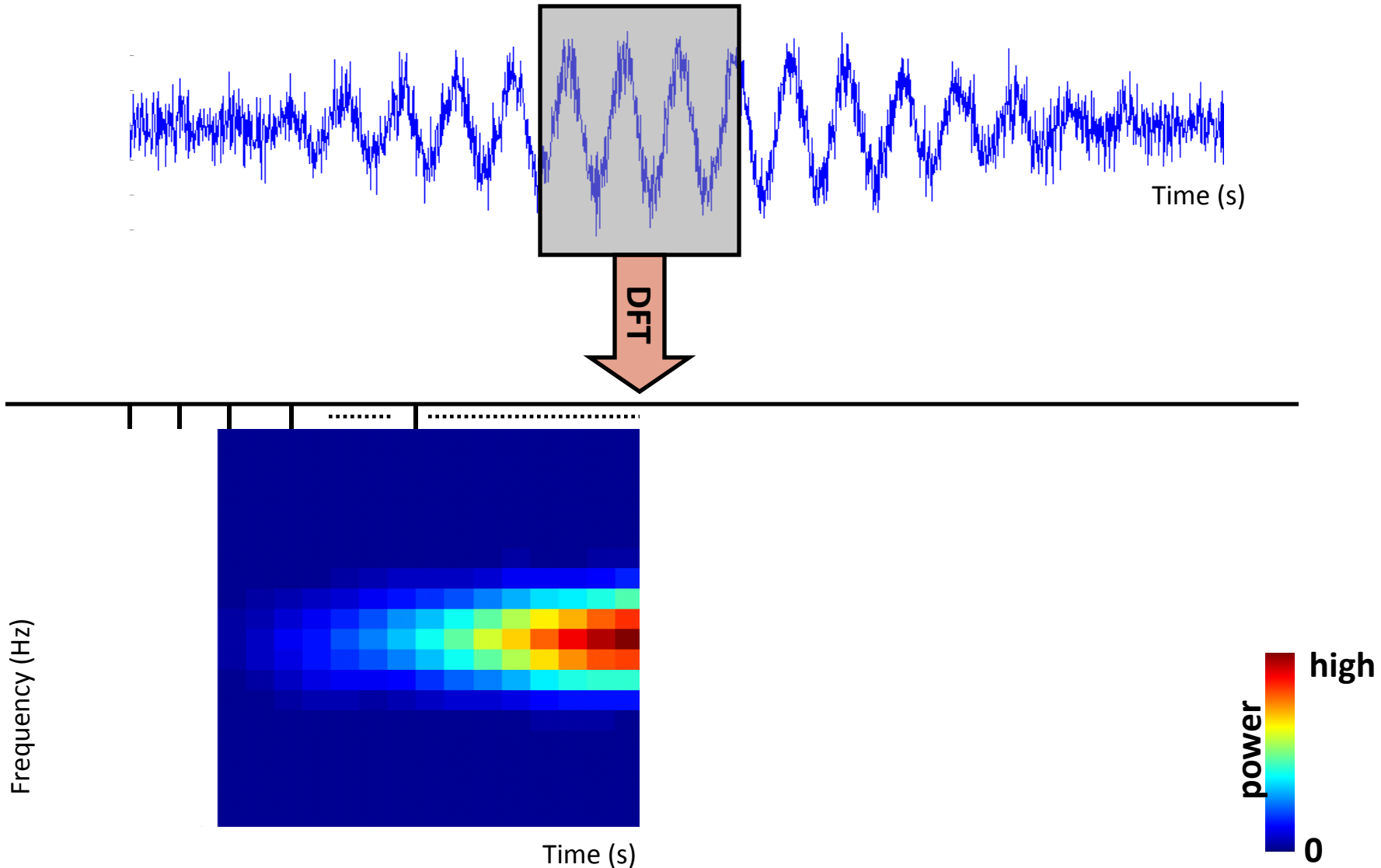
Time frequency analysis



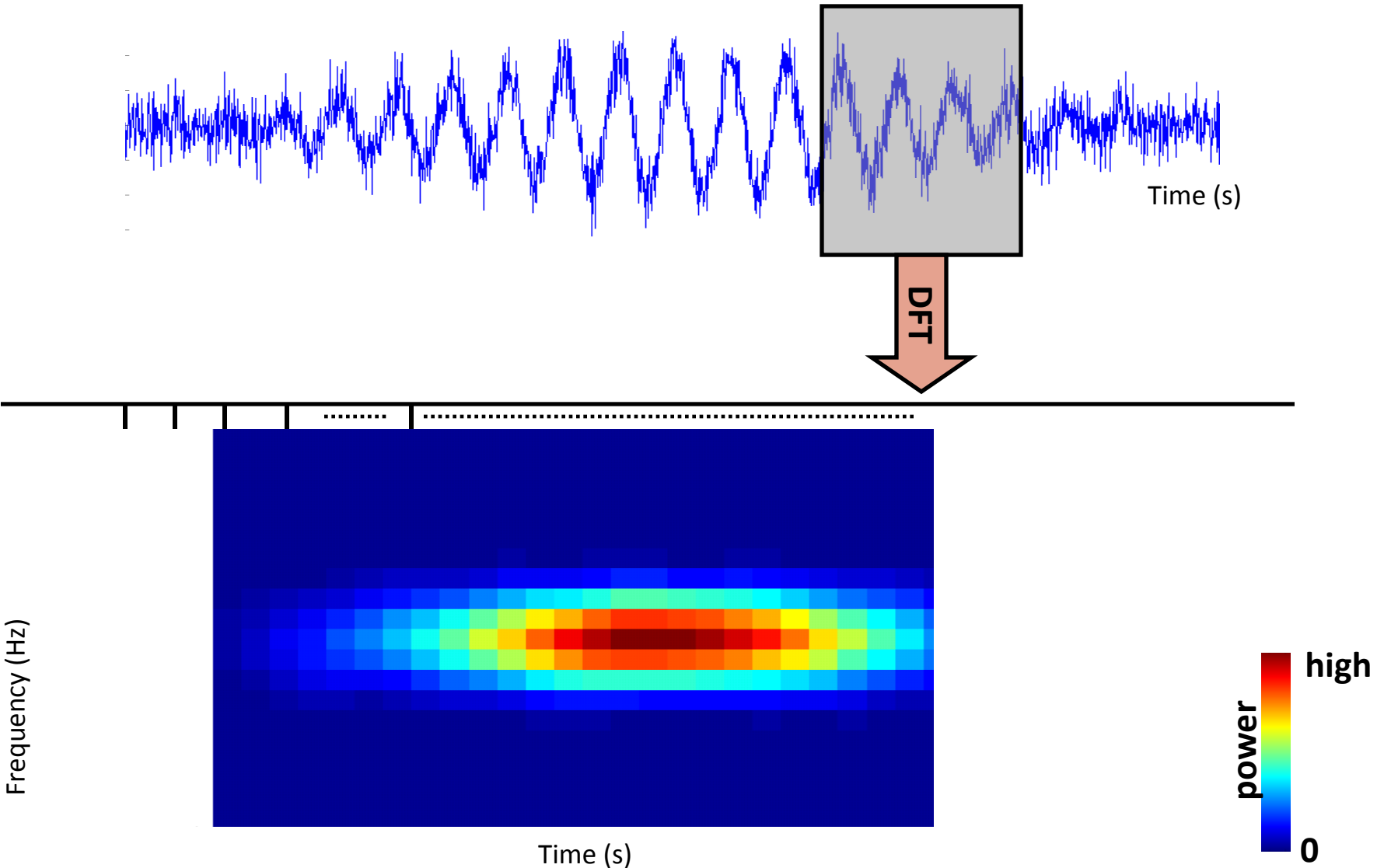
Time frequency analysis



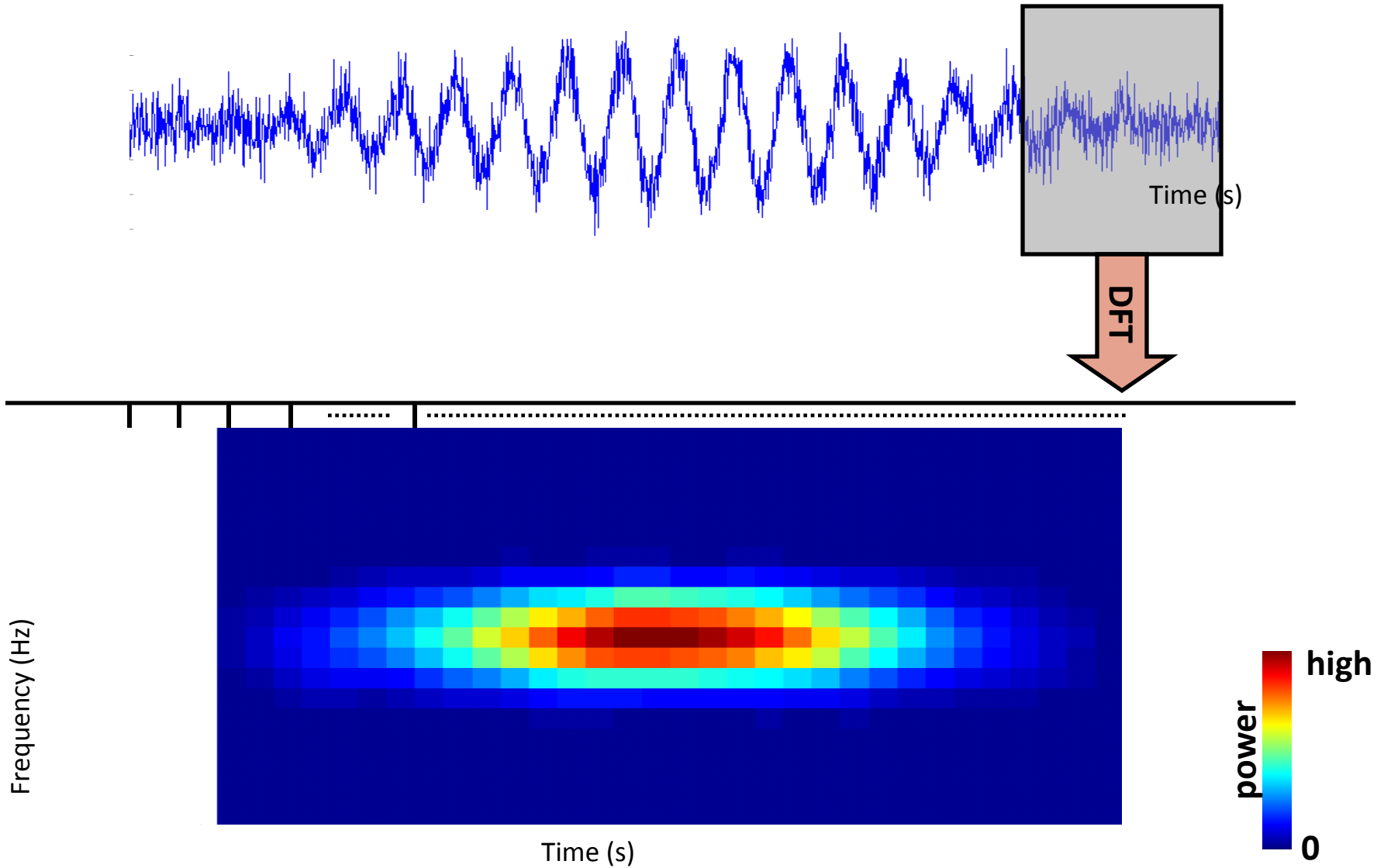
Time frequency analysis



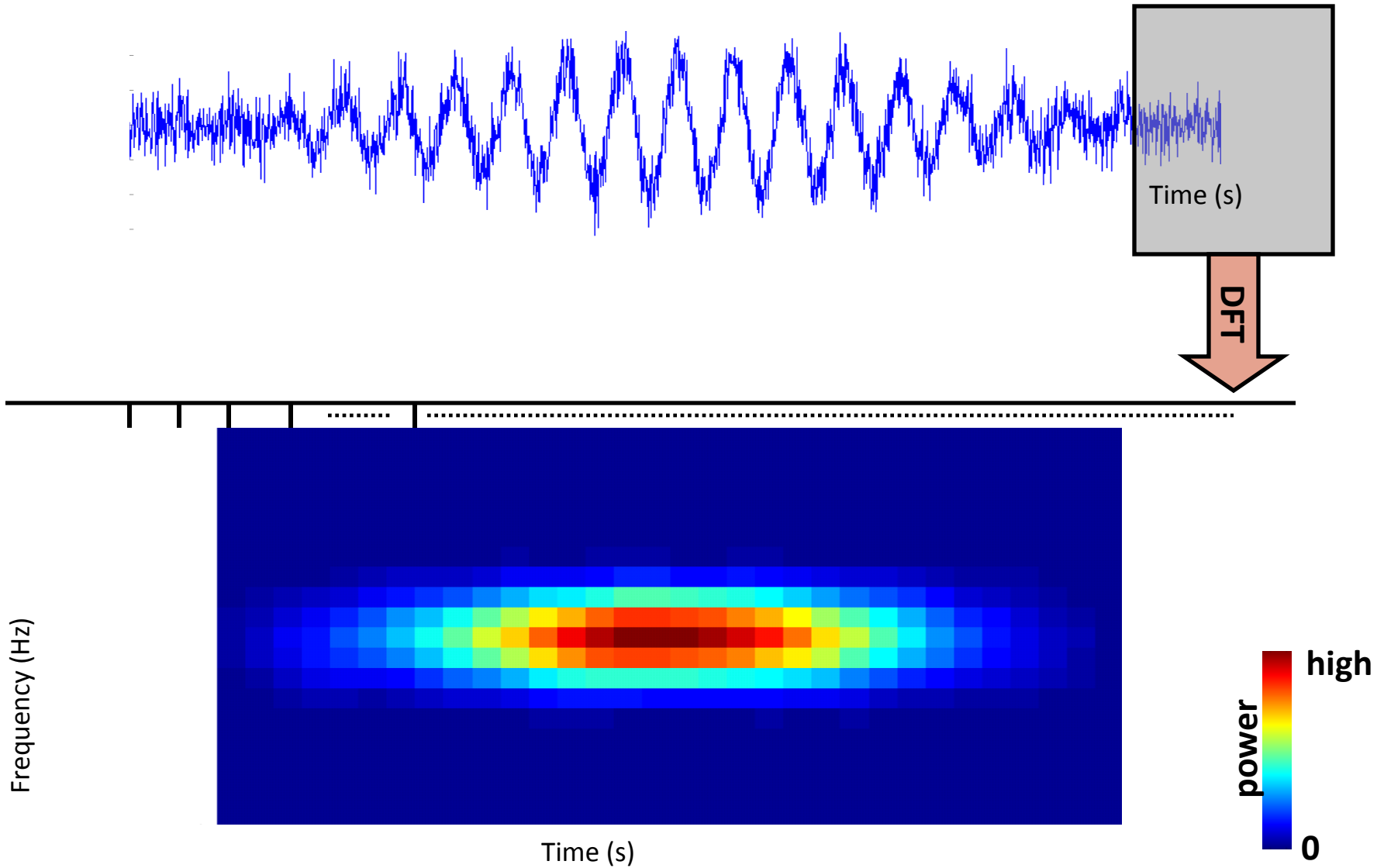
Time frequency analysis



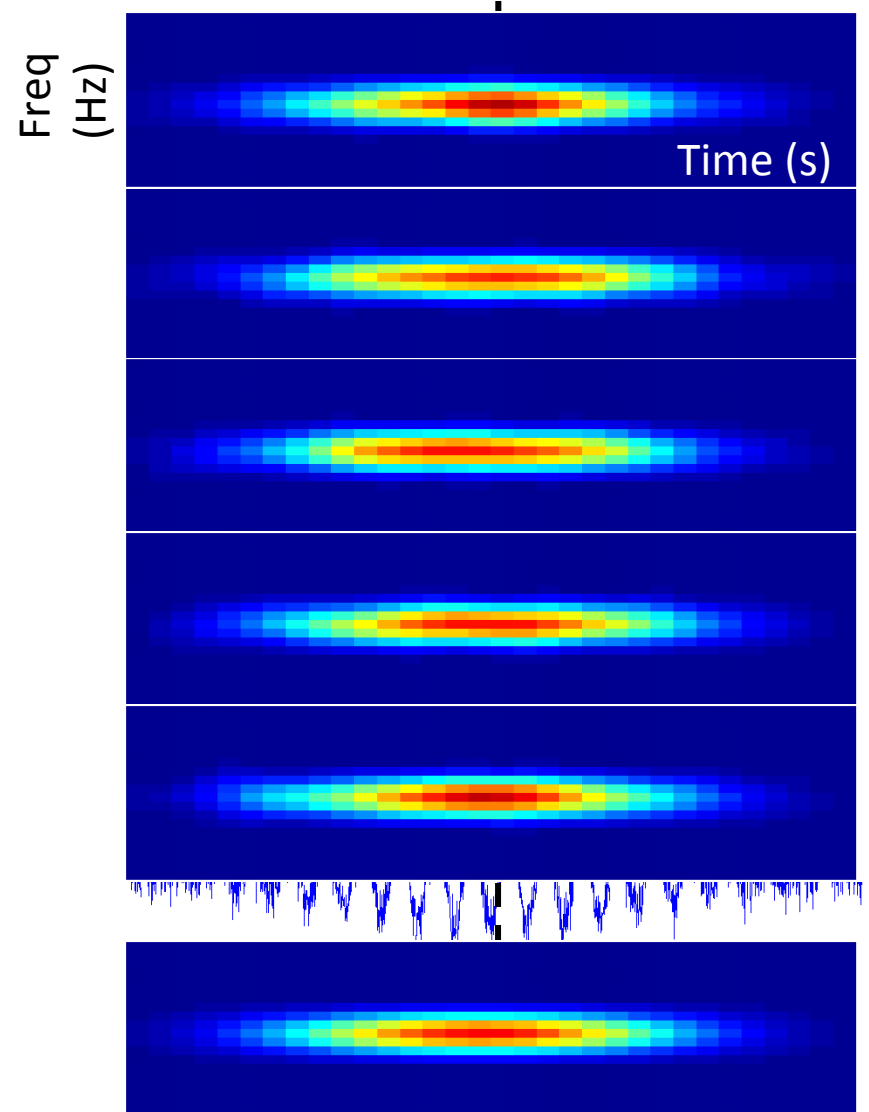
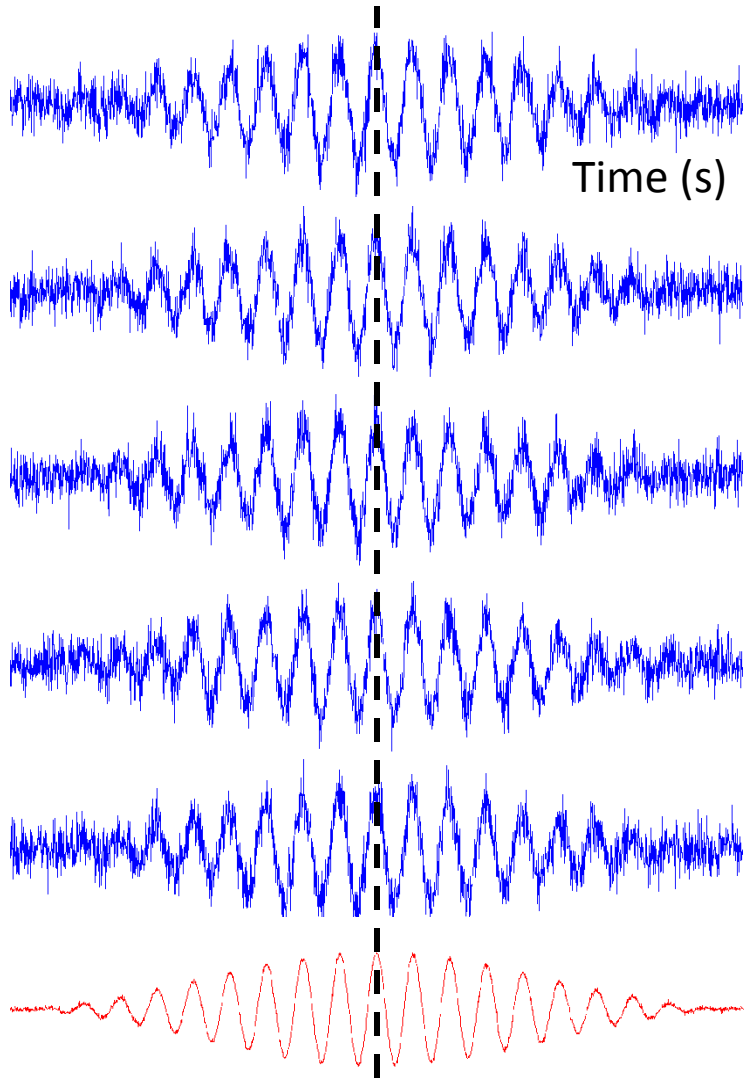
Time frequency analysis



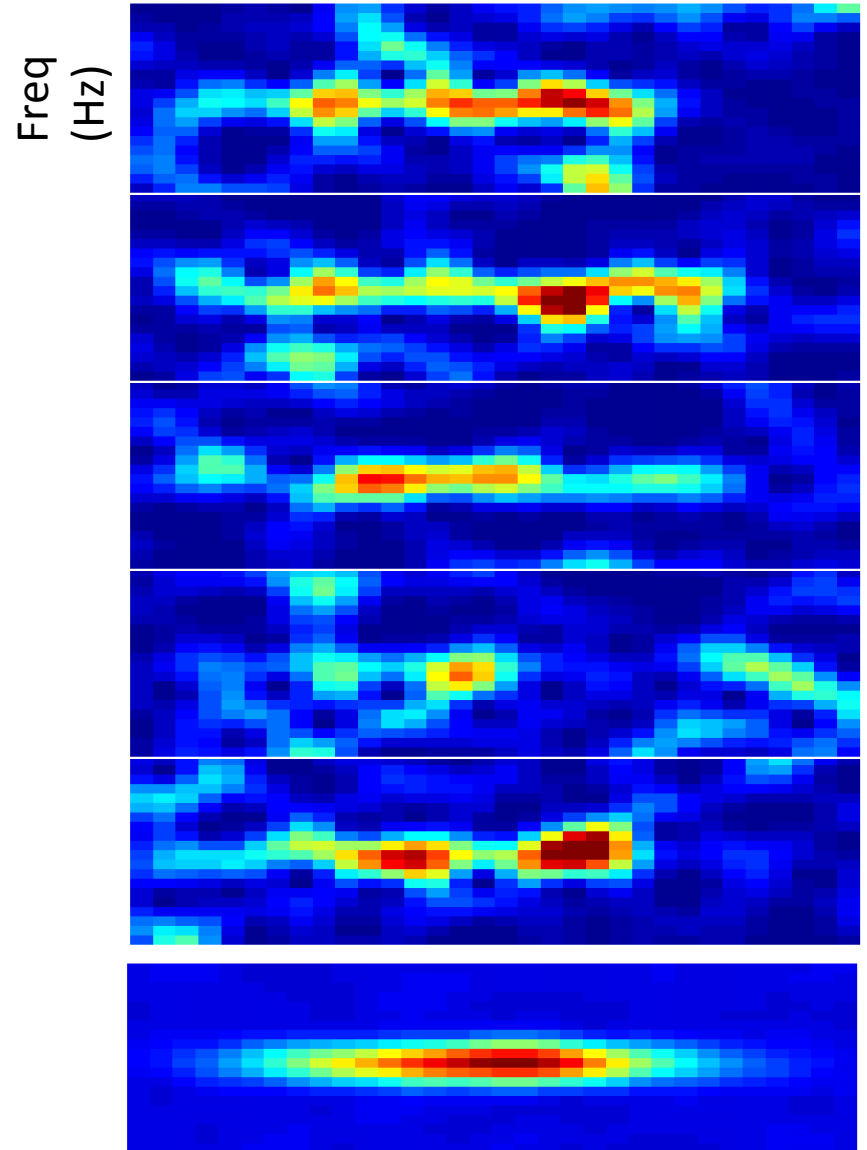
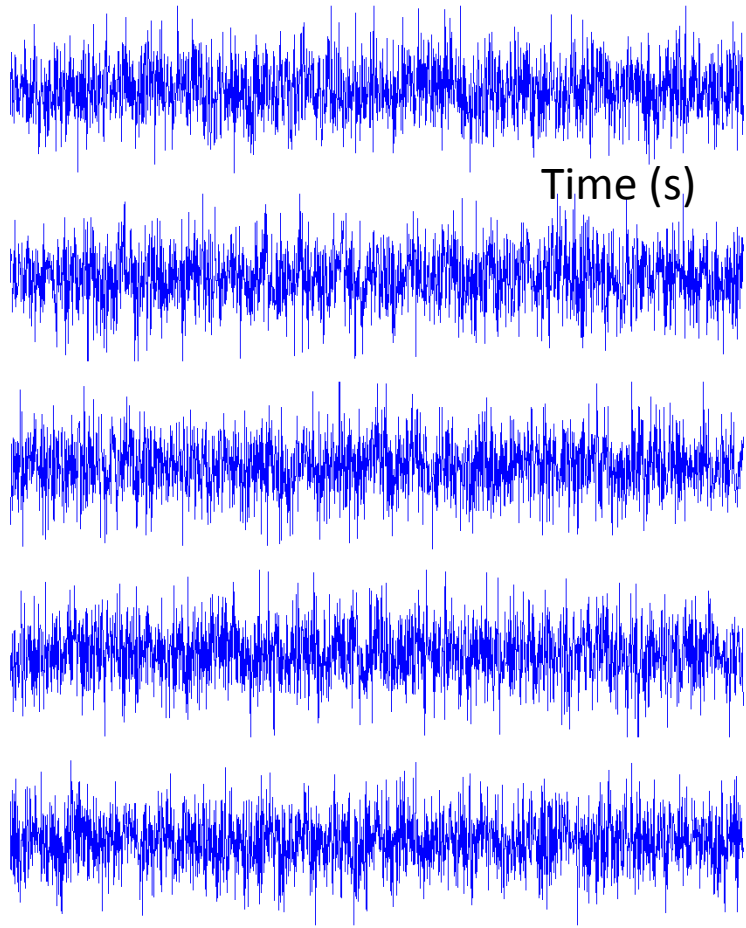
Time frequency analysis



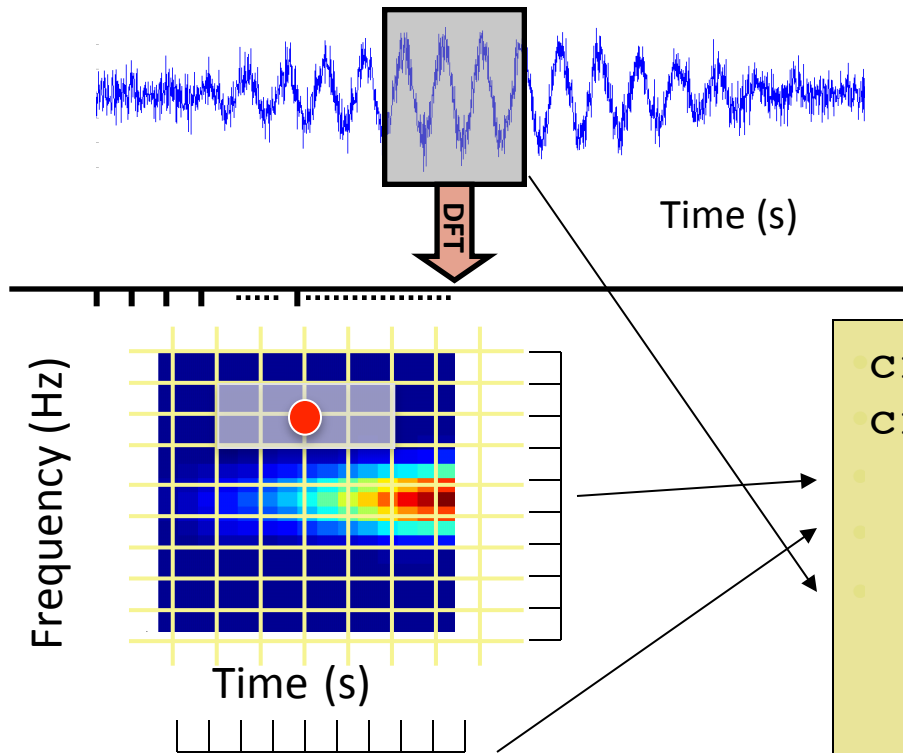
Evoked versus induced activity



Noisy signal -> many trials needed



The time-frequency plane



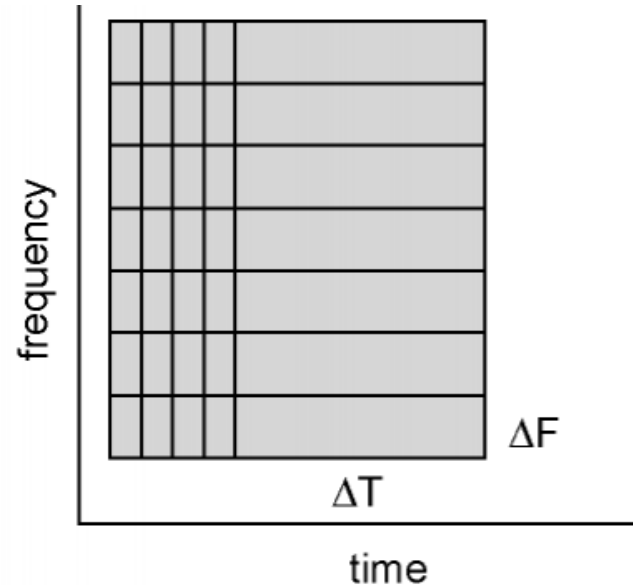
```
cfg = [];  
cfg.method = 'mtmconvol';  
.  
.  
.  
freq = ft_freqanalysis(cfg,data);
```

The time-frequency plane

The division is ‘up to you’

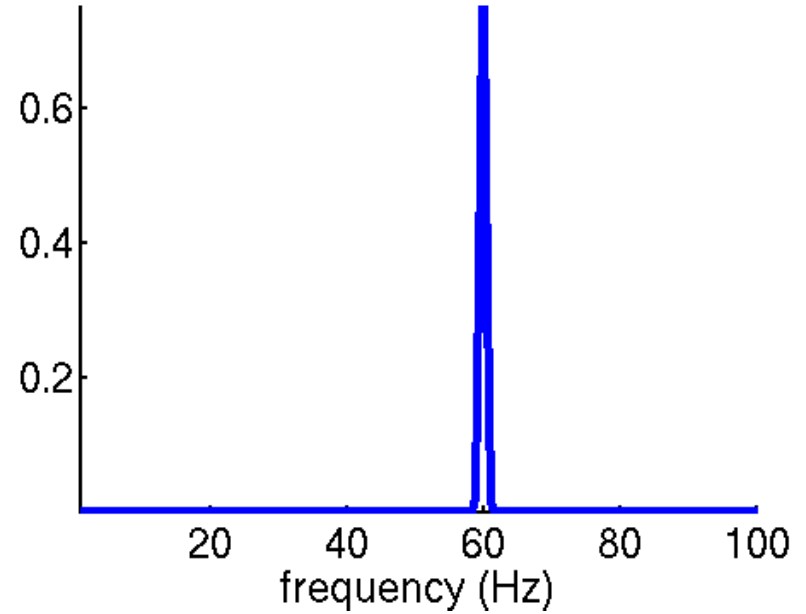
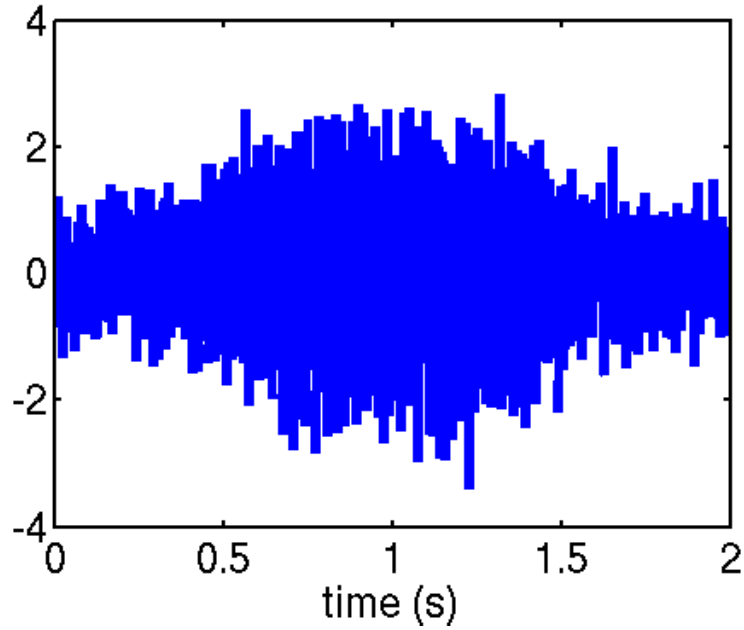
Depends on the phenomenon
you want to investigate:

- Which frequency band?
- Which time scale?



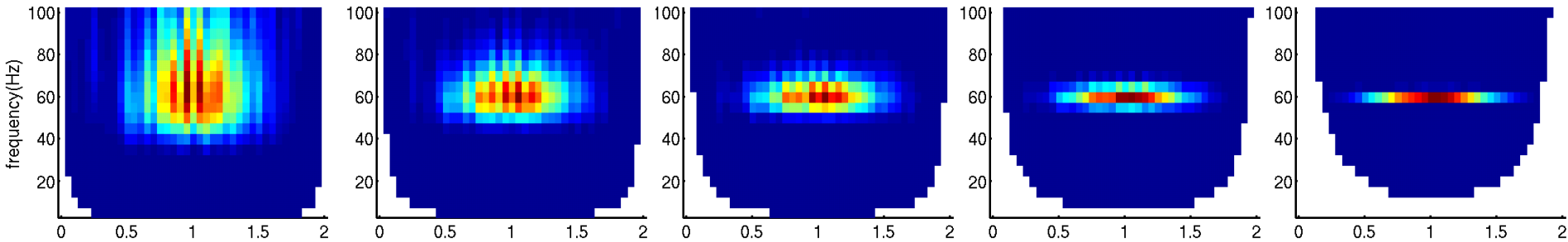
```
cfg = [];  
cfg.method      = 'mtmconvol';  
cfg.foi         = [2 4 ... 40];  
cfg.toi         = [0:0.050:1.0];  
cfg.t_ftimwin  = [0.5 0.5 ... 0.5];  
cfg.tapsmofrq  = [ 4  4 ... 4 ];  
.  
.  
freq = ft_freqanalysis(cfg,data);
```

Time versus frequency resolution



short timewindow

long timewindow



Interim summary

Time frequency analysis

Fourier analysis on shorter sliding time window

Evoked & Induced activity

Time frequency resolution trade off

Wavelet analysis

Popular method to calculate time-frequency representations

Is based on convolution of signal with a family of 'wavelets' which capture different frequency components in the signal

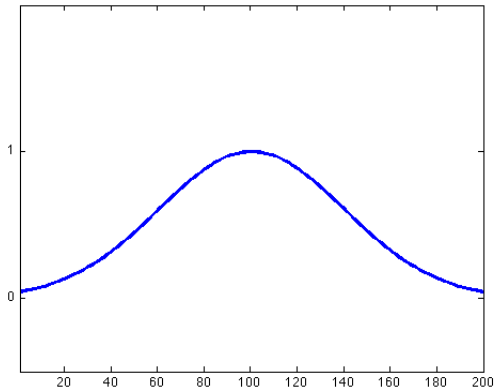
Convolution \sim local correlation

Wavelet analysis

```
cfg = [];  
cfg.method = 'wavelet';  
.  
.  
.  
freq=ft_freqanalysis(cfg, data);
```

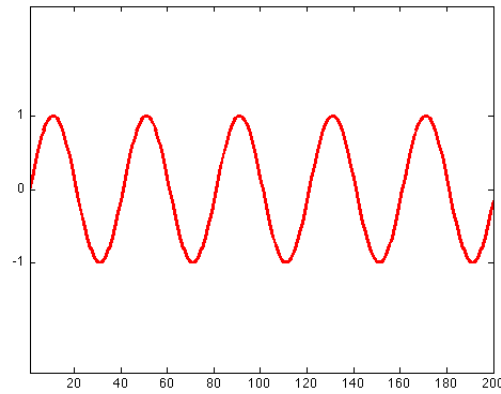
Wavelets

Taper

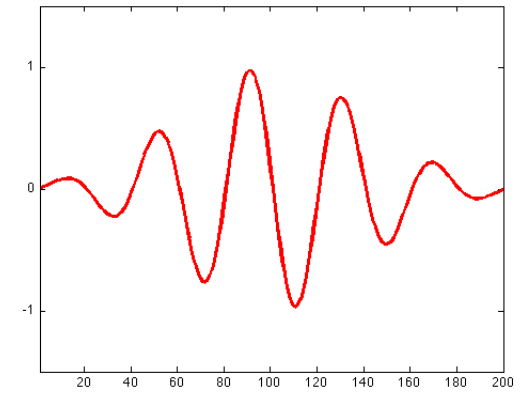


X

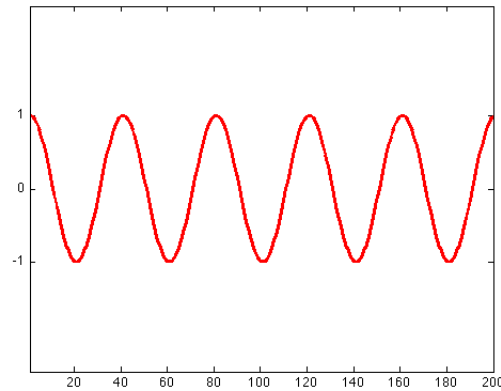
Sine wave



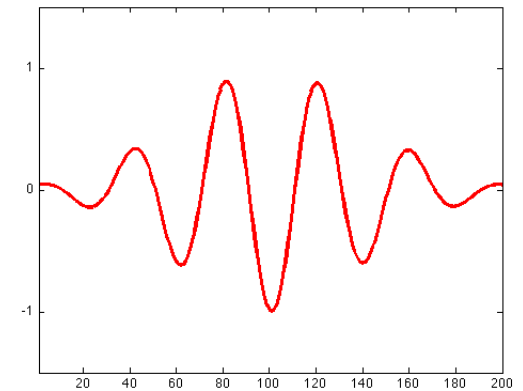
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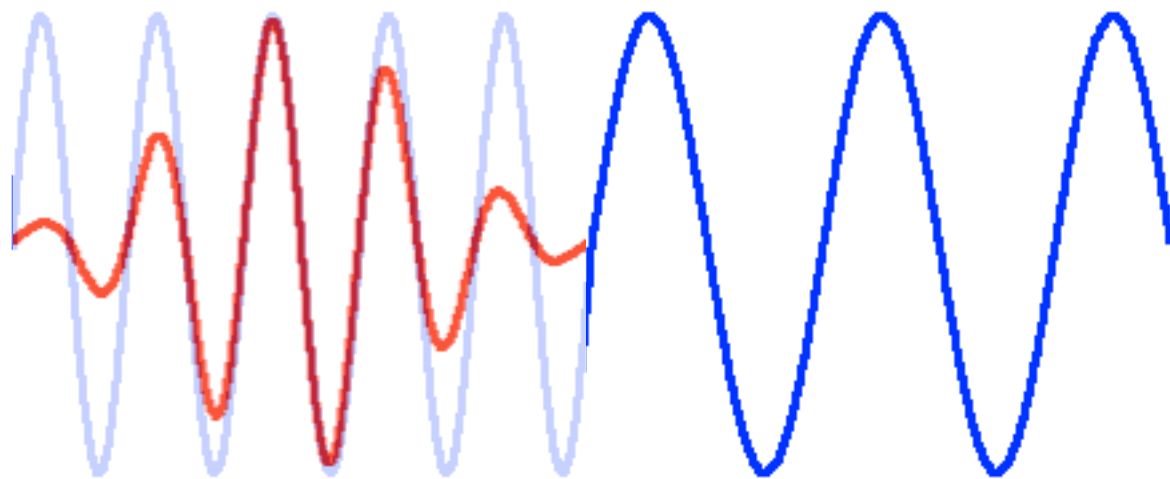


Cosine wave



=





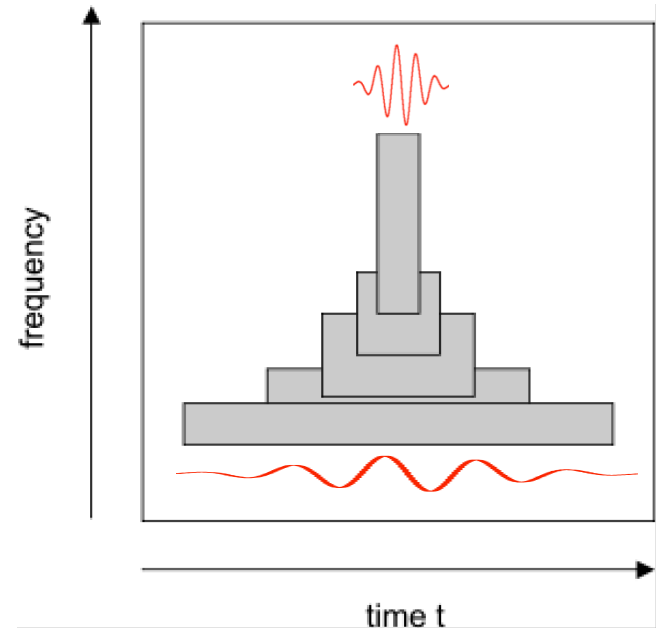
Wavelet analysis

Wavelet width determines the time-frequency resolution

Width is a function of frequency (often 5 cycles)

‘Long’ wavelet at low frequencies leads to relatively narrow frequency resolution but poor temporal resolution

‘Short’ wavelet at high frequencies leads to broad frequency resolution but more accurate temporal resolution



Wavelet analysis

Similar to Fourier analysis, but

Can be computationally slower

Tiles the time frequency plane in a particular way
with fewer degrees of freedom

```
%time frequency analysis with  
%multitapers  
  
cfg = [];  
cfg.method      = 'mtmconvol';  
cfg.toi         = [0:0.05:1];  
cfg.foi         = [ 4   8   ...  80];  
cfg.t_ftimwin   = [0.5 0.5 ... 0.5];  
cfg.tapsmofrq   = [ 2   2   ...  10];  
    .  
    .  
freq=ft_freqanalysis(cfg, data);
```

```
%time frequency analysis with  
%wavelets  
  
cfg = [];  
cfg.method      = 'wavelet';  
cfg.toi         = [0:0.05:1];  
cfg.foi         = [4 8 ... 80];  
cfg.width       = 5;  
    .  
    .  
    .  
freq=ft_freqanalysis(cfg, data);
```

Summary

Spectral analysis

Relation between time and frequency domains

Tapers

Time frequency analysis

Time vs frequency resolution

Wavelets

Hands-on

Go to

<http://www.fieldtriptoolbox.org/workshop/marseille>

and then

http://www.fieldtriptoolbox.org/tutorial/sensor_analysis

This will cover

ERFs following visual stimulus onset

oscillatory activity following visual stimulation

coherence between the cortex and the muscle